

# Dynamic Spectrum Access in Multi-Channel Cognitive Radio Networks

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**Abstract**—In this paper, dynamic spectrum access (DSA) in multi-channel cognitive radio networks (CRNs) is studied. The two fundamental issues in DSA, *spectrum sensing* and *spectrum sharing*, for a general scenario are revisited, where the channels present different usage characteristics and the detection performance of individual secondary users (SUs) varies. First, spectrum sensing is investigated, where multiple SUs are coordinated to cooperatively sense the channels owned by the primary users (PUs) for different interests. When the PUs' interests are concerned, cooperative spectrum sensing is performed to better protect the PUs while satisfying the SUs' requirement on the expected access time. For the SUs' interests, the objective is to maximize the expected available time while keeping the interference to PUs under a predefined level. With the dynamics in the channel usage characteristics and the detection capacities, the coordination problems for the above two cases are formulated as nonlinear integer programming problems accordingly, which are proved to be NP-complete. To find the solution efficiently, for the former case, the original problem is transformed into a variant of convex bipartite matching problem by constructing a complete bipartite graph and defining proper weight vectors. Based on the problem transformation, a channel selection algorithm is proposed to compute the solution. For the latter case, the deterministic optimization problem is first transformed to an associated stochastic optimization problem, which is then solved by cross-entropy (CE) method of stochastic optimization. Then, the sharing of the available channels by SUs after sensing is modeled by a channel access game, based on the framework of weighted congestion game. An algorithm for SUs to select access channels to achieve Nash equilibrium (NE) is proposed. Simulation results are presented to validate the performance of the proposed algorithms.

**Index Terms**—Dynamic spectrum access, multiple channels, cooperative spectrum sensing, spectrum sharing.

## I. INTRODUCTION

THE BOOMING wireless applications and services demands more spectrum bands, which consequently results in the spectrum scarcity. On the other hand, recent studies reveal that the allocated spectrum is largely underutilized [1]. With the development of cognitive radio (CR) technologies, dynamic spectrum access (DSA) has been envisaged to be a promising solution to improve the spectrum utilization, which allows unlicensed/secondary users (SUs) to utilize the

unused spectrum owned by licensed/primary users (PUs) in an opportunistic fashion [2]–[6]. The Notice of Proposed Rule Making of Federal Communications Commission (FCC) has indicated CR as the candidate to implement opportunistic spectrum sharing. Moreover, IEEE has proposed the first standard IEEE 802.22 to utilize CR for reuse the unused TV spectrum on a non-interfering basis [7].

To facilitate DSA in CRNs, spectrum sensing is of significance which has to be performed by SUs to detect idle spectrum bands before commencing transmission. SUs can access the spectrum bands for transmission only when no active PU is detected in the spectrum band of interest. However, the performance of spectrum sensing can be severely degraded due to the adverse effects of fading and shadowing, which consequently interferes with the PUs [8], [9]. To address these issues, cooperative spectrum sensing is proposed to improve the sensing performance and reduce the chance of interfering with PUs, where multiple SUs share the sensing results to make a combined decision. Based on spatial diversity and multiuser diversity, cooperative spectrum sensing can improve the detection performance in terms of increasing the detection probability and reducing the false-alarm probability [10]. In the literature, cooperative spectrum sensing for the single channel case has been extensively studied [11]–[13]. Since there usually exist multiple channels in the system, DSA in multi-channel CRNs has drawn increasing attentions recently, which is more challenging due to multiple channels and multiple SUs. For DSA in multi-channel CRNs, two fundamental issues have to be addressed well: i) how to coordinate SUs for multi-channel sensing; and ii) how to share the available channels, which correspond to the issues of *spectrum sensing* and *spectrum sharing*, respectively [14].

For *spectrum sensing* in multi-channel scenarios, from the single user's perspective, the quickest detection is studied with the objective of finding an idle period from multiple channels as fast as possible using the theory of partially observable Markov decision process (POMDP) in [15] and dynamic programming in [16], respectively. Besides that, from the system's perspective, the issue regarding how to assign SUs to different channels for maximizing the system performance are studied in [17]–[20]. In [17], heuristic channel selection algorithms are designed for cooperative spectrum sensing to maximize the number of available channels. In [19], the authors study this issue to maximize the throughput of SUs. However, a common assumption is made that all the SUs have the same sensing performance for all channels. In practice, the sensing

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performance of SUs depends on the channel conditions from the PUs to the SUs, which usually differs from user to user. Moreover, the channel usage characteristics of PUs are not taken into consideration in the existing literature.

For *spectrum sharing*, diverse approaches have been proposed in the literature. In [21], the auction game is utilized, where SUs, PUs, and spectrum bands, are modeled as auctioneers, bidders and bidding articles, respectively. In [22], SUs share the available channels by accessing the channel with equal probability. In [23], the spectrum access based on multi-channel ALOHA protocol is studied using theory of potential games, without considering available duration of channels. In [24], channel allocation is studied using stable marriage game, which aims to find the most stable pairings between the users and channels. Recently, congestion game has gained much attentions, which is a prominent approach to model the scenario where multiple rational users share a set of common resource. It has been utilized to solve the issue of spectrum sharing in [25]–[27], where congestion game is utilized for SUs to share the channels and each SU chooses one channel for accessing to maximize its own utility. However, all SUs are treated equally, ignoring their channel conditions.

In this paper, we study the two aforementioned fundamental issues for a multi-channel CRN, where cooperative sensing is performed among SUs. Due to hardware limitation, each SU can only choose one channel in spectrum sensing and access one channel at a time for spectrum sharing. Two cases are investigated for the channel selection problem in spectrum sensing. From the point of view of the PUs' interests, SUs act conservatively in spectrum sensing, with the objective of minimizing the interference to the PUs, while satisfying the SUs' requirement on the available time they expect to achieve through sensing. From the point of view of the SUs' interests, SUs behave aggressively in spectrum sensing, aiming at maximizing the expected available time of all the channels, under the constraint that the PUs are sufficiently protected. To achieve the objectives, SUs decide which channels to be sensed. Different from the existing works, a more general scenario is considered in this paper, where the main differences are: i) the detection performance of individual SU depends on the channel condition, which may differ from user to user; and ii) the channels are considered to present different usage characteristics, such as average sojourn idle time and the probability of being idle. Due to those factors, the channel selection problem becomes more challenging. For both cases, we formulate the channel selection problems as nonlinear integer programming problems which are proved to be NP-complete. Depending on the problem formulation, we apply different approaches to solve them accordingly. To efficiently solve the problem for the first case, we further investigate the problem and transform it into a convex bipartite matching problem by constructing a complete bipartite graph and defining proper weight vectors. Based on the problem transformation, a channel selection algorithm is proposed. For the second case, we first define an associated stochastic optimization problem of the original deterministic optimization problem. Then, we apply the cross-entropy (CE) method of stochastic optimization to find the channel selection solution efficiently. Finally, we study spectrum sharing and model it using a more

general game based on the framework of weighted congestion game. SUs with different channel conditions are assigned different weights, with the purpose of favoring SUs with better channel conditions. In the proposed game, each SU chooses a channel from the available channel set to maximize their own interests. An algorithm that can help SUs to achieve Nash Equilibrium (NE) is proposed. It is proved that the algorithm can achieve NE. Simulation results are provided to show the performance of the proposed algorithms.

The remainder of the paper is organized as follows. The detailed description of the system model is given in Section II. The problem formulations and the proposed approaches for spectrum sensing are presented in Section III, while spectrum sharing is studied in Section IV. Simulation results are provided in Section V, followed by concluding remarks in Section VI.

## II. SYSTEM MODEL

### A. Network Architecture

We consider a cognitive radio network which composes of two types of users: the primary users (PUs) and secondary users (SUs). The PUs own certain licensed spectrum bands where they can operate. The SUs do not own any spectrum and can only opportunistically access the unused spectrum for transmission. The amount of spectrum accessible to the SUs is further divided into a set of channels, each of which has a fixed amount of frequency bandwidth.

In the network, there exist  $K$  licensed bands (channels) which allow PUs to transmit simultaneously. Suppose that a PU operates in a channel, which can be either active or inactive. In the same area,  $N$  SUs ( $N \geq K$ ) seek for transmission opportunities. In order to avoid interference to the PUs, the SUs perform spectrum sensing before transmission to detect the unused channels.

### B. Channel Usage Characteristics

Similar to [28], an ON-OFF channel usage model is applied to model the status of each channel. The status of the channel alternates between ON (busy) and OFF (idle). The SU can access the channel only when it is in the state OFF. Suppose that  $PU_j$  operates over channel  $j$  and the state of each channel changes independently. Denote by  $\alpha_j$  the transition rate for channel  $j$  ( $1 \leq j \leq K$ ) from state ON to state OFF and  $\beta_j$  vice versa. Then, the two-state Markov chain in Fig. 1 can describe the status of a given channel. Note that the channel usage characteristics may not be the same for all the channels. In other words,  $\alpha_i$  and  $\beta_i$  for channel  $i$  are not necessarily the same as  $\alpha_j$  and  $\beta_j$  for channel  $j$ .

### C. Individual Spectrum Sensing

Spectrum sensing is carried out to detect the status of the channels. Let  $H_1$  denote the state that the PU is present in the channel of interest and  $H_0$  denote the state that the PU is absent. In the literature, popular detection techniques include energy detection, cyclostationary detection, and matched filtering. In this work, we adopt energy detection due to its simplicity and minimal time overhead (typically less than 1

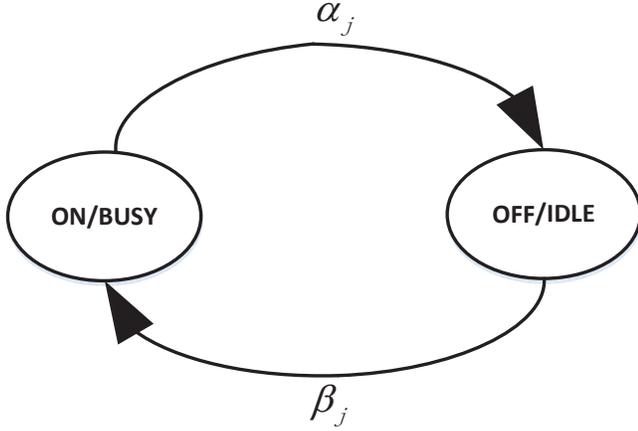


Figure 1. ON-OFF model for a given channel.

ms). When energy detector is adopted in spectrum sensing, the detection probability  $p_d$  and the false alarm probability  $p_f$  are defined as

$$p_d = Pr(D > \delta | H_1), \quad p_f = Pr(D > \delta | H_0) \quad (1)$$

where  $\delta$  is the detection threshold and  $D$  is the test statistic. Particularly,  $D = \frac{1}{M} \sum_{n=1}^M |y(n)|^2$ , where  $M$  is the number of samples in an observation period and  $y(n)$  is the  $n$ -th sample of the received signal.

Without loss of generality, similar to [3], we focus on the case of the complex-valued PSK signal and Circular Symmetric Complex Gaussian (CSCG) noise. According to [3], the false alarm probability of  $SU_i$  for channel  $j$  can be given by

$$p_f(i, j) = Q\left(\left(\frac{\delta}{\sigma^2} - 1\right)\sqrt{M}\right) \quad (2)$$

where  $Q(\cdot)$  is the complementary distribution function of the standard Gaussian. We consider the Neyman-Pearson criterion [29], where the false alarm probability is fixed. In other words, the false alarm probabilities for all SUs are the same and denoted by  $p_f$  for simplicity. Therefore, all SUs have the same value of  $\delta$ .

The detection probability of  $SU_i$  for channel  $j$  is calculated as follows:

$$p_d(i, j) = Q\left(\left(\frac{\delta}{\sigma^2} - \bar{\gamma}_{i,j} - 1\right)\sqrt{\frac{M}{2\bar{\gamma}_{i,j} + 1}}\right) \quad (3)$$

where  $\bar{\gamma}_{i,j}$  is the average received signal-to-noise ratio (SNR) from  $PU_j$  at  $SU_i$ . Particularly,  $\bar{\gamma}_{i,j} = \frac{P_{PU} h_{i,j}}{\sigma^2}$ , where  $P_{PU}$  is the transmission power of the PU,  $h_{i,j}$  is the average channel gain from  $PU_j$  to  $SU_i$ , and  $\sigma^2$  is the variance of the Gaussian noise.

Given  $p_f(i, j)$ , based on (2) and (3), the detection probability  $p_d(i, j)$  can be calculated as follows:

$$p_d(i, j) = Q\left(\frac{1}{\sqrt{2\bar{\gamma}_{i,j} + 1}}(Q^{-1}(p_f(i, j)) - \sqrt{M}\bar{\gamma}_{i,j})\right). \quad (4)$$

#### D. Cooperative Spectrum Sensing

In cooperative spectrum sensing, SUs cooperate with each other to improve the sensing performance. Specifically, SUs

share the sensing results to output a combined decision on whether the PU is present or absent using a decision fusion rule. The decision rules include AND rule, OR rule, the soft combination rule, or the majority rule. In order to minimize the communication overhead and transmission delay, SUs only share their final 1-bit decisions (e.g., bit 0 and 1 represent the idle and busy states, respectively) rather than their decision statistics. When OR rule is adopted, PUs are considered to be present if at least one SU claims the presence of PUs. Suppose that each SU selects a channel for sensing at one time and let  $\mathbf{S}_j$  be the set of SUs selecting channel  $j$ . Then, the cooperative detection probability and the cooperative false alarm probability can be given as follows:

$$F_d^j = 1 - \prod_{i \in \mathbf{S}_j} (1 - p_d(i, j)) = 1 - \prod_{i \in \mathbf{S}_j} p_m(i, j) \quad (5)$$

$$F_f^j = 1 - \prod_{i \in \mathbf{S}_j} (1 - p_f(i, j)) = 1 - \prod_{i \in \mathbf{S}_j} p_s(i, j) \quad (6)$$

where  $p_m(i, j) = Pr(D < \delta | H_1) = 1 - p_d(i, j)$  and  $p_s(i, j) = Pr(D < \delta | H_0) = 1 - p_f(i, j)$ . The cooperative misdetection probability  $F_m^j$  is defined as the probability that the presence of the PU is not detected, i.e.,  $F_m^j = 1 - F_d^j$ .

If AND rule is adopted, PUs are considered to be present if all the SUs report the result of presence. The cooperative detection probability and the cooperative false alarm probability are respectively given by

$$F_d^j = \prod_{i \in \mathbf{S}_j} p_d(i, j), \quad F_f^j = \prod_{i \in \mathbf{S}_j} p_f(i, j). \quad (7)$$

Note that in spectrum sensing, adopting AND rule is more aggressive for SUs, while adopting OR rule is more conservative. Adopting AND rule leads to a smaller false alarm probability, which means SUs are more aggressive to explore the spectrum access opportunities, while adopting OR rule results in a greater detection probability, which means SUs are more conservative to explore the spectrum access opportunities [30].

### III. SPECTRUM SENSING IN MULTI-CHANNEL CRNs

In this section, spectrum sensing is studied for the following two cases. For the first case, SUs act conservatively in spectrum sensing, and aim at minimizing the interference to the PUs, while satisfying the SUs' requirement on the expected available access time. For the second case, SUs behave aggressively in spectrum sensing, and try to maximize the expected available time of all the channels, under the constraint that the PUs are sufficiently protected. For different objectives, the channel selection problems are formulated, and the approaches are proposed accordingly.

#### A. From the point of view of the PUs' interests

When the PUs' interests are concerned, SUs act more conservatively in spectrum sensing and OR rule is adopted. The objective is to minimize the interference to the PUs, while satisfying the SUs' requirement on the available time they expect to achieve through sensing. In the following, the channel selection problem is formulated first, followed by the proposed approach.

1) *Problem Formulation:* Denote the sojourn times of ON state and OFF state for channel  $j$  by  $T_{ON}^j$  and  $T_{OFF}^j$ , respectively, which are assumed following exponential distributions with means given by

$$\bar{T}_{ON}^j = \frac{1}{\alpha_j}, \quad \bar{T}_{OFF}^j = \frac{1}{\beta_j}. \quad (8)$$

The probabilities that channel  $j$  is in ON state and OFF state are denoted by  $P_{ON}^j$  and  $P_{OFF}^j$ , respectively.  $P_{ON}^j$  and  $P_{OFF}^j$  can be calculated as

$$P_{ON}^j = \frac{\beta_j}{\alpha_j + \beta_j}, \quad P_{OFF}^j = \frac{\alpha_j}{\alpha_j + \beta_j}. \quad (9)$$

If channel  $j$  is sensed to be in OFF state when it is actually idle, the SUs have an average period of  $\bar{T}_{OFF}^j$  to access. If channel  $j$  is sensed to be in OFF state but it is busy in fact, the SUs will access the channel, interfering with the PUs.

Consider that PUs are sensitive to the interference caused by SUs due to the misdetections and there exists a threshold value  $\bar{P}_{rm}$ . To measure the experience feeling of PUs, satisfaction is adopted as a subjective metric, which is similar to the conception of quality of experience (QoE) if we regard the spectrum sensing as a service to PUs. Specifically, when the misdetection probability  $F_m(j)$  is greater than  $\bar{P}_{rm}$ , the satisfaction of PUs will decrease dramatically. When  $F_m(j)$  is lower than  $\bar{P}_{rm}$ , the satisfaction of PUs will increase slowly. The misdetection probability for a given channel depends on the number of sensing SUs and the channel conditions associated with these SUs. To estimate the satisfaction of the PU on channel  $j$ , the satisfaction is defined as  $s = \log \frac{\bar{P}_{rm}}{F_m(j)}$ . The satisfaction is positive when  $F_m(j) < \bar{P}_{rm}$ , while it is negative when  $F_m(j) > \bar{P}_{rm}$ . Moreover, the satisfaction of PUs increases as  $F_m(j)$  decreases. However, the marginal rate of the satisfactory improvement is diminishing, following the law of the diminishing marginal benefit in economics [31]. Suppose that SUs have a requirement on the expected available duration  $T_r$  for the channel which they select to sense. For example, if SUs select channel  $j$  to perform spectrum sensing, it should be satisfied that  $T_{OFF}^j P_{OFF}^j (1 - F_f(j)) \geq T_r$ . Further, we define a channel selection matrix  $\mathbf{I} = (I_{i,j})_{N \times K}$ , where  $I_{i,j} = \{0, 1\}$  indicates whether or not  $SU_i$  selects channel  $j$  for sensing. When  $I_{i,j} = 1$ ,  $SU_i$  selects channel  $j$  for sensing, and vice versa. Based on  $\mathbf{I}$ , the set of SUs choosing channel  $j$  can be determined by  $\mathbf{S}_j = \{SU_i, I_{i,j} = 1\}$ .

The objective of the channel selection is to maximize the PUs' satisfaction while meeting SUs' requirements of the average available period of the channels they sense. The channel selection problem can be formulated as follows:

$$(\mathcal{P}1): \quad \max_{\mathbf{I}} \sum_{j=1}^K \log \frac{\bar{P}_{rm}}{F_m(j)} \\ \text{s.t.} \quad \sum_{j=1}^K I_{i,j} \leq 1, i \in \{1, 2, \dots, N\} \quad (10) \\ \bar{T}_{OFF}^j P_{OFF}^j (1 - F_f(j)) \geq T_r \\ I_{i,j} = \{0, 1\}.$$

The problem  $\mathcal{P}1$  is a nonlinear integer programming problem and is NP-complete. The proof can be found in Appendix

VII-A. To solve the problem  $\mathcal{P}1$ , we will transform it into a convex bipartite matching problem in the next section.

2) *Convex Bipartite Matching Approach:* In this section, a brief review of nonlinear bipartite matching is first presented. Then, the problem  $\mathcal{P}1$  is transformed into a convex bipartite matching problem, which belongs to a class of nonlinear bipartite matching problem [32]. Finally, an efficient algorithm is proposed to find the optimal channel selection.

**Nonlinear Bipartite Matching:** Denote  $\Theta_{n,n}$  to be a complete bipartite graph, with the edges  $E := \{(i, j) : 1 \leq i \leq n, 1 \leq j \leq n\}$ . Given  $d$  integer weight vectors  $w^1, w^2, \dots, w^d$  on the edges  $E$ , the objective of the nonlinear bipartite matching is to maximize (or minimize)  $f(w^1(\mathbf{M}), w^2(\mathbf{M}), \dots, w^d(\mathbf{M}))$  by finding a perfect matching  $\mathbf{M}$ , where  $f$  is an arbitrary mapping function from  $\mathbb{R}^d$  to  $\mathbb{R}$ , and  $w^n(\mathbf{M}) := \sum \{w^n(i, j) : (i, j) \in \mathbf{M}\}$ . It can also be formulated as

$$(\mathcal{P}2): \quad \max_{\mathbf{x} \in \mathbb{N}^{n \times n}} \quad \text{or} \quad \min_{\mathbf{x} \in \mathbb{N}^{n \times n}} \quad f(w^1(\mathbf{x}), w^2(\mathbf{x}), \dots, w^d(\mathbf{x})) \\ \text{s.t.} \quad \sum_{i=1}^n x_{i,j} = 1 \\ \sum_{j=1}^n x_{i,j} = 1 \\ x_{i,j} = \{0, 1\} \quad (11)$$

where  $\mathbb{N}$  refers to the nonnegative integers. If  $f(\cdot)$  is convex, the problem  $\mathcal{P}2$  becomes a convex bipartite matching problem. The conventional bipartite matching is to find the best matching such that the sum of edges can be maximized or minimized. In contrast, the objective function of nonlinear bipartite matching can be an arbitrary function. Therefore, nonlinear bipartite matching can be applied to more broader and general scenarios.

**Variant of Convex Bipartite Matching:** To transform the problem  $\mathcal{P}1$  into a convex bipartite matching problem, a complete bipartite graph is constructed as follows. Given  $K$  channels and  $N$  SUs, let  $n := NK$ ,  $A := \{(j, i) : j = 1, \dots, K, i = 1, \dots, N\}$ , and  $B := \{1, \dots, N\} \uplus B_v$ , where  $B_v$  is a set of  $(K-1)N$  dummy vertices. As shown in Fig. 2, the constructed bipartite graph  $E_{n \times n}$  has the vertex  $A \uplus B$ .

Define  $K$  weight vectors  $w^1, w^2, \dots, w^K$  on the edges of  $E_{n \times n}$  by

$$w^j(a, b) = \begin{cases} \log \frac{1}{1 - p_d(i, j)}, & \text{if } a = \{j, i\} \text{ and } b = i \\ 0, & \text{otherwise.} \end{cases} \quad (12)$$

The weight vector  $w^j$  can be given as follows:

$$\begin{pmatrix} 0 & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ \log \frac{1}{1 - p_d(1, j)} & 0 & 0 & \cdots \\ 0 & \log \frac{1}{1 - p_d(2, j)} & 0 & \cdots \\ 0 & 0 & \log \frac{1}{1 - p_d(3, j)} & \cdots \\ \vdots & \ddots & \ddots & \cdots \end{pmatrix} \quad (13)$$

Then, the objective function in the problem  $\mathcal{P}1$  can be

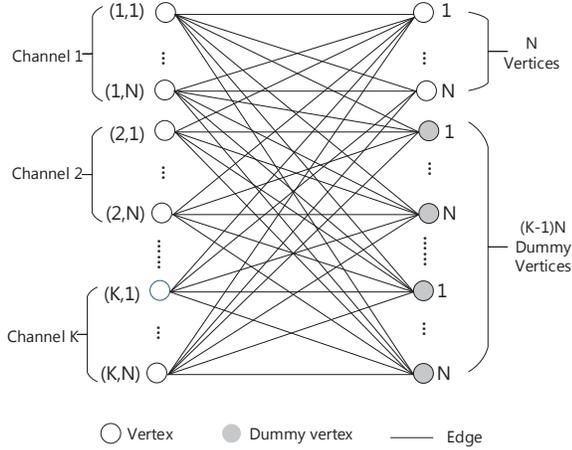


Figure 2. The complete bipartite graph.

rewritten as

$$\begin{aligned}
 \max_{\mathbf{I}} \sum_{j=1}^K \log \frac{\bar{P}_{rm}}{F_m(j)} &= \sum_{j=1}^K [\log \bar{P}_{rm} - \log F_m(j)] \\
 &= \sum_{j=1}^K [\log \bar{P}_{rm} - \sum_{i \in \mathcal{S}_j} \log(1 - p_d(i, j))] \\
 &= \sum_{j=1}^K \log \bar{P}_{rm} + \sum_{j=1}^K \sum_{i \in \mathcal{S}_j} \log \frac{1}{1 - p_d(i, j)} \\
 &= \sum_{j=1}^K \log \bar{P}_{rm} + \sum_{j=1}^K w^j \mathbf{x}.
 \end{aligned} \tag{14}$$

Note that given  $w^j$  in (13) and a potential matching  $\mathbf{x}$ ,  $\sum_{j=1}^K \sum_{i \in \mathcal{S}_j} \log \frac{1}{1 - p_d(i, j)} = \sum_{j=1}^K w^j \mathbf{x}$ .

Based on the constructed bipartite graph  $E_{n \times n}$ , the original problem  $\mathcal{P}1$  can be transformed into the problem  $\mathcal{P}3$  as follows:

$$\begin{aligned}
 (\mathcal{P}3) : \quad & \max_{\mathbf{x} \in \mathbb{N}^{n \times n}} \sum_{j=1}^K w^j \mathbf{x} \\
 \text{s.t.} \quad & \sum_{i=1}^n x_{i,j} = 1, \quad \sum_{j=1}^n x_{i,j} = 1 \\
 & \bar{T}_{OFF}^j P_{OFF}^j (1 - F_f(j)) \geq T_r \\
 & x_{i,j} \in \{0, 1\},
 \end{aligned} \tag{15}$$

which is linear and thus convex. Due to the additional constraint regarding the false alarm probability. The problem  $\mathcal{P}3$  is a variant of the convex bipartite matching problem [32].

**Channel Selection Algorithm:** Based on the approach in [32], a channel selection algorithm is proposed to solve the problem  $\mathcal{P}3$ . The basic idea of the proposed solution is to first find the smallest grid containing vert, order the grid points, find the vertex of the fiber, and then check whether the obtained fiber satisfies the constraint to find the permutation

matrix<sup>1</sup>.

First, we define the matrix  $\Pi^n$  as follows:

$$\Pi^n = \{ \tilde{\mathbf{x}}_{i,j} \in \mathbb{R}_+^{n \times n} : \sum_i \tilde{x}_{i,j} = 1, \sum_j \tilde{x}_{i,j} = 1 \} \tag{16}$$

where  $\mathbb{R}_+$  corresponds to the nonnegative reals.

Let  $\Pi_w^n$  be the projection of  $\Pi^n$  under  $\mathbf{w}$ , which is given by

$$\Pi_w^n := \{ \mathbf{w}\tilde{\mathbf{x}} : (w^1 \tilde{\mathbf{x}}, w^2 \tilde{\mathbf{x}}, \dots, w^K \tilde{\mathbf{x}}) : \mathbf{w} \in \mathbb{R}^{n \times n} \} \subset \mathbb{R}^n \tag{17}$$

where the projection  $\mathbf{w} : \mathbb{R}^{n \times n} \rightarrow \mathbb{R}^n$  is defined to map matrix  $\tilde{\mathbf{x}}$  to vectors  $\mathbf{w}\tilde{\mathbf{x}}$  as follows:

$$\mathbf{w}\tilde{\mathbf{x}} := \left( \sum_{i,j} w_{i,j}^1 \tilde{x}_{i,j}, \sum_{i,j} w_{i,j}^2 \tilde{x}_{i,j}, \dots, \sum_{i,j} w_{i,j}^K \tilde{x}_{i,j} \right). \tag{18}$$

We further define the fiber of any  $\mathbf{y} = (y_1, \dots, y_K), \mathbf{y} \in \mathbb{Z}^K$  as a polytope which can be represented as follows:

$$\begin{aligned}
 \Pi^n \cap \mathbf{w}^{-1}(\mathbf{y}) := \{ \tilde{\mathbf{x}} \in \mathbb{R}_+^{n \times n} : & \sum_i \tilde{x}_{i,j} = 1, \\
 & \sum_j \tilde{x}_{i,j} = 1, w^m \tilde{\mathbf{x}} = y_m \}.
 \end{aligned}$$

The channel selection algorithm consists of the following four main steps.

- 1) Solve the following two linear programs to find  $b_j$  and  $d_j$  for  $j = 1, 2, \dots, K$ :

$$\begin{aligned}
 b_j &:= \min \{ w^j x : \sum_i \tilde{x}_{i,j} = \sum_j \tilde{x}_{i,j} = 1, x \geq 0 \} \\
 d_j &:= \max \{ w^j x : \sum_i \tilde{x}_{i,j} = \sum_j \tilde{x}_{i,j} = 1, x \geq 0 \}.
 \end{aligned} \tag{19}$$

Then, we define a grid  $G := \{ y \in \mathbb{Z}^K : b_j \leq y_j \leq d_j, j = 1, 2, \dots, K \}$ , where  $y := \{ y_1, y_2, \dots, y_K \}$ ;

- 2) Calculate the values of  $f$  for all the possible  $y$ . Arrange all the possible  $y$  in a nonincreasing order according to the values of  $f$ , i.e.,  $y^1, y^2, \dots, y^{|G|}$  where  $f(y^1) \geq f(y^2) \geq \dots \geq f(y^{|G|})$ ;
- 3) Check the fibers of each  $y^i$  in order to find the first  $y^k$  such that vertex  $\mathbf{x}$  of fiber  $\Pi^n \cap \mathbf{w}^{-1}(y^k)$  is a permutation matrix and satisfies  $\bar{T}_{OFF}^j P_{OFF}^j \prod_{i=1}^N w_t^{j,i} \cdot \mathbf{x} \geq T_r$ , where  $w_t^{j,i}$  is given by

$$w_t^{j,i}(a, b) = \begin{cases} 1 - p_f(i, j) & \text{if } a = \{j, i\} \text{ and } b = i \\ 1 & \text{if } a = \{j, i\} \text{ and } b = k \neq i \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

- 4) Return the perfect matching corresponding to the permutation matrix  $\mathbf{x}$  as the optimal channel selection.

### B. From the point of view of the SUs' interests

From the point of view of the SUs' interests, SUs act more aggressively in spectrum sensing and hence AND rule is adopted. The objective is to maximize the expected available

<sup>1</sup>Note that the weight vectors should only contain integers for a convex bipartite matching problem. To transform the original problem into a convex bipartite matching problem, we approximate (or scale) the components in  $w^j$  as integers.

time of all the channels, under the constraint that the PUs are sufficiently protected. In the following, the channel selection problem is formulated first. Then, the problem is solved based on a cross-entropy (CE) approach.

1) *Problem Formulation*: When channel  $j$  is detected to be in OFF state and it is actually idle, on average, the SUs have  $\bar{T}_{OFF}^j$  for access. The objective for SUs is to maximize the total average available time, which can be formulated as follows:

$$\begin{aligned}
 (\mathcal{P}4) : \quad & \max_{\mathbf{I}} \sum_{j=1}^{j=K} \bar{T}_{OFF}^j P_{OFF}^j (1 - F_f(j)) \\
 & s.t. \quad \sum_{j=1}^{j=K} I_{i,j} \leq 1, i \in \{1, 2, \dots, N\} \quad (21) \\
 & (1 - F_d(j)) P_{ON}^j \leq P_i \\
 & I_{i,j} = \{0, 1\}
 \end{aligned}$$

By using exterior point method which permits the variables to violate the inequality constraint during the iterations, the constraint that  $(1 - F_d(j)) P_{ON}^j \leq P_i$  can be removed. Then, the above problem can be transformed into the format as shown in (22) on the top of the following page, where  $U_0 > 0$  is a linear penalty factor when the constraint  $(1 - F_d(j)) P_{ON}^j \leq P_i$  is violated.  $A(F_d(j))$  is the indicator function, where  $A(F_d(j)) = 1$  when  $(1 - F_d(j)) P_{ON}^j \geq P_i$ , and  $A(F_d(j)) = 0$ , otherwise.

Since the above problem is non-convex integer programming, which cannot be solved by the previous convex bipartite matching approach. Therefore, we apply the C-E method of stochastic optimization, which can provide an efficient solution for solving combinatorial optimization problem. In the following, we first give a brief review of the C-E method and then propose the solution to solve the above optimization problem.

2) *Cross-Entropy Based Approach*:

**Cross-Entropy**: The Cross-Entropy (C-E) method was first introduced to estimate the probabilities of rare events in complex stochastic networks [33]. It was realized that a simple cross-entropy modification of C-E method could also be used to solve difficult combinatorial optimization problems. In C-E method, deterministic optimization problem should be translated into a related stochastic optimization problem, where the rare event simulation techniques similar to [33] can be utilized. In other words, the main idea behind the C-E method is to define for the original optimization problem an associated stochastic problem (ASP) and then efficiently solve the ASP based on an adaptive scheme. It sequentially generates random solutions which converge stochastically to the optimal or near-optimal one.

**C-E algorithm**: The basic idea of C-E algorithm is to generate a random data sample according to a specified stochastic policy, and update the stochastic policy based on the outcome of the sample to produce a "better" sample in the next iteration. Algorithm 1 presents the detailed procedure of channel selection, which consists of five main steps as follows.

Define the strategy space  $\mathbb{S}$  for SUs as follows:

$$\mathbb{S} := \{ch_1, ch_2, \dots, ch_K\}, \quad (23)$$

where each SU can only choose one channel from  $\mathbb{S}$ . Define the probability vector associated with the strategy space as follows:

$$\mathbb{P}_t^i := \{p_{1,t}^i, p_{2,t}^i, \dots, p_{K,t}^i\}, \quad \sum_{j=1}^K p_{j,t}^i = 1, \quad (24)$$

where  $\mathbb{P}_t^i$  denotes the stochastic policy of  $SU_i$  on the strategy space  $\mathbb{S}$  at  $t$ -th iteration, and  $p_{j,t}^i$  denotes the probability that  $SU_i$  chooses channel  $j$  at  $t$ -th iteration.

- 1) (Initialization). Set the iteration counter  $t := 1$ . Set the initial stochastic policy  $\mathbb{P}_0^i$  of all SUs to be the uniform distribution on the strategy space  $\mathbb{S}$ . In other words, for each SU, it picks the strategy from the strategy space uniformly, with equal probability  $1/K$ .
- 2) (Generation samples). Based on the stochastic policy of all SUs,  $Z$  samples of the strategy vector are generated, which can be given as follows:

$$\mathbb{S}^i(z) := \{I_{i,1}(z), I_{i,2}(z), \dots, I_{i,K}(z)\}, \quad (25)$$

where  $\mathbb{S}^i(z)$  is the  $z$ -th strategy vector of  $SU_i$  with only one element to be "1" and the rest are "0". The probability for the  $I_{i,j}$  to be "1" is  $p_{j,t}^i$ .

- 3) (Performance evaluation). Substitute the samples into the objective function of (22) to calculate the utilities  $U(z)$ . Arrange the  $U(z)$  in a nonincreasing order according to the values, i.e.,  $U^1 \geq U^2 \geq \dots \geq U^Z$ . Let  $v$  be the  $(1-\rho)$ -th sample, i.e.,  $v = U^{\lceil (1-\rho)Z \rceil}$ , where  $\rho$  is the percentage of samples are obsolete at each iteration and  $\lceil \cdot \rceil$  is the ceiling function.
- 4) (Stochastic policy update). Based on the same sample, calculate  $\mathbb{P}_t^i := \{p_{1,t}^i, p_{2,t}^i, \dots, p_{K,t}^i\}$ , using the following equation:

$$p_{j,t}^i = \frac{\sum_{z=1}^N X_{U^z \geq v} I_{i,j}(z) = 1}{\sum_{z=1}^N X_{U^z \geq v}}, \quad (26)$$

where  $X_{U^z \geq v}$  is defined as follows:

$$X_{U^z \geq v} = \begin{cases} 1 & U^z \geq v \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

- 5) If the stopping criterion is met, which is the maximum number of iterations (i.e.,  $T$ ), then stop; otherwise increase the iteration counter  $t$  by 1, and reiterate from step 2.

#### IV. SPECTRUM SHARING IN MULTI-CHANNEL CRNS

After spectrum sensing, available channels can be detected. Subsequently, SUs start the process of spectrum sharing. In this section, based on weighted congestion game, a channel access game is utilized to model the behavior of SUs during spectrum sharing. A brief review of congestion game is given first, followed by the proposed channel access game. Finally, a channel access algorithm is proposed for SUs to achieve NE in spectrum sharing.

##### A. Congestion Game

Congestion game is a prominent approach to model the scenario where multiple rational users share a set of common

$$\begin{aligned}
& \max_{\mathbf{I}} \sum_{j=1}^{j=K} [\bar{T}_{OFF}^j P_{OFF}^j (1 - F_f(j)) - A(F_d(j)) U_0 (1 - F_d(j)) P_{ON}^j] \\
& \text{s.t.} \sum_{j=1}^{j=K} I_{i,j} \leq 1, i \in \{1, 2, \dots, N\} \\
& I_{i,j} = \{0, 1\}
\end{aligned} \tag{22}$$

---

**Algorithm 1** Channel Selection Algorithm

---

```

1: // Initialization
2:  $p_{j,t}^i = 1/K$ .
3: for  $t = 1 : T$  do
4:   for  $z = 1 : Z$  do
5:     for  $n = 1 : N$  do
6:       Generate samples of the strategy vector.
7:     end for
8:   end for
9:   for  $z = 1 : Z$  do
10:    Calculate the utilities  $U(z)$ .
11:   end for
12:   Order the utilities  $U(z)$  in a nonincreasing manner.
13:   for  $j = 1 : N$  do
14:     for  $k = 1 : K$  do
15:       Update  $\mathbb{P}_t^i$  using (26)
16:     end for
17:   end for
18: end for
19: return

```

---

resource. In congestion game, each individual player strives to maximize its own utility by selecting a set of resources. The share of each resource is a non-increasing function with respect to the number of players choosing it. The formal definition of congestion game is given as follows.

The standard congestion game is defined by the tuple  $\{\mathcal{N}, \mathcal{R}, (\sum i)_{i \in \mathcal{N}}, (U_j^r)_{j \in \mathcal{R}}\}$ , where  $\mathcal{N} = \{1, 2, \dots, N\}$  denotes the set of players,  $\mathcal{R} = \{1, 2, \dots, R\}$  denotes the set of resources,  $(\sum i)$  represents the strategy space of player  $i$ , and  $U_j^r$  is the payoff associated with resource  $j$ , which is a function of the total number of players sharing it.  $U_j^r$  is a decreasing function due to competition or congestion, e.g.,  $U_j^r = 1/n_j$ , where  $n_j$  is the total number of players choosing resource  $j$ . Denote by  $S = (s_1, s_2, \dots, s_N)$  the strategy profile of the game, where  $s_i \in \sum i_{i \in \mathcal{R}}$  and  $s_i$  corresponds to the strategy of the player  $i$ . Denote by  $n = \{n_1, n_2, \dots, n_R\}$  the congestion vector, where  $n_j$  represents the total number of players sharing resource  $j$ . The utility of player  $i$  is given as follows:

$$U_i = \sum_{j \in s_i} U_j^r(n_j(S)). \tag{28}$$

A more general version of congestion game is the weighted congestion game, where each player is assigned a weight. Denote by  $w = (w_1, w_2, \dots, w_N)$  the weight vector of the players, where  $w_i$  is the weight of player  $i$ . Different from the standard congestion game, the payoff associated with resource

$j$  is a function of the total weights of players sharing resource  $i$ . It has been proved in [34] that every standard congestion game admits an NE. However, the weighted congestion games do not necessarily possess an NE.

### B. Channel Access Game

We model the channel access procedure of SUs based on weighted congestion game, where SUs with good channel conditions are favored by being assigned a higher weight. The channel access game  $\Gamma$  is defined by  $\{\mathcal{N}, \mathcal{K}, (w_i)_{i \in \mathcal{N}}, (\sum i)_{i \in \mathcal{N}}, (U_j^i)_{i \in \mathcal{N}, j \in \mathcal{K}}\}$ , where  $\mathcal{N} = \{1, 2, \dots, N\}$  denotes the set of SUs,  $\mathcal{K} = \{1, 2, \dots, K\}$  denotes the set of channels,  $w_i$  denotes the weight associated with  $SU_i$ ,  $\sum i$  represents the strategy space of  $SU_i$ , and  $U_j^i$  is the utility function of  $SU_i$  for selecting channel  $j$ .  $U_j^i$  is a function of the sum of weights of SUs choosing the same channel, which is a decreasing function. Each SU aims to maximize its utility by deciding which channel to be accessed and the utility function of  $SU_i$  can be given by

$$U_i^j = \frac{w_i \Psi_j}{\sum_{j \in s_i} w_i} = w_i \zeta_j(W_j) \tag{29}$$

where  $\Psi_j$  is the average sojourn time of state OFF of channel  $j$ ,  $W_j$  is the sum of weights of SUs choosing channel  $j$ , and  $\zeta_j(W_j) = \frac{\Psi_j}{\sum_{j \in s_i} w_i}$  is the payoff function of resource  $j$ , which depends on the sum of weights of channel  $j$ . Therefore,  $U_j^i$  represents the access time that  $SU_i$  can obtain. Note that when  $w_i = 1$  for all SUs, the channel access game  $\Gamma$  becomes a standard congestion game, i.e., all the SUs are equally treated to share the common resource and select the access channel to maximize their own interests. Thus, a higher fairness can be achieved. On the other hand, the overall throughput of the secondary network needs to be considered when sharing the available channels. In order to favor the users with good channel conditions, greater weights can be assigned to them such that they have higher priority in the resource sharing procedure. In other words, the SUs with greater weights can have longer average time for transmissions, which consequently increases the overall throughput of the secondary network. To this end, the channel is considered to be in a good or bad state, when compared with a predefined threshold. The weights  $w'$  and  $w$  ( $w' > w$ ) are assigned to the SUs with good channel and bad channel conditions, respectively.

In this game, each SU chooses a single channel to access for maximizing its utility. The solution of this game is Nash Equilibrium (NE). If each one has chosen a strategy and no SU can increase its utility by changing strategy while the strategies of others keep unchanged, then the current set of strategies

constitutes an NE.

*Definition 3:* A strategy profile  $S^* = (s_1^*, s_2^*, \dots, s_M^*)$  is an NE if and only if

$$U_j(s_i^*, s_{-i}^*) \geq U_j(s_i', s_{-i}^*), \forall i \in \mathcal{N}, s_i' \in S_i, \quad (30)$$

where  $s_i$  and  $s_{-i}$  are the strategies selected by  $SU_i$  and all of its opponents, respectively. NE means no one can increase its utility unilaterally.

The potential function approach is a well-known method to prove the existence of NE in the congestion games. We can define a potential function with respect to the strategies of players, in which every strictly improving move by a player will improve the value of this function. If there exists a potential function for a game, then it is guaranteed that the game exists an NE. In the following, we will prove that the channel access game  $\Gamma$  is a weighted potential game, and there exists an NE.

*Definition 1:* A game  $\Upsilon$  is an ordinal potential game if there exists an ordinal potential function  $P$  which satisfies the following condition:

$$U^i(s_{-i}, s_i') - U^i(s_{-i}, s_i) > 0 \text{ iff } P(s_{-i}, s_i') - P(s_{-i}, s_i) > 0.$$

*Definition 2:* A game  $\Upsilon$  associated with a weight vector  $w = (w_1, w_2, \dots, w_N)$  is a weighted potential game if there exists a weighted potential function  $P$  satisfying the following condition:

$$U^i(s_{-i}, s_i') - U^i(s_{-i}, s_i) = w_i [P(s_{-i}, s_i') - P(s_{-i}, s_i)]. \quad (31)$$

To prove the existence of NE in the channel access game  $\Gamma$ , we use Rosenthal's potential function  $\mathfrak{R}(S)$  [25], which is defined as follows:

$$\begin{aligned} \mathfrak{R}(S) &= \sum_{j \in \mathcal{K}} \sum_{i=w}^{W_j} \zeta_j(i) \\ &= \sum_{i=1}^N \sum_{j \in s_i} \zeta_j(W_j^i), \end{aligned} \quad (32)$$

where  $W_j^i$  is the sum of weights of SUs selecting channel  $j$  whose indices do not exceed  $i$ .

Suppose  $SU_i$  unilaterally deviates from strategy  $s_i$  to  $s_i'$ . The change in the potential  $\mathfrak{R}(S)$  can be obtained as follows:

$$\begin{aligned} \Delta \mathfrak{R}(s_i \rightarrow s_i') &= \zeta_{j \in s_i'}(W_j + w_i) - \zeta_{k \in s_i}(W_k) \\ &= \zeta_{j \in s_i'}(W_j(s_{-i}, s_i') - \zeta_{k \in s_i}(W_k(s_{-i}, s_i)) \\ &= \frac{1}{w_i} [U^i(s_{-i}, s_i') - U^i(s_{-i}, s_i)], \end{aligned} \quad (33)$$

where  $U^i$  is the utility function of  $SU_i$ . Therefore, the channel access game  $\Gamma$  is a weighted potential game.

In [35], for every finite ordinal potential game, there exists an Nash Equilibrium (NE). Since weighted potential game is a subset of ordinal potential games, there exists an NE in the weighted potential game. Therefore, an NE exists in the channel access game  $\Gamma$ . It is well known that an NE can be achieved when each SU strives to optimize their own utilities after a finite number of steps [25]. Therefore, we propose a channel access algorithm, the Algorithm 2. The main idea of the proposed algorithm is that each SU aims at improving

Table I  
PARAMETERS USED IN THE SIMULATIONS

Parameters	Value
Number of channels	[3, 4, 5, 6, 7]
Number of samples	6000
Transmission power of PUs	10mw
Variance of noise	-80dB
Path loss exponent $\mu$	3.5
$P_{rm}$	0.1
$p_f$	0.1
$T_r$	[1, 2]
$T_{OFF}$	[4, 4, 5, 5]
$\alpha$	[0.6, 0.8, 1, 1.2]
$U_0$	2
Simulation times	200

its own utility and then they end up optimizing the global objective, i.e., the potential function. By doing so, the NE can be obtained. The proof that the algorithm can achieve NE is given in Appendix VII-B.

### Algorithm 2

```

1: // Channel Access Algorithm
2: Set congestion vector  $W(S) = (W_1, \dots, W_K) = (0, 0, \dots, 0)$ .
3: for each  $SU_i \subseteq \mathcal{N}$  do
4:   if Channel  $i$  for  $SU_i$  pair is in good state then
5:      $w_i \leftarrow w'$  and put  $SU_i$  into the set  $U_{G1}$ 
6:   else
7:      $w_i \leftarrow w$  and put  $SU_i$  into the set  $U_{G2}$ 
8:   end if
9: end for
10: Order the rewarding periods on each channel  $[\Psi_1, \Psi_2, \dots, \Psi_K]$  decreasingly according to the length.
11: Order the SUs in the user set  $U_{G1}$  and  $U_{G2}$  decreasingly according to the channel gain
12: for each  $SU_i \subseteq U_{G1}$  do
13:    $SU_i$  calculates  $w_i \zeta_j(W_j + w')$ 
14:    $SU_i$  selects the channel with maximum  $w_i \zeta_j(W_j + w')$ 
15:    $W_j \leftarrow W_j + w'$ 
16: end for
17: for each  $\Psi_j$ , where  $j \subseteq U_{G2}$  do
18:    $SU_i$  calculates  $w_i \zeta_j(W_j + w)$ 
19:    $SU_i$  selects the channel with maximum  $w_i \zeta_j(W_j + w)$ 
20:    $W_j \leftarrow W_j + w$ 
21: end for
22: return

```

## V. SIMULATION RESULTS

In this section, the simulation results are provided to validate the performance of the proposed algorithms. The simulation is set up as follows. In a 2 km $\times$ 2 km area, there is a set of PUs located inside the circle with 1 km radius, while a group of SUs seeking for transmission opportunities is randomly distributed outside the circle. The transmission power of PUs is set to 10 mw, while the variance of noise is set to -80 dB. The channel gain between a given SU and a PU is calculated by  $h = \frac{k}{d^\mu}$ , where  $k = 1$  and  $\mu = 3.5$ . The value of  $p_f$  is set to 0.1 for all SUs. For simplicity, let  $w' = 2w$  and

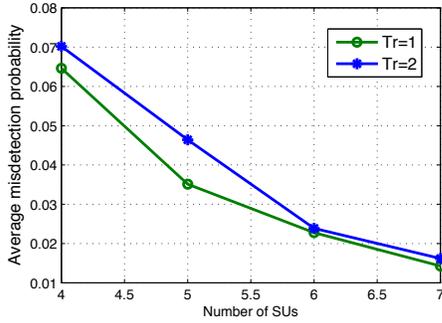


Figure 3. Average misdetection probability versus the number of SUs.

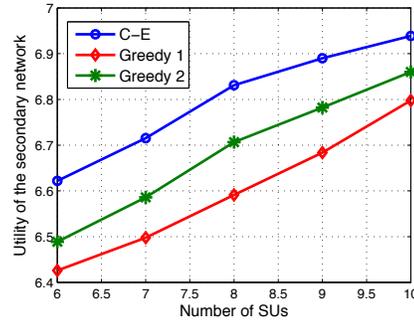


Figure 5. Utility of SUs versus the number of SUs.

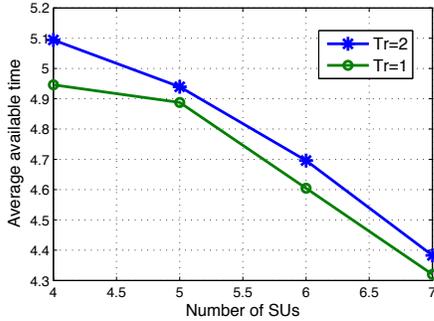


Figure 4. Average available time through sensing versus the number of SUs.

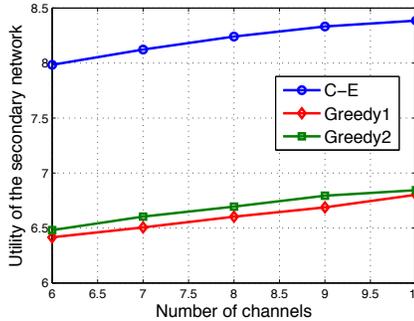


Figure 6. Utility of SUs versus the number of channels.

$w = 1$ . Detailed simulation parameters are shown in Table II. We obtain the average results using Monte Carlo simulation.

To evaluate the performance of the convex bipartite matching approach, the number of channels is set to 4, i.e., 4 PUs are considered in the area of interest, while  $\bar{P}_{rm}$  is set to 0.1. According to our case study, a good balance between the accuracy and complexity of channel selection can be achieved by first rounding the components in  $w^j$  to 2 decimal places and then multiply the results by 100. Fig. 3 shows the average misdetection probability of PUs with respect to the number of SUs. It can be seen that the average misdetection probability decreases as the number of SUs increases. This is mainly because of the OR rule in cooperative spectrum sensing, where PUs are considered to be absent only when all SUs report the result of absence. It can also be seen a lower required available time leads to a lower misdetection probability.

Fig. 4 shows the average available time of SUs with respect to the number of SUs. It can be seen that the average available time decreases as the number of SUs increases. This is mainly because the OR rule is adopted in cooperative spectrum sensing. As the number of SUs increases, the false alarm probability increases, which consequently reduces the chance to detect the available channels. Moreover, it can also be seen that a higher required available time also leads to a longer available time for the SUs.

Fig. 5 shows the utility of the secondary network with respect to the number of SUs for different approaches when the number of channels is 4. We compare the proposed C-E algorithm with the greedy algorithm in [19]. Greedy 1 algorithm does not consider the dynamics of channels and detection probabilities of SUs, while Greedy 2 algorithm does. It can be seen that Greedy 2 algorithm can achieve higher

utility than Greedy 1 algorithm. In C-E algorithm,  $\rho$  and  $Z$  are set to 0.2 and 100, respectively. It can also be seen that C-E algorithm can achieve higher utility than Greedy algorithms.

Fig. 6 shows the utility of the secondary network with respect to the number of channels for different approaches when the number of SUs is 10. It can be seen that as the number of channels increases, the utility of the secondary network increases. It can also be seen that Greedy 2 algorithm performs slightly better than Greedy 1 algorithm, while the proposed C-E algorithm can achieve the highest utility among these algorithms.

Fig. 7 shows the throughput of the secondary network with respect to the number of SUs by using the weighted congestion game and standard congestion game when the number of channels is set to 5. The throughput is calculated using the Shannon capacity formula. For each SU, the channel condition is randomly generated, which takes value from [15dB, 35dB] using uniform distribution. The threshold is set to 25dB. If the channel gain is greater than the threshold, it is treated as good channel and the SU will be assigned a larger weight in weighted congestion game. From the figure, it can be seen that the weighted congestion game can achieve higher throughput compared with the standard congestion game. This is because the SUs with good channel conditions are favored, which can obtain a relatively larger share of the available channel.

Fig. 8 shows the average throughput per user for the proposed sensing and access strategy and random channel access strategy, respectively, when the number of channels is set to 5. With the random channel access strategy, SUs randomly choose a channel for sensing and access the channel when it is detected to be idle. From the figure, it can be seen that the proposed sensing and access strategy can achieve higher

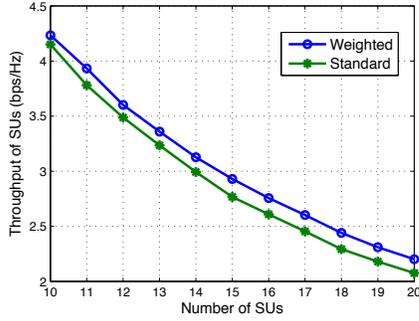


Figure 7. Throughput of SUs versus the number of SUs.

throughput per user. It implies that SUs have the incentive to participate in the proposed sensing and access strategy since they can achieve higher utility.

## VI. CONCLUSION

In this paper, we have investigated dynamic spectrum access in multi-channel CRNs. Depending on their different interests, spectrum sensing has been investigated, considering both the diverse channel usage characteristics and the diverse sensing performance of individual SUs. To minimize the interference to PUs while satisfying the required access time, an efficient channel selection algorithm has been proposed. To maximize the expected available time of all the channels, a cross-entropy based approach has been proposed. For spectrum sharing, a channel access game has been formulated based on weighted congestion game. An channel access algorithm has been proposed to achieve NE. Simulation results have been presented to validate the proposed algorithms.

For the future work, we will consider a scenario where SUs can adjust their own detection thresholds. In addition, the scenario that SUs can sense multiple channels simultaneously will also be considered.

## VII. APPENDIX

### A. Problem $\mathcal{P}1$ is NP-Complete

The objective function can be rewritten as follows:

$$\begin{aligned}
 & \max_{\mathbf{I}} \sum_{j=1}^K \log \frac{\bar{P}_{rm}}{F_m(j)} \\
 \Rightarrow & \max_K \bar{P}_{rm} - \sum_{j=1}^K \log F_m(j) \\
 \Rightarrow & \min \sum_{j=1}^K \log [\prod p_m(i, j)] \\
 \Rightarrow & \min \sum_{j=1}^K \sum_{i=1}^N I_{i,j} \cdot \log p_m(i, j) \\
 \Rightarrow & \min \sum_{j=1}^K \sum_{i=1}^N I_{i,j} \cdot f(i, j)
 \end{aligned} \tag{34}$$

The constraint  $\bar{T}_{OFF}^j P_{OFF}^j (1 - F_f(j)) \geq T_r$  can be

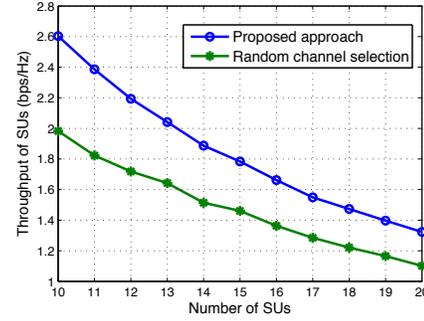


Figure 8. The proposed framework versus random channel access.

rewritten as follows:

$$\begin{aligned}
 1 - F_f(j) & \geq \frac{T_r}{\bar{T}_{OFF}^j P_{OFF}^j} \\
 \Rightarrow F_f(j) & \leq 1 - \frac{T_r}{\bar{T}_{OFF}^j P_{OFF}^j} \\
 \Rightarrow 1 - \prod P_s(i, j) & \leq 1 - \frac{T_r}{\bar{T}_{OFF}^j P_{OFF}^j} \\
 \Rightarrow \prod P_s(i, j) & \geq \frac{T_r}{\bar{T}_{OFF}^j P_{OFF}^j} \\
 \Rightarrow \sum_{i=1}^N I_{i,j} \cdot \log P_s(i, j) & \geq \log \frac{T_r}{\bar{T}_{OFF}^j P_{OFF}^j} \\
 \Rightarrow \sum_{i=1}^N I_{i,j} \cdot g_{i,j} & \geq C
 \end{aligned} \tag{35}$$

Then the optimization problem has the following format:

$$\begin{aligned}
 & \min \sum_{j=1}^K \sum_{i=1}^N I_{i,j} \cdot f(i, j) \\
 & \text{s.t.} \sum_{j=1}^K I_{i,j} \leq 1, i \in \{1, 2, \dots, N\} \\
 & \sum_{i=1}^N I_{i,j} \cdot g_{i,j} \geq C \\
 & I_{i,j} = \{0, 1\}.
 \end{aligned} \tag{36}$$

When the variable  $K$  is reduced to 1, we have

$$\begin{aligned}
 & \min \sum_{i=1}^N I_i \cdot f(i) \\
 & \text{s.t.} \sum_{i=1}^N I_i \cdot g_i \geq C \\
 & I_i = \{0, 1\}
 \end{aligned} \tag{37}$$

The above problem is a classic Knapsack problem, which is considered to be NP-Complete. Therefore, the original problem is NP-Complete.

### B. Proof of Nash Equilibrium

For an NE, it should satisfy the following requirement:

$$w_i \zeta_{j \in s_i} (W_j) \geq w_i \zeta_k (W_k + w_i), \forall k \in \mathcal{K}, j \neq k, i = 1, \dots, N. \tag{38}$$

To constitute an NE, for any two arbitrary users  $i$  and  $k$ , according to (38), we have

$$w_i \zeta_{j \in s_i}(W_j) \geq w_i \zeta_{j \in s_k}(W_j + w_i) \text{ and} \\ w_k \zeta_{j \in s_k}(W_j) \geq w_k \zeta_{j \in s_i}(W_j + w_k)$$

Suppose that  $SU_1$  chooses channel  $k$  with the maximum  $w' \zeta_k(W_k + w')$ , since  $w' \zeta_k(W_k + w') > w' \zeta_m(W_m + w')$ ,  $m \neq k, m \in K$ . For  $SU_2$ , it chooses channel  $l$  with the maximum  $w' \zeta_l(W_l + w')$ , since  $w' \zeta_l(W_l + w') > w' \zeta_s(W_s + w')$ ,  $s \neq l, s \in K$ . Since  $w' \zeta_k(W_k + w') > w' \zeta_l(W_l + w')$ , we have  $w' \zeta_k(W_k + w') > w' \zeta_l(W_l + w' + w')$ . Also, we have  $w' \zeta_l(W_l + w') > w' \zeta_k(W_k + w')$ . Thus, none of them are willing to change their strategies, and hence their strategies constitute an NE.

For the subsequent users in  $UG_1$ , they choose their best strategies and then all the strategy files constitute an NE for the new users and existing users. For a new user  $SU_n$ , it chooses channel  $q$  with the maximum  $w' \zeta_q(W_q + w')$ , since  $w' \zeta_q(W_q + w') > w' \zeta_m(W_m + w')$ ,  $m \neq q, m \in K$ . Before  $SU_n$  joining, all the former users's strategies constitute an NE. i.e., for  $SU_p$  choosing channel  $j$ ,  $w' \zeta_j(W_j) > w' \zeta_m(W_m + w')$ ,  $m \neq j, m \in K$ . Then, we have  $w' \zeta_j(W_j) > w' \zeta_q(W_q + w')$   $> w' \zeta_q(W_q + w' + w')$ . It also holds that  $w' \zeta_q(W_q + w') > w' \zeta_m(W_m + w')$ ,  $m \neq q, m \in K$ . Thus, the strategies of all the users constitute an NE.

For the user set  $UG_2$ , each user is assigned a weight of  $w$ . For a new user  $SU_j$ , it chooses channel  $x$  with the maximum  $w \zeta_x(W_x + w)$ , since  $w \zeta_x(W_x + w) > w \zeta_m(W_m + w)$ ,  $m \neq x, m \in K$ . Before  $SU_j$  joining, all the former users are in NE. Taking an arbitrary user  $SU_i$  as an example, if  $SU_i$  has chosen channel  $j$  rather than channel  $x$ ,  $w_i \zeta_j(W_j) > w_i \zeta_m(W_m + w_i)$ ,  $m \neq j, m \in K$ . Then, we have  $w_i \zeta_j(W_j) > w_i \zeta_x(W_x + w_i) > w_i \zeta_x(W_x + w_i + w_i)$ . Therefore, for those SUs choosing channel  $j$  rather than channel  $x$ , they should stay in their current channel and do not change their strategies. If  $SU_i$  has chosen channel  $x$ , since  $w_i \zeta(W_x + w) > w_i \zeta(W_m + w)$ ,  $m \neq x, m \in K$ ,  $w_i \zeta(W_x + w) > w_i \zeta(W_m + w) > w_i \zeta(W_m + w + w)$ ,  $m \neq x, m \in K$ . Then, we have  $\Psi_x \zeta(W_x + w) > \Psi_j \zeta(W_j + w + w')$ . Those users do not have the motivation to change their strategies. Therefore, for all users, their strategies constitute an NE.

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