RESEARCH ARTICLE

Performance analysis of spectrally efficient amplify-and-forward opportunistic relaying scheme for adaptive cooperative wireless systems

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ABSTRACT

In this paper, we propose an adaptive amplify-and-forward (AF) relaying scheme that selects the best relay among the available relay nodes opportunistically to cooperate with a source node for improvement of the spectral efficiency. This improvement can be achieved by introducing a policy that gives the useful cooperative regions and defines a switching threshold signal-to-noise ratio that guarantees the bit error rate (BER) of cooperative transmission is below the target. We model all links as independent non-identically distributed Rayleigh fading channels. We then derive closed-form expressions for the average spectral efficiency, average BER, and outage probability when an upper bound for the signal-to-noise ratio of the end-to-end relay path is applied and adaptive discrete rate is considered. Numerical and simulation results show that the proposed scheme, compared with the outage-based AF incremental relaying, AF fixed relaying, and the conventional direct transmission, can achieve the maximum average spectral efficiency while maintaining the average BER and outage probability. Copyright © 2013 John Wiley & Sons, Ltd.

KEYWORDS

cooperative diversity; relay selection; adaptive modulation; spectral efficiency

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1. INTRODUCTION

Cooperative diversity (CD) is well-known as an alternative solution of achieving spatial diversity in wireless communication systems when there is a limitation in implementing multiple antennas at the transmitter and/or receiver [1–4]. It basically enables intermediate node(s), either part of the infrastructure of the network or existing users, to forward the received information signal from a source node to the corresponding destination node. Cooperative transmission has recently become a key technology for modern wireless networks such as 3GPP long-term evolution, WiMax, vehicular networks, and so on [5–8], because in such networks, the transmission rate, communication reliability, and coverage problems could be solved in an efficient and cost-effective manner.

Cooperative diversity can be classified into three main approaches as follows: fixed relaying, selection relaying, and incremental relaying. In fixed relaying, the relay node always forwards the received information signal all the time that represents the simplest cooperative diversity scheme [9]. In selection relaying, the relay node forwards the received information signal when the channel quality exceeds a predefined limit, otherwise, the relay node keeps silent [10]. In incremental relaying, the relay node is activated if a request is made from the destination node [11]. The two well-known cooperative protocols are the amplify-and-forward (AF) and decode-and-forward (DF), where the main difference between them is the processing type at the relay node.

In multiple-relay cooperative system, the relay nodes usually transmit over orthogonal channels to avoid interference. This operation mode increases the operation complexity and may reduce the spectral efficiency. One approach to reduce this complexity is to use relay selection methods where only one relay node is selected from the
available relay nodes using a pre-defined policy [11–19]. Opportunistic relaying (OR) [12] is a simple and efficient relay selection method that was proposed to reduce the complexity of the distributed space time coding (DSTC) system [20,21]. Another relay selection scheme, called selection cooperation (SC), proposed in [13], outperforms the DSTC system in terms of outage probability under high signal-to-noise ratio (SNR) assumption. In [14], comparison between OR and SC schemes is provided in terms of outage and bit error probability (BEP) in the DF scheme. Numerical results show that SC has slightly lower outage probability. However, for BEP, both schemes may outperform one another. In [15] and [16], AF cooperative network with OR is investigated in terms of outage probability and symbol error probability, respectively. Chen et al. [17] also considers the outage probability of AF OR but for high SNR scenario. In [18], a new relay selection method is proposed on the basis of a threshold value. This threshold value is exploited to decide whether to use a direct link or to let the best relay cooperate with the source node. The drawback of this method is the difficulty in optimizing the threshold value, which makes it unrealistic to implement.

Recently, several cooperative schemes with variable-rate transmission have appeared. In [9], the performance of multiple-relay AF fixed relaying is analyzed. Results show a reduction in the spectral efficiency as the number of relay nodes increases. This problem is solved partially in [19] where opportunistic AF fixed relaying scheme is proposed. Another work considers also opportunistic AF incremental relaying to improve the spectral efficiency in which the relay node is activated only if the source node experiences an outage [11]. This mode of operation is not optimal in terms of maximizing spectral efficiency. In [22], an efficient AF adaptive cooperative scheme is proposed when identically distributed links are assumed. Relay selection strategy based on OR and variable-rate transmission are used to improve the spectral efficiency. In [23], DF scheme is investigated with adaptive modulation. The work assumed that the destination node may receive two different modulation modes that increases the detection complexity. This assumption is relaxed in [24] by incorporating the variable-rate transmission in DF OR in order to utilize the degree of freedom of the channels.

This paper focuses on the design and analysis of efficient cooperative scheme that performs variable-rate transmission and relay selection over multi-relay systems. It aims to improve the spectral efficiency while maintaining the target quality in terms of bit error rate (BER) and outage probability. Similar to other opportunistic relaying-based schemes (e.g., [11,12,19,22,23]), the proposed scheme uses the OR strategy to select the best relay among the available relay nodes utilizing only local channel knowledge of each relay node [12]. Although the fixed relaying scheme is simple [9,19], the reduction of the spectral efficiency is noticeable at low average SNR of the relay path. On the other hand, the outage-based cooperative scheme lacks to improve the spectral efficiency at average SNR because cooperative transmission is not activated. This motivates the design of an adaptive scheme that has the capability to maximize the spectral efficiency at all time by adapting the transmission rate and the mode of transmission (i.e., cooperative or direct). To achieve this, a discrete adaptive modulation is introduced as a practical implementation of variable-rate transmission. Then, a switching policy is defined to switch between cooperative and non-cooperative transmissions, which depends on the definition of the switching thresholds of each modulation level as well as a switching threshold for the mode of transmission. It is shown that this switching threshold can be approximated from its original definition without affecting the performance. Furthermore, the work of [22] is extended for the case of independent non-identically distributed Rayleigh fading channels in which closed-form expressions for the average spectral efficiency, average BER (ABER), and outage probability are derived.

The remainder of this paper is organized as follows. Section 2 describes the system model and its parameters, which includes the channel model and the adaptive modulation. Section 3 presents the adaptive AF OR scheme and its mode of operation. Performance analysis is given in Section 4. Numerical examples are presented in Section 5. Finally, in Section 6, the paper is concluded and future research is presented.

2. SYSTEM DESCRIPTION

A cooperative wireless system is considered consisting of $L$ relay nodes ($R_i, i = 1, 2, ..., L$), where only one relay is selected to cooperate with source node $S$ to transfer the information signal to destination node $D$, as shown in Figure 1. All relay links are modeled as independent non-identically distributed block Rayleigh fading channels. The transmit power from the source and relay nodes are kept constant because the variable rate is used [25].

The transmission of information signal takes place after deciding the mode of operation, whether to transmit only over the direct link or to select the best relay node among the available relay nodes to cooperate. If the cooperative mode is activated, the first phase of transmission starts with broadcasting the source information signal $x(t)$ to the best relay node, $R_b$, and the destination node. The received signals at the relay and destination nodes are

$$y_{SR_b}(t) = h_{SR_b}x(t) + n_{SR_b}(t), \quad t = 1, 2, ..., T/2$$  \hspace{1cm} (1)

$$y_{SD}(t) = h_{SD}x(t) + n_{SD}(t), \quad t = 1, 2, ..., T/2$$  \hspace{1cm} (2)

respectively, where $h_{SR_b}$ and $h_{SD}$ are the fading coefficients between the source and best relay nodes, and the source and destination nodes, respectively. $n_{SR_b}$ and $n_{SD}$ are the additive white Gaussian noise terms at the best relay and destination nodes, respectively, with a variance of $N_0$ for all links, and $T$ is a time-slot duration. In the
second phase, the best relay node amplifies the received signal, $y_{SR}(t)$, and transmits it to the destination node. The received signal at the destination node is

$$y_{RD}(t) = G_{RD} h_{RD} y_{SR}(t) + n_{RD}(t), \quad t = T, 2T, \ldots, T$$

(3)

where $h_{RD}$ is the fading coefficient between the best relay and destination nodes, $n_{RD}$ is the additive white Gaussian noise term at the destination node, and $G_{RD}$ is the amplifying gain that can be defined as [2]

$$G_{RD}^2 = \frac{E_s}{E_s |h_{SR}|^2 + N_o}$$

(4)

where $E_s$ is the average symbol energy. After the completion of the two phases, destination node detects the two received signals from the source and best relay nodes. One way of detection is to use maximal ratio combining that maximizes the overall received SNR. As a result, the maximum AF effective received SNR becomes

$$\gamma_{AF}^h = \gamma_{SD} + \max_{i \in \{1, 2, \ldots, L\}} \left[ \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \right]$$

(5)

where $\gamma_{SR}$ and $\gamma_{RD}$ are the instantaneous SNRs between the source and the $i$-th relay nodes, and the $i$-th relay and destination nodes, respectively. For derivation tractability, many research works such as [9,11,19,26,27] approximate (5) by its upper bound as

$$\gamma_{AF}^h \leq \gamma_{SD} + \max_{i \in \{1, 2, \ldots, L\}} \left[ \min \left( \frac{\gamma_{SR} \gamma_{RD}}{\gamma_{SR} + \gamma_{RD}} \right) \right]$$

(6)

where $\gamma_{SR}$ is the upper bound of the end-to-end SNR of the best relay path. If the cooperative mode is not activated, the source node transmits the information signals using the full time-slot duration, $T$.

For independent Rayleigh fading channels, the cumulative distribution function (CDF) of the output SNR for each link from node $i$ to node $j$ is given by [11,19]

$$F_{\gamma_{ij}}(x) = 1 - e^{-x/\bar{\gamma}_{ij}}$$

(7)

where $\bar{\gamma}_{ij}$ is the average SNR. The corresponding probability density function (PDF), $f_{\gamma_{ij}}(x)$, can be obtained by differentiating (7) with respect to $x$.

For practical implementation of adaptive transmission, an adaptive M-ary Quadrature Amplitude Modulation (M-QAM) with constellation size of $2^n$, $n = 2, 3, \ldots, N$ is considered, where $N$ is the maximum spectral efficiency. The modulation mode selection depends on a predesigned target performance that can be represented by the target BER (BER$_T$). This target performance actually divides the range of the SNR into regions $[\gamma_n, \gamma_{n+1})$, each region corresponds to a modulation mode. The minimum switching threshold is $\gamma_2$ in which the transmission is declared to be under outage condition if $\gamma_n < \gamma_2$, where $n = 0$. In M-QAM modulation with coherent detection and Gray coding [28], BER can be approximated as

$$\text{BER}(n, \gamma) \approx \frac{2(\sqrt{2^n} - 1)}{n\sqrt{2^n}} Q\left(\sqrt{\frac{3\gamma}{2^n - 1}}\right), \quad n \geq 2$$

(8)

where $Q(.)$ is the Gaussian Q-function. Then, the switching thresholds are given by

$$\gamma_n \approx \frac{2^n - 1}{3} \left( Q^{-1} \left( \frac{n\sqrt{2^n} \text{BER}_T}{2(\sqrt{2^n} - 1)} \right) \right)^2, \quad 2 \leq n \leq N$$

(9)

and $\gamma_{N+1} = \infty$. Note that if $n = 0$, $[\gamma_n, \gamma_{n+1}) = [0, \gamma_2)$ because $n = 1$ is not considered.

3. PROPOSED ADAPTIVE AMPLIFY-AND-FORWARD OPPORTUNISTIC RELAYING SCHEME

In multiple-relay cooperative system, the number of relay nodes used for cooperative transmission has an impact on the spectral efficiency. Although the diversity is high (i.e., $L+1$ diversity order can be achieved), the operation becomes very complicated and the system suffers low spectral efficiency due to the transmission over orthogonal channels [9]. Also, the characteristics of the channel are usually under continuous change due to multipath fading. Therefore, in order to maximize the spectral efficiency, it is important to use a relay selection technique.
and let the cooperative transmission be adaptive in terms of both cooperative and non-cooperative transmissions and rate selection. The switching policy is defined as

1. If \( \gamma_{SD} \geq \gamma_2 \) do not cooperate, where \( \lfloor k \rfloor \) is the largest integer less than or equal to \( k \).
2. If \( \gamma_{SD} < \gamma_2 \) cooperate if \( \gamma_{AF}^b \geq T_n \),
3. else do not cooperate,

where \( T_n \) is the switching threshold SNR used to guarantee that the cooperative transmission can maximize the spectral efficiency. The output SNR of this scheme can be defined as

\[
\gamma_{OUT} = \begin{cases} 
\gamma_{SD}, & \text{if } \left( \gamma_{SD} \geq \gamma_2 \right), \\
\gamma_{SD}^b, & \text{if } \left( \gamma_{SD} \leq \gamma_{n+1}^b \right) \text{ and } \gamma_{AF}^b < T_n, n = 2, \ldots, \left\lfloor \frac{N}{2} \right\rfloor, \\
\gamma_{AF}^b, & \text{otherwise}
\end{cases}
\] (10)

When \( \gamma_{SD} \geq \gamma_2 \), the spectral efficiency of the direct transmission, \( n \), is above the mid spectral efficiency, \( N/2 \), then the cooperative transmission cannot maximize spectral efficiency. On the other hand, when the spectral efficiency of the direct transmission is less than \( N/2 \), cooperative transmission is activated if the spectral efficiency is larger than \( 2n \). In addition, the spectral efficiency of cooperative transmission should still satisfy the target BER. Hence, the BER of the cooperative transmission should be less or equal the BER of the direct transmission, that is,

\[
\text{BER}
(2n, \gamma_{AF}^b) \leq \text{BER}(n, \gamma_{SD}), \quad 2 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor
\] (11)

by substituting (8) into (11) and doing some manipulations, \( \gamma_{AF}^b \) is lower bounded by

\[
\gamma_{AF}^b \geq \frac{2^n - 1}{3} \left\{ Q^{-1} \left( \frac{2^n + 1}{2^n + \sqrt{2^n}} \right) \right\}^2 \times Q \left( \sqrt{\frac{3 \gamma_{SD}}{2^n - 1}} \right), \quad 2 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor
\] (12)

For \( n = 0 \), the minimum requirement of \( \gamma_{AF}^b \) is to be larger than or equal to \( \gamma_2 \); otherwise, the cooperative transmission is under outage as well. This condition represents the policy proposed in [11]. The mathematical model of the switching threshold SNR can be represented by

\[
T_n = \begin{cases} 
\gamma_2, & n = 0, \\
\frac{2^n - 1}{3} \left\{ Q^{-1} \left( \frac{2^n + 1}{2^n + \sqrt{2^n}} \right) \right\}^2 \times Q \left( \sqrt{\frac{3 \gamma_{SD}}{2^n - 1}} \right), & 2 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor
\end{cases}
\] (13)

Furthermore, the expression of \( T_n \) can be approximated for the analysis tractability and simplify the calculation at the relay node. In a successful wireless transmission, the target BER is usually less than \( 10^{-3} \), so the Q-function inside the inverse of the Q-function as shown in (13) is very small compared with the fraction multiplied with it, (i.e., \( Q(.) \ll 1 \) and \( \frac{2^n + 1}{2^n + \sqrt{2^n}} > 1 \)). Therefore, this fraction can be approximated to be equal to 1. Then, (13) can be approximated by

\[
T_n \approx \gamma_2, \quad (2^n + 1)\gamma_{SD}, \quad 2 \leq n \leq \left\lfloor \frac{N}{2} \right\rfloor
\] (14)

Figure 2 shows the comparison of the approximation (14) to the exact (13). Notice that the gap slightly increases as the SNR increases because the approximate expression of the BER in (8) is not as accurate at high SNR. The effect of this approximation on the performance of the proposed scheme is further analyzed in Section 5 (see Figure 5).

The decision to cooperate or not and selecting the best relay node under variable-rate transmission requires that each relay node estimates the relay links by receiving a ready-to-send signal from the source node and a clear-to-send signal from the destination node, which includes the SNR of the direct link, \( \gamma_{SD} \). On the basis of this information, each relay node is able to apply the proposed policy as shown in Figure 3. Each relay node, \( R_i \), finds the spectral efficiency of the direct link, \( n \), calculates \( \gamma_{SR_i} D \) and \( \gamma_{AF} \). If \( n \) is below the mid spectral efficiency and \( \gamma_{AF} > T_n \), the relay node sets a timer that is inversely

Figure 2. Switching threshold signal-to-noise ratio (SNR), \( T_n \), and its upper bound approximation.
Special cases. Various schemes can be obtained on the basis of the values of $\gamma_{SD}$ and $\gamma_{AF}$ and mode of operation as follows:

- $\gamma_{AF}^b = 0$ (i.e., no cooperation) $\Rightarrow$ The conventional direct transmission [28].

- $\left(\gamma_{SD} > T\right) / \gamma_{AF}^b < \infty$ (i.e., no restriction on their values), cooperate all the time $\Rightarrow$ AF fixed relaying scheme [9,19].

- if $\gamma_{SD} < \gamma_2$ (i.e., the direct transmission experiences an outage), cooperate $\Rightarrow$ outage-based AF incremental relaying scheme [11].

4. PERFORMANCE ANALYSIS

4.1. Average spectral efficiency

On the basis of the mode of operation, the average spectral efficiency of the proposed scheme can be expressed as [22]

$$\eta = \frac{1}{\gamma_{SR_D}} \left[ a(n) + \sum_{n=2}^{N} b(n) + \sum_{n=1}^{N} \sum_{m=0}^{\left\lfloor \frac{N}{2} \right\rfloor} m c(n,m) \right]$$

where $m$ is the spectral efficiency of the cooperative transmission and divided by 2 due to the half duplex constraint. $a(n)$ is the probability that $\gamma_{SD}$ falls in region $n$, $b(n)$ is the probability when $\gamma_{SD} \leq \gamma_{SD}^b$ and $\gamma_{AF}^b < \gamma_T$, and $c(n,m)$ is the probability when $\gamma_2 < \gamma_{SD} \leq \gamma_2 + \frac{N}{2}$ and $\gamma_{AF}^b \geq \gamma_T$, also, $I_n = 2$ if $n = 0$ and $I_n = 2n$ if $n \neq 0$. $a(n)$ can be obtained as

$$a(n) = \int_{\gamma_n}^{\gamma_n+1} f_{\gamma_{SD}}(x) dx = F_{\gamma_{SD}}(\gamma_n+1) - F_{\gamma_{SD}}(\gamma_n)$$

where $f_{\gamma_{SD}}(x)$ is the PDF of $\gamma_{SD}$, and $F_{\gamma_{SD}}(x)$ is its CDF. Let $\psi(\alpha, \beta) = e^{-\beta x} - e^{-\beta x+1}$, then $a(n)$ can be rewritten as $a(n) = \psi(n, 1/\gamma_{SD})$.

$b(n)$ can be represented by

$$b(n) \approx \int_{\gamma_n}^{\gamma_n+1} F_{\gamma_{SR_D}}(T_n-x) f_{\gamma_{SD}}(x) dx$$
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where \( F_{YSR_bD}(\cdot) \) is the CDF of the upper bound of the best-relay end-to-end path SNR. This can be given by

\[
F_{YSR_bD}(\gamma) = \Pr\left\{ \max_{i \in \{1,2,\ldots,L\}} \min_{(\gamma_{SR_i}, \gamma_{R_i}, D)} \left[ (\gamma_{SR_i} \cdot \gamma_{R_i} \cdot D) \right] \leq \gamma \right\}
\]

\[
= \prod_{i=1}^{L} \left[ 1 - (1 - F_{YSR_i}(\gamma))(1 - F_{Y_{R_i}D}(\gamma)) \right]
\]

\[
= \prod_{i=1}^{L} \left( 1 - e^{-\frac{\gamma}{\gamma_{SR_i}D}} \right)
\]  \hspace{1cm} (18)

where \( \tilde{\gamma}_{SR_i}D = \tilde{\gamma}_{SR_i} \tilde{\gamma}_{R_i}D / (\tilde{\gamma}_{SR_i} + \tilde{\gamma}_{R_i}D) \). \( F_{YSR_bD}(\gamma) \) can be rewritten as

\[
F_{YSR_bD}(\gamma) = 1 - \sum_{i=1}^{L} (-1)^{i-1} \sum_{k_1=1}^{L-i+1} \sum_{k_2=k_1+1}^{L-i+2} \cdots \sum_{k_i=k_{i-1}+1}^{L} \left( -\gamma \sum_{d=1}^{i} \frac{1}{\gamma_{SR_{kd}}D} \right)
\]  \hspace{1cm} (19)

The corresponding PDF is obtained by differentiating (19) with respect to \( \gamma \), yielding

\[
f_{YSR_bD}(\gamma) = \sum_{i=1}^{L} (-1)^{i-1} \sum_{k_1=1}^{L-i+1} \sum_{k_2=k_1+1}^{L-i+2} \cdots \sum_{k_i=k_{i-1}+1}^{L} \frac{1}{\tilde{\gamma}_{SR_{kd}}D} \times e^{-\frac{\gamma}{\gamma_{SR_{kd}}D}}
\]

\[
= \psi\left( n, \frac{1}{\gamma_{SD}} \right) \times \psi\left( m, \frac{1}{\gamma_{SD}} \right)
\]  \hspace{1cm} (20)

substituting (19) into (17) and solving the integral, \( b(n) \) is given by (21). Finally, \( c(n, m) \) can be represented by

\[
b(n) \approx \psi\left( n, \frac{1}{\gamma_{SD}} \right) \times \psi\left( m, \frac{1}{\gamma_{SD}} \right)
\]

\[
= \sum_{i=1}^{L} (-1)^{i-1} \sum_{k_1=1}^{L-i+1} \sum_{k_2=k_1+1}^{L-i+2} \cdots \sum_{k_i=k_{i-1}+1}^{L} \frac{1}{\gamma_{SR_{kd}}D} \times \psi\left( n, \frac{1}{\gamma_{SD}} + 2^n \sum_{d=1}^{i} \frac{1}{\gamma_{SR_{kd}}D} \right)
\]  \hspace{1cm} (21)

On the basis of the conditions of the cooperative regions, \( c(n, m) \) has two values, \( c_1(n, m) \) and \( c_2(n, m) \). For \( CR = [\gamma_{m}, \gamma_{m+1}] \), substituting (20) into (22) and solving the double integral, \( c_1(n, m) \) is given by (24). Similarly, For \( CR = [T_n, \gamma_{m+1}] \), \( c_2(n, m) \) is given by (25). Substituting (16), (21), (24), and (25) into (15), a closed-form expression for the average spectral efficiency of the proposed scheme can be obtained.

4.2. Average bit error rate

In general, the average BER (ABER) can be defined as the average number of bits in error divided by the total average number of transmitted bits. Therefore, the ABER of the proposed scheme can be expressed as [22]
\[
c_2(n, m) \approx \sum_{i=1}^{L} (-1)^{i-1} \sum_{k_1=1}^{L-i+1} \sum_{k_2=k_1+1}^{L-i+2} \sum_{k_i=k_{i-1}+1}^{L} \frac{1}{1 + 2^n \gamma SD \sum_{d=1}^{i} \frac{1}{RS_{Rkd} D}} \\
\times \psi \left( n, \frac{1}{\gamma SD} + 2^n \sum_{d=1}^{i} \frac{1}{RS_{Rkd} D} \right) \\
\times e^{-\gamma_{n+1} \sum_{d=1}^{i} \frac{1}{RS_{Rkd} D}} \\
\times \left[ n, \frac{1}{\gamma SD} - \sum_{d=1}^{i} \frac{1}{RS_{Rkd} D} \right] \right] (25)
\]

Solving the integral using integration by parts, (29) can be given by

\[
\rho(s_1, s_2, \gamma, B) = \frac{Q(\sqrt{B s_1}) e^{-s_1/\gamma}}{\sqrt{B s_1} + B} - \frac{Q(\sqrt{B s_2}) e^{-s_2/\gamma}}{\sqrt{B s_2} + B} + \frac{1}{\gamma SD} \int_{s_1}^{s_2} Q(\sqrt{B x}) e^{-x/\gamma} dx (30)
\]

then, \(P_e^{a(n)}\) becomes

\[
P_e^{a(n)} = A_n \rho(\gamma_n, \gamma_{n+1}, \gamma SD, B_n) (31)
\]

The second part of the ABER of the direct transmission, \(P_e^{b(n)}\), can be written as

\[
P_e^{b(n)} \approx \int_{\gamma_n}^{\gamma_{n+1}} \text{BER}(n, x) f_{\gamma SD}(\gamma) d\gamma (32)
\]

where \(P_e^{a(n)}\) and \(P_e^{b(n)}\) are the ABER when the direct transmission is activated, whereas \(P_e^{c(n,m)}\) is the ABER when cooperative transmission is activated. \(P_e^{a(n)}\) can be written as

\[
P_e^{a(n)} = \frac{\gamma_{n+1}}{\gamma_n} \text{BER}(n, \gamma) f_{\gamma SD}(\gamma) d\gamma (27)
\]

where \(\text{BER}(n, \gamma)\) can be rewritten as

\[
\text{BER}(n, \gamma) \approx \frac{2 \left(\sqrt{2^n} - 1\right)}{n \sqrt{2^n}} Q\left(\sqrt{\frac{3 \gamma}{2^n} - 1}\right) = A_n Q\left(\sqrt{B_n \gamma}\right), \quad n \geq 2 (28)
\]

where \(A_n = 2 \left(\sqrt{2^n} - 1\right) / \left(n \sqrt{2^n}\right)\) and \(B_n = 3/(2^n - 1)\). To simplify the derivation, a common finite integral can be defined as

\[
\rho(s_1, s_2, \gamma, B) = \frac{s_2^2}{s_1} Q\left(\sqrt{B s_1} x e^{-x/\gamma}\right) dx (29)
\]

Substituting (19) and (28) into (32) and solving the integral, \(P_e^{b(n)}\) is given by (33).

The ABER of the cooperative transmission, \(P_e^{c(n,m)}\), can be written as

\[
P_e^{c(n,m)} \approx \int_{\gamma_n}^{\gamma_{n+1}} \text{BER}(m, u) f_{\gamma SD}(u - x) d\gamma (34)
\]
Then, the outage probability can be written as

\[ P_{\text{out}}(n,m) \approx A_{m} \sum_{i=1}^{L} (-1)^{i-1} \left( \sum_{k_{1}=i}^{L-i} \sum_{k_{2}=k_{1}+1}^{L-i+2} \cdots \sum_{k_{i}=k_{i-1}+1}^{L} \right) \]

\[ \times \left[ \frac{1}{1-\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \right] \]

\[ \times \left( \frac{1}{\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \right) \]

\[ \times \rho \left( \nu m, \frac{1}{\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \right) \]

\[ (35) \]

Substituting (20) into (37) and solving the double integral, the outage probability of the proposed scheme can be given by

\[ P_{\text{out}} \approx \sum_{i=1}^{L} (-1)^{i-1} \left( \sum_{k_{1}=i}^{L-i} \sum_{k_{2}=k_{1}+1}^{L-i+2} \cdots \sum_{k_{i}=k_{i-1}+1}^{L} \right) \]

\[ \times \left[ \frac{1}{1-\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \right] \]

\[ \times \rho \left( \nu n, \frac{1}{\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \right) \]

\[ \times \frac{1}{\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \]

\[ + \frac{1}{\gamma SD} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D} \]

\[ e^{-\gamma_{2} \sum_{d=1}^{i} \frac{1}{\gamma SR_{k_{d}} D}} \]

\[ (38) \]

The proposed scheme has the same outage probability as the opportunistic outage-based AF incremental relaying [11] and the opportunistic AF fixed relaying [19]. This is because that if \( \gamma_{AF}^b < \gamma_{2} \), \( \gamma SD \) is also less than \( \gamma_{2} \) because \( \gamma SD \leq \gamma_{AF}^b \), then \[ \text{Pr} \left[ \gamma SD < \gamma_{2}, \gamma_{AF}^b < \gamma_{2} \right] = \text{Pr} \left[ \gamma_{AF}^b < \gamma_{2} \right] \]. Even though the proposed scheme aims to improve the spectral efficiency, the outage probability can still be maintained. Further evaluation of the performance of the proposed scheme is presented in the following section.

### 5. NUMERICAL EXAMPLES

In this section, some numerical examples are presented to evaluate the performance of the proposed scheme in terms of average spectral efficiency, ABER, and outage probability. The proposed scheme is compared with outage-based opportunistic AF incremental relaying [11], opportunistic...
SNRs, signal-to-noise ratios.

Table I. Setting of the number of relay nodes in the system and their links average SNRs.

<table>
<thead>
<tr>
<th>No. of relay nodes</th>
<th>Relay path average SNRs</th>
</tr>
</thead>
<tbody>
<tr>
<td>L = 5</td>
<td>( \bar{\gamma}_{SR} = [5.0, 4.5, 4.0, 4.1, 3.7] )</td>
</tr>
<tr>
<td>L = 3</td>
<td>( \bar{\gamma}_{SR} = [4.6, 4.1, 3.8, 3.4, 3.1] )</td>
</tr>
<tr>
<td>L = 1</td>
<td>( \bar{\gamma}_{SR} = 5.0 )</td>
</tr>
</tbody>
</table>

AF fixed relaying [19], and direct transmission. Table I gives the average SNR for each relay link (i.e., \( \bar{\gamma}_{SR} \) and \( \bar{\gamma}_{RD} \)) as a function of the average SNR, \( \zeta \), in which all the performance measures are depicted versus \( \zeta \). It is assumed that all relay nodes are within the range of the source and destination nodes, and the topology is as shown in Figure 1. The setting of the average SNRs is chosen to reflect the non-identically distributed relays [11]. Without loss of generality, other parameters are set as follows: \( \bar{\gamma}_{SD} = 13dB \), the maximum spectral efficiency, \( N \), to be equal to 8, and the target BER, \( BER_T \), to be equal to \( 10^{-3} \) and \( 10^{-6} \).

Figure 4 shows the average spectral efficiency of the proposed scheme, the outage-based AF incremental relaying, the AF fixed relaying, and the conventional direct transmission. Setting \( BER_T = 10^{-3} \) and \( L = 1 \) and 5, at low average SNR (i.e., average SNR < 10 dB), the average spectral efficiency of the proposed scheme is similar to the outage-based AF incremental relaying due to rarely activation of cooperative transmission. In contrast, AF fixed relaying experiences a reduction in the spectral efficiency due to the continuous use of cooperative transmission even though the channel gain is low. On the other hand, at high average SNR (i.e., average SNR > 25 dB), the average spectral efficiency of the proposed scheme converges to that of the AF fixed relaying because the direct transmission is rarely activated. It is clear that at this region, the spectral efficiency saturates to the half of the maximum spectral efficiency due to the half-duplex transmission mode. Furthermore, at moderate average SNR, the proposed scheme has its maximum gain because it has the capability to optimize the mode of operation efficiently. The gain ranges from 1 to 3 dB compared with the same average spectral efficiency of the outage-based and fixed relaying. Finally, the average spectral efficiency benefits from increasing the number of available relay nodes. For instance, the proposed scheme can achieve 4 dB gain when increasing the number of relay node from 1 to 5.

Figure 5 shows the average spectral efficiency of the proposed scheme when the switching threshold SNR, \( T_R \), is either calculated by (13) or by its approximated expression given by (14). Setting \( L = 3 \) and using two different target BERs, \( 10^{-3} \) and \( 10^{-6} \), it is observed that the approximated value of \( T_R \) has no impact on the average spectral efficiency for both target BERs. This verifies that the approximate expression of \( T_R \) is simple and yet accurate.

Figure 6 shows the ABER of the proposed scheme, the outage-based AF incremental relaying, the AF fixed relaying, and the conventional direct transmission. Setting \( L = 1 \) and \( BER_T = 10^{-3} \) and \( 10^{-6} \), at low and moderate average SNRs, all schemes have same and almost constant ABER due to the use of adaptive modulation. At high average SNR, the ABER of the proposed and AF fixed relaying schemes outperform the outage-based and direct transmission due to diversity gain improvement. Furthermore, for
6. CONCLUSION AND FUTURE WORK

In this paper, we have proposed an adaptive AF OR scheme with variable-rate transmission. The scheme provides a solution for enhancing the spectral efficiency of the AF cooperative wireless systems. For many applications, it is not necessary to minimize the BER as long as the error rate is kept below a certain level. Therefore, the advantage of the proposed scheme is due to (i) exploiting the variable-rate transmission in which the maximum rate is selected all the time; (ii) selecting the best relay node that has the best end-to-end path between the source and destination nodes; (iii) the flexibility to avoid cooperative transmission whenever it is not beneficial; and (iv) the ability to maintain the target quality in terms of BER.

Future research will investigate on the optimal placement of the relay nodes for a predefined traffic demand and target quality of service because the placement of the relay nodes and the type of environment have direct impact on the performance.

REFERENCES


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