

Packet Level Performance Analysis in Wireless User-Relaying Networks

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Abstract—In this paper, the impact of user relaying on the behavior of a relay node, which acts as the source node at the same time, is analyzed in a wireless relay network at the packet level. The analysis process models the behavior of the relay node as a queueing system and represents the service for its own packet transmission as an M/G/1-type Markov chain. By considering the fact that the maximum number of packet arrivals is ordinarily limited in a practical system, the M/G/1-type Markov chain is further reformatted into a quasi-birth-death (QBD) process through re-blocking so as to simplify the analysis and obtain the associated performance, such as average packet transmission delay. As an application of the results arising from the analysis, a new relay node selection scheme, based on a utility function approach that jointly considers the channel and the queue conditions at the relay node, is proposed. Numerical results show that the proposed analysis model is quite accurate and the proposed relay node selection scheme is effective in balancing cooperative diversity gain and packet transmission delay.

Index Terms—User relaying, cooperative diversity, M/G/1, quasi-birth-death (QBD), cross-layer design, wireless communications.

I. INTRODUCTION

IN the past couple of decades, wireless communications have gained dramatic development and have been recently considered as an alternative to wireline networks in providing the last-mile broadband services. Such development further stimulates the emergence of multimedia applications, which require wireless networks to support broader bandwidth, higher transmission rates, and lower end-to-end delay. For wireless communications, the challenge to provide multimedia services stems from the hostile wireless channel conditions. Besides channel noise, the time-variant channel fluctuation (i.e., channel fading) severely affects the transmission accuracy and the provision of quality of service (QoS). In

order to combat channel fading, diversity technologies have been proposed, which provide multiple independently-faded channels for the same information transmission so that the probability that all received signals suffer deep fading will be significantly reduced. Among different diversity technologies, user relaying is a promising one.

User relaying or cooperative diversity is a newly introduced technique to improve system capacity by achieving multiple transmit-antenna spatial diversity. The basic idea of user relaying results from the broadcasting property of wireless transmissions so that other nodes in the same network can overhear the information from the source node and can then forward what is received to the destination. Since the same information reaches the destination through spatially independent transmission paths, certain kind of spatial diversity can be achieved if relaying is appropriately applied. Compared to the traditional multiple transmit-antenna techniques which achieve spatial diversity by equipping the transmitter with multiple antennas, user relaying can provide similar diversity gain with lower complexity. Because of its advantage in capacity improvement, user relaying has been considered in a number of current standards, such as IEEE 802.11, IEEE 802.15.4a, and IEEE 802.16 [1]–[3]. Research work in this area has been attracting more and more attention from both academia and industry.

The effectiveness of user relaying on network capacity improvement has been demonstrated in [4]–[8]. However, analytical results also indicate that such performance improvement heavily depends on selecting suitable relay nodes [4]. Otherwise, applying user relaying may not achieve performance improvement, or maybe even worse than direct transmission without relaying. Therefore, designing effective algorithms for optimal relay node selection becomes critical in wireless relay networks. For example, in [9], space-time coding algorithms are proposed to achieve fully distributed relay node selection. However, the algorithms require network-wide channel state information and introduce high computational complexity due to encoding and decoding processes. Explicit relay node selection is discussed in [4], [10]–[14]. Both centralized and distributed algorithms are proposed. The centralized algorithms focus on solving complicated optimization problems by assuming complete channel state information, while the distributed algorithms are based on opportunistic cooperation, which may induce high system overhead due to contention. In [15], a semi-distributed relay node selection algorithm is proposed based on a derived sufficient condition for determining

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eligibility of relay nodes. Relay node selection, referred to as partner matching, is also studied in [16]. Recently, cross-layer design methods have also been proposed in wireless relay networks, which jointly consider cooperative diversity gain at the physical layer and power allocation or medium access control at higher layers [17]–[19].

Although many research works on partner selection have appeared in the literature, in general, they have the following limitations.

- 1) The selection criteria used focus only on optimizing parameters at the physical layer, for example, maximizing the average mutual information or maximizing the combined signal-to-noise ratio (SNR) at the receiver end. The effects of user relaying on the higher layers are not considered. However, for future wireless networks, packet level performance, such as average packet delay, is quite important.
- 2) A common assumption is that the available relay nodes only perform relaying functions. However, in practice, relay nodes can also act as source nodes or destination nodes. Providing relaying can block the transmission of the relay nodes' own packets. Therefore, nodes which operate relaying functions may become the bottleneck of the system and any node already under heavy traffic load should not be selected as a partner. Otherwise, cooperative transmission may be detrimental to system performance.

In summary, when selecting suitable relay nodes for a given source and destination pair, care should be taken to jointly consider the channel and the relay nodes' traffic load conditions.

In order to facilitate the design of relay node selection algorithms, in this paper, the effects of user relaying on the transmission of the relay node are analyzed at the packet level. By modelling the behavior of the relay node as a queueing system, the service for a given relay node's own packet transmission can be represented as an M/G/1-type Markov chain. Since the maximum number of packet arrivals is ordinarily limited in practical systems, the M/G/1-type Markov chain can be reformatted into a quasi-birth-death (QBD) process through re-blocking, and matrix-analytic methods can be applied to obtain the desired network performance, such as average packet transmission delay. Based on the analytical results and using the cross-layer design principle, a novel relay node selection scheme is proposed. A utility function is introduced by jointly considering channel conditions at the physical layer and queue length at the link layer. Numerical results are given to demonstrate the accuracy of the proposed analytical models and the effectiveness of the proposed relay node selection scheme.

To our best knowledge, this is the first work which considers the performance of relay node in wireless relay networks at the packet level. Moreover, the proposed analytical model provides a general framework for performance analysis in user relaying networks.

The remainder of this paper is organized as follows. In Section II, the system model of the wireless relay network under consideration is defined. Section III discusses the queue modelling and provides detailed analytical procedures. A

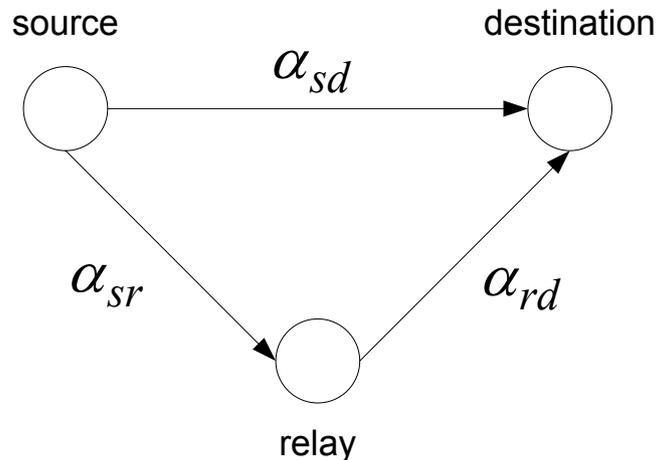


Fig. 1. System model of a user relay system.

new relay node selection scheme is proposed in Section IV. Section V presents simulation results, with conclusions given in Section VI.

II. SYSTEM MODEL

Consider a time division duplex (TDD) wireless relay network, which has a single destination locating at the center of the covered area surrounded by multiple source nodes [15]. One interpretation of such model is the IEEE 802.16 multiple-point-to-point (MPP) network, where the source nodes represent the users with packets waiting for transmission, and the destination node represents the access point (AP). In this study, the inter-user interference is omitted by allocating orthogonal channels to different source nodes. Consider one pair of source and destination nodes, as shown in Fig. 1. For the source node, besides the direct connection to the destination node, another node can be assigned to construct a relay path, where the assigned node is called relay node. In this paper, the relay node works under the amplify-and-forward mode, i.e., the relay node simply amplifies the received signal from the source node and then forwards it to the destination. The selection of amplify-and-forward results from the fact that it has advantages in simple implementation and low computation load on the relay node.

The transmission in the time domain is on a frame-by-frame basis. Each frame is further partitioned into two consecutive time slots. For example, frame n consists of a pair of slots, denoted as $((n, 1), (n, 2))$. The first time slot is used for the transmissions of the source node while the second one is used for either the relay node or the source node depending on whether user relaying is applied. In each slot, only one packet can be served. Since our emphasis is on the performance of relay node, we consider the worst case scenario where there are always packets waiting for transmission from the source node.

Each pair of nodes, i and j , experiences a flat fading channel with channel gain α_{ij} [4]. The channel gain may integrate the effects from both propagation path loss and fading, and is slowly time-varying so that it is approximately unchanged during at least one frame interval. Therefore, for frame n , the

received signals at relay node r and destination node d at the end of the slot $(n, 1)$ can be respectively written as

$$y_{sr}[n, 1] = \alpha_{sr}x_s[n, 1] + Z_r[n, 1], \quad (1)$$

$$y_{sd}[n, 1] = \alpha_{sd}x_s[n, 1] + Z_d[n, 1] \quad (2)$$

where α_{sr} and α_{sd} denote the channel gains from the source node to the relay node and from the source node to the destination node, respectively. $x_s[n, 1]$ denotes the information signal transmitted from the source node at time slot $(n, 1)$. $Z_r[n, 1]$ and $Z_d[n, 1]$ denote the background noise at relay node r and destination node d , respectively, which are independent and identically distributed (i.i.d.) complex Gaussian random variables with a common variance σ_n^2 .

If user relaying is applied, then at the end of time slot $(n, 2)$, the received signal at destination node d from relay node r is given by

$$\begin{aligned} y_{rd}[n, 2] &= \alpha_{rd}\lambda_r y_{sr}[n, 1] + Z_d[n, 2] \\ &= \alpha_{rd}\lambda_r\alpha_{sr}x_s[n, 1] + \alpha_{rd}\lambda_r Z_r[n, 1] + Z_d[n, 2] \end{aligned} \quad (3)$$

where α_{rd} is the channel gain between the relay node and the destination node. λ_r is an amplification factor, which is used to guarantee the transmission power of the relay node and satisfies

$$\begin{aligned} \lambda_r^2(|\alpha_{sr}|^2 P_s + \sigma_n^2) &= P_r \\ \Rightarrow \lambda_r^2 &= \frac{P_r}{|\alpha_{sr}|^2 P_s + \sigma_n^2} \end{aligned} \quad (4)$$

where P_s and P_r denote the transmission power of the source node and the relay node, respectively. Since our discussion focuses on the relay node selection only, we let $P_s = P_r = P/2$, where P is the accumulated power constraint.

If α_{sr} , α_{rd} and α_{sd} are known, the channel combining both the direct path and the relay path can be modelled as an equivalent one-input, two-output complex Gaussian noise channel, which has the maximum average mutual information I_{AF} given by [4]

$$I_{AF} = \frac{1}{2} \log\left(1 + SNR\left(\beta_{sd} + \frac{\beta_{sr}\beta_{rd}SNR}{(\beta_{sr} + \beta_{rd})SNR + 1}\right)\right) \quad (5)$$

where $\beta_{sr} = |\alpha_{sr}|^2$, $\beta_{sd} = |\alpha_{sd}|^2$, and $\beta_{rd} = |\alpha_{rd}|^2$.

If no relaying is applied and both time slots in the frame contribute to the source nodes' transmissions, i.e., direct transmission, the achievable average mutual information can be calculated as

$$I_D = \frac{1}{2} \log(1 + \beta_{sd}SNR)^2. \quad (6)$$

Combining (5) and (6), the maximum average mutual information (or capacity) for a pair of cooperating users equals

$$C = \max\{I_D, I_{AF}\}. \quad (7)$$

A relay node is defined as feasible if user relaying via it can provide better capacity performance than direct transmission, i.e., $I_{AF} > I_D$. Then, the decision on whether user relaying should be applied is equivalent to the decision on the relay node's feasibility. Our previous work in [15] has derived a tight sufficient condition for feasibility as

$$\beta_{sr} > C_s \quad \text{and} \quad \beta_{rd} > C_s \quad (8)$$

where

$$\begin{aligned} C_s &= \left(1 + \sqrt{1 + \frac{1}{SNR \cdot \beta_{sd}(1 + \beta_{sd}SNR)}}\right) \\ &\quad \times \beta_{sd}(1 + \beta_{sd}SNR). \end{aligned} \quad (9)$$

Therefore, by collecting all channel conditions, the destination node can make decision and notify both the source node and the relay node. From the relay node point of view, if the sufficient condition is satisfied, it will participate in the relaying process and performs the relaying function. Otherwise, it will pick the packets from its own buffer for transmission.

The estimates of β_{sd} and β_{rd} are ordinarily easy to obtain by measuring the periodic pilot signal from the destination node (the access point). However, the channel condition, β_{si} , can be estimated by the relay node only after the source node's transmission has been initiated. Therefore, according to the time when the information on β_{si} is available, we consider the following three cases:

Case 1: The channel estimate of β_{si} is available at the end of the first slot in each frame by measuring the information transmission from the source node. Thus, in order to avoid the possible missing of cooperative diversity gain, the relay node needs to always receive signal from the source node in the first slot. In the second slot, based on the decision of the destination node, the relay node may transmit its own packets if relaying is not feasible.

Case 2: In Case 1, possible waste of system resource exists if the relay node is not feasible. To avoid such waste, one possible solution is to allow full duplex transmission at the relay node so that in the first slot, the relay node can transmit and receive simultaneously. However, by assuming the relay node will use the same radio channel as the source-to-destination channel for information relaying, when a relay node forwards information from the source to the destination, the transmission of the source node must be stopped since both transmissions will be on the same channel and will otherwise conflict with each other. The cost for improved resource utilization is the increased complexity at the relay node, which requires the equipment of multiple radios.

Case 3: If the transmission from the source node in the previous frames can be used to predict the channel condition in the current frame, at the beginning of the new frame, β_{si} becomes available and the decision on relay node feasibility can be made. Therefore, in Case 3, the whole frame will either be used or not used by the relay node for its own transmission.

Define $\alpha = Pr\{\beta_{sr} < C_s \text{ or } \beta_{rd} < C_s\}$. Then, in each frame, the probability that the relay node participates in user relaying equals $1 - \alpha$.

III. PERFORMANCE ANALYSIS

This section presents the analysis of the effect of a node participating in user relaying.

The boundary equations for the process are

$$[\mathbf{x}_0, \mathbf{x}_1] = [\mathbf{x}_0, \mathbf{x}_1]B[R] \quad (15)$$

where

$$B[R] = \begin{bmatrix} F & C \\ E & D_1 + RD_2 \end{bmatrix}.$$

Since $\mathbf{x}\mathbf{1} = 1$, according to (14), we have

$$\mathbf{x}_0\mathbf{1} + \mathbf{x}_1(I - R)^{-1}\mathbf{1} = 1. \quad (16)$$

Combining (14), (15), and (16), the stationary probability \mathbf{x} can be obtained.

For practical applications, however, the solution to equation (13) may involve high computational complexity. In order to simplify the calculations, we introduce a matrix G , which is given by the minimal mean solution to

$$G = D_2 + D_1G + D_0G^2. \quad (17)$$

Since D_2 has a favor structure in our problem, the calculation of G will be much easier than directly computing R . It has been shown that G is stochastic if the stability condition is satisfied [21], [22]. According to (13) and (17), R and G are related as follows:

$$R = D_0(I - D_1 - D_0G)^{-1}. \quad (18)$$

Let R be written as

$$R = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,K} \\ R_{2,1} & R_{2,2} & \cdots & R_{2,K} \\ \vdots & \vdots & \cdots & \vdots \\ R_{K,1} & R_{K,2} & \cdots & R_{K,K} \end{bmatrix}$$

where $R_{i,j}$ is a 2×2 matrix for Cases 1 and 2, and it is a 4×4 matrix for Case 3. Then, we have

$$\mathbf{x}_{i+1}^j = \sum_{v=1}^K \mathbf{x}_i^v R_{v,j}, \text{ for } i \geq 1. \quad (19)$$

Let $\mathbf{y}_{K(i-1)+j} = \{y_{K(i-1)+j,k}\} = \mathbf{x}_i^j$. Then, the probability that there are i packets in the system at time slot k is given as $y_{i,k}$, and the mean number of packets in the system at arbitrary time, $E[L]$, can be calculated as

$$\begin{aligned} E[L] &= \sum_{i=1}^{\infty} \sum_{k=1}^w y_{i,k} \\ &= \sum_{i=1}^{\infty} [(i-1)K\mathbf{x}_i\mathbf{1}_{wK} + \mathbf{x}_i(V \otimes \mathbf{1}_w)] \end{aligned} \quad (20)$$

where $V = [1, 2, \dots, K]^T$ and $\mathbf{1}_k$ is a $k \times 1$ column vector of ones. For Cases 1 and 2, $w = 2$ while for Case 3, $w = 4$. After routine algebraic manipulations, we have

$$E[L] = \mathbf{x}_1[KR(I - R)^{-2}\mathbf{1}_{wK} + (I - R)^{-1}(V \otimes \mathbf{1}_w)]. \quad (21)$$

According to Little's Law, the mean waiting time equals

$$E[W] = \lambda^{-1}E[L]. \quad (22)$$

Next, we present detailed block elements of each of the three Markov chains.

C. Case 1

In case 1, all A_k and C_k for $0 \leq k \leq K$ are 2×2 matrices, which indicate the transition between neighboring slots. The block matrices for Case 1 can be summarized as

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & \alpha a_0 \\ 0 & 0 \end{bmatrix}, \\ A_k &= \begin{bmatrix} 0 & \alpha a_k + (1 - \alpha)a_{k-1} \\ a_{k-1} & 0 \end{bmatrix}, \quad 1 \leq k \leq K, \\ A_{K+1} &= \begin{bmatrix} 0 & (1 - \alpha)a_K \\ a_K & 0 \end{bmatrix}, \\ C_k &= \begin{bmatrix} 0 & a_k \\ a_k & 0 \end{bmatrix}, \quad 0 \leq k \leq K. \end{aligned} \quad (23)$$

The detailed derivation is given in Appendix I.

Since only a fraction α of each frame can be contributed by the transmission of the relay node, it is clear that the system is stable iff

$$\frac{2\lambda}{\alpha} < 1. \quad (24)$$

For Case 1, since D_2 in (11) has all zero columns except the last one, the stochastic matrix G can be directly written out as

$$G = \mathbf{e}^T(2K)_{2K} \otimes \mathbf{1} \quad (25)$$

where $\mathbf{e}(2K)_{2K}$ denotes a $2K \times 1$ column vector of zeros with a 1 in the $(2K)^{th}$ location. Therefore, from (18), we have

$$R = D_0(I - D_1 - D_0(\mathbf{e}^T(2K)_{2K} \otimes \mathbf{1}))^{-1}. \quad (26)$$

Since no calculation is required for deriving the matrix G , calculating R from G is much simpler than the direct computation of R .

Let

$$\tilde{D} = \begin{bmatrix} A_1 & A_2 & \cdots & A_{K-1} & \sum_{j=0}^1 A_{K+j} \\ A_0 & A_1 & \cdots & A_{K-2} & \sum_{j=-1}^1 A_{K+j} \\ & A_0 & \cdots & A_{K-3} & \sum_{j=-2}^1 A_{K+j} \\ & & \ddots & \ddots & \vdots \\ & & & A_0 & \sum_{j=-K+1}^1 A_{K+j} \end{bmatrix}.$$

Then R can be represented in terms of \tilde{D} as

$$R = D_0(I - \tilde{D})^{-1}. \quad (27)$$

Noticing that $RD_2 = D_0G$, we have the boundary equations for Case 1 as

$$[\mathbf{x}_0, \mathbf{x}_1] = [\mathbf{x}_0, \mathbf{x}_1]B[R]$$

where $B[R] = \begin{bmatrix} F & C \\ E & \tilde{D} \end{bmatrix}$.

There exist many algorithms to solve the boundary condition $[\mathbf{x}_0, \mathbf{x}_1]$. In this paper, the iterative algorithm, presented in Appendix II, will be used in the simulation to calculate $[\mathbf{x}_0, \mathbf{x}_1]$. Although the algorithm may not be the most efficient one, it is simple and is easy to implement.

D. Case 2

In Case 2, A_k and C_k are 2×2 matrices. Following a similar analysis procedure as in Case 1, the block matrices for this case are

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & \alpha a_0 \\ a_0 & 0 \end{bmatrix}, \\ A_k &= \begin{bmatrix} 0 & \alpha a_k + (1-\alpha)a_{k-1} \\ a_k & 0 \end{bmatrix}, \quad 1 \leq k \leq K, \\ A_{K+1} &= \begin{bmatrix} 0 & (1-\alpha)a_K \\ 0 & 0 \end{bmatrix}, \\ C_k &= \begin{bmatrix} 0 & \alpha a_k \\ a_k & 0 \end{bmatrix}, \quad 0 \leq k \leq K. \end{aligned}$$

For Case 2, it is clear that the system is stable iff

$$\frac{2\lambda}{1+\alpha} < 1.$$

Although (25) does not hold for Case 2, the structure of D_2 can still provide opportunities to find a simple algorithm for calculating G . According to (11), the matrix D_2 can be rewritten as

$$D_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & A_0 \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \quad (28)$$

where $\mathbf{0}$ denotes a 2×2 all zero matrix. Then according to (17), the matrix G has the following format

$$G = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & G_1 \\ \mathbf{0} & \mathbf{0} & \cdots & G_2 \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & G_K \end{bmatrix} \quad (29)$$

where G_i , $i = 1, 2, \dots, K$, is a 2×2 matrix. Obviously, we have

$$G^2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \cdots & G_1 G_K \\ \mathbf{0} & \mathbf{0} & \cdots & G_2 G_K \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & G_K^2 \end{bmatrix}. \quad (30)$$

Substituting (29) and (30) into (17), we have

$$G_1 = A_0 + \sum_{i=1}^K A_i G_i + A_{K+1} G_1 G_K \quad (31)$$

$$\begin{aligned} G_j &= \sum_{i=0}^{K-j+1} A_i G_{i+j-1} + \sum_{i=0}^{j-1} A_{K-j+i+2} G_{i+1} G_K, \\ j &= 2, \dots, K. \end{aligned} \quad (32)$$

According to (31) and (32), an iterative algorithm can be derived by letting the initial state $G_i = 0$, $\forall i = 1, 2, \dots, K$. The details of the iterative algorithm are summarized in Appendix III. Compared to the cyclic reduction procedure [23], which is another commonly used algorithm to calculate matrix G and treats all entries of G as non-zero values, the introduced iterative algorithm is simpler, since it only involves low-dimensional matrices in the calculation. However, the algorithm needs longer iteration steps (25 in our simulation) to achieve the same accuracy as the cyclic reduction procedure (with 10 iteration steps).

E. Case 3

Different from the previous two Cases, the state space of Case 3 is doubled, i.e., the dimensionality of A_k and C_k is 4×4 , and the block matrices are

$$\begin{aligned} A_0 &= \begin{bmatrix} 0 & 0 & a_0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha a_0 & 0 & 0 & 0 \\ \alpha a_0 & 0 & 0 & 0 \end{bmatrix}, \\ A_k &= \begin{bmatrix} 0 & 0 & a_k & 0 \\ 0 & 0 & 0 & a_{k-1} \\ \alpha a_k & (1-\alpha)a_{k-1} & 0 & 0 \\ \alpha a_k & (1-\alpha)a_{k-1} & 0 & 0 \end{bmatrix}, \quad 1 \leq k \leq K, \end{aligned}$$

$$A_{k+1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_K \\ 0 & (1-\alpha)a_K & 0 & 0 \\ 0 & (1-\alpha)a_K & 0 & 0 \end{bmatrix},$$

$$C_k = \begin{bmatrix} 0 & 0 & a_k & 0 \\ 0 & 0 & 0 & a_k \\ a_k & 0 & 0 & 0 \\ 0 & a_k & 0 & 0 \end{bmatrix}, \quad 0 \leq k \leq K.$$

For Case 3, it is clear that the system is stable iff

$$\frac{\lambda}{\alpha} < 1,$$

and both algorithms in the Appendices can be used for calculating matrix G and the boundary conditions $[\mathbf{x}_0, \mathbf{x}_1]$.

IV. UTILITY FUNCTION BASED RELAY NODE SELECTION METHOD

According to the analysis provided in the previous section and the numerical results shown in Section V, it can be observed that applying user relaying will significantly affect the transmission performance of the relay node, especially when the relay node is already under heavy traffic load. As an example of applying this observation, in this section, we propose a new relay node selection method, which can jointly take into consideration the channel conditions and the relay node traffic status.

Consider a given source and destination pair. According to (5), the diversity gain heavily depends on the channel gains β_{sr} and β_{rd} , and both gains play a same role in determining the diversity gain. On the other hand, the relay node selection should avoid the nodes under heavy traffic load, which can be represented by their instantaneous queue lengths, Q_r . Therefore, in order to reflect the effects of β_{sr} , β_{rd} , and Q_r , a utility function, proportional to the product of β_{sr} and β_{rd} , and inversely proportional to Q_r , is introduced as follows:

$$U = \frac{\beta_{sr}\beta_{rd}}{Q_r}. \quad (33)$$

In (33), we define $1/Q_r = 1$ if $Q_r = 0$, i.e., the nodes with empty queue or queue occupancy of one packet are given same preference for relay node selection.

Given the utility function, relay node selection is based on the following criteria.

TABLE I
COMPARISON BETWEEN NUMERICAL AND ANALYTICAL RESULTS.

		$E[L]$	$E[W]$
Case 1	Numerical	12.78	40.16
	Analytical	12.5	38.46
Case 2	Numerical	0.72	2.20
	Analytical	0.77	2.36
Case 3	Numerical	1.07	3.30
	Analytical	1.04	3.21

Relay Node Selection Method:

Given a pair of source and destination nodes, and a set of available relay nodes Γ . Relay node i , $i \in \Gamma$, will be selected if

- 1) node i is feasible, i.e.,

$$\beta_{si} > C_s \text{ and } \beta_{id} > C_s; \quad (34)$$

- 2) node i can maximize the utility function, i.e.,

$$i = \arg \max_{i \in \Gamma} U = \arg \max_{i \in \Gamma} \frac{\beta_{si}\beta_{id}}{Q_i}. \quad (35)$$

V. NUMERICAL RESULTS

In this section, numerical results are presented to demonstrate the accuracy of our analytical model and the behavior of packet queueing in the relay node with respect to different system parameters. The performance of the proposed relay node selection method will also be evaluated by simulation.

In our simulation, the relay network shown in Fig. 1 is considered. The relay node maintains its own queue with an infinite buffer size and there are always packets waiting for transmission from the source to the destination. The maximum number of arriving packets in each time slot is truncated to $K = 3$ and the packet arrival satisfies the following distribution

$$a_0 = 0.8, a_1 = 0.1, a_2 = 0.075, a_3 = 0.025,$$

which results in the average arrival rate

$$\lambda = 0.325.$$

Since our analysis model is independent of the packet arrival process, in the simulation, a discrete random variable satisfying the defined distribution is used to describe the arrival process. The parameter α is determined by the channel models of β_{sr} , β_{rd} , and β_{sd} . For simplicity, in our simulation, α is fixed at 0.7. In practice, the value of α can be obtained by observing multiple frame transmissions. Based on the discussions in the previous section, the parameter settings can guarantee the stability condition for all three cases.

Table I shows the performance comparison resulting from both simulation and analysis in terms of average number of packets in the system ($E[L]$) and average transmission delay ($E[W]$). From the table, it is observed that the analytical results approximate the simulation ones very well, i.e., our proposed analysis model is quite accurate. Case 1 has the worst performance while Case 2 is the best. This is a consequence of the fact that in the Case 1, only a fraction ($\alpha/2$) of each frame on average can be used for its own transmission, while in Case 2, any value of α can guarantee stability.

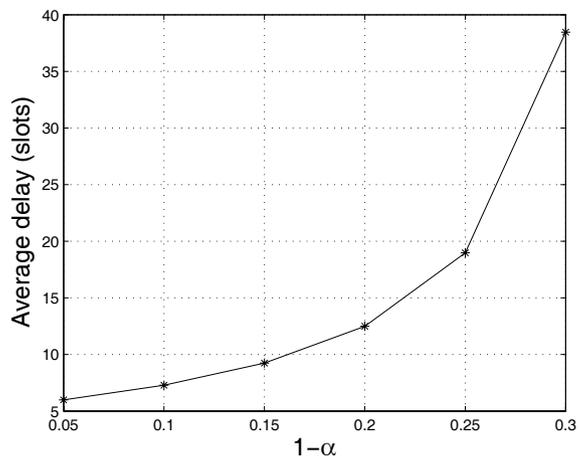


Fig. 2. Average transmission delay in Case 1.

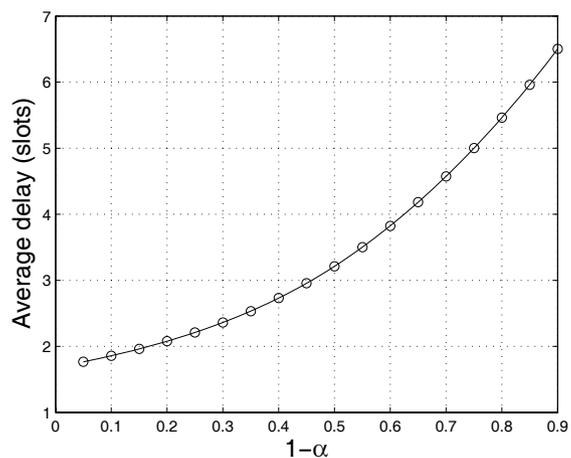


Fig. 3. Average transmission delay in Case 2.

If no user relaying is applied, then under the same system parameters, $E[L] = 0.54$ and $E[W] = 1.68$. Compared to the results shown in Table I, it can be concluded that participating in user relaying will greatly affect the transmission of the relay node.

Figs. 2, 3, and 4 show the analytical results of average transmission delay as a function of the probability ($1-\alpha$) for Cases 1, 2, and 3, respectively. Since the stability conditions are different for the three cases, different ranges of ($1-\alpha$) are used in the simulation. From the three figures, it can be observed that the effect of participating in user relaying on delay performance is not linear. If the relay traffic load is light, i.e., ($1-\alpha$) is small, the average delay increases almost linearly with respect to $1-\alpha$. However, if the relay traffic load is heavy, i.e., ($1-\alpha$) \rightarrow 1, the average delay increases exponentially, which indicates a significant performance degradation of the relay node. Therefore, if the relay node is already under heavy traffic load, the chance of its participating in user relaying should be reduced so that the overall network performance can be balanced.

To evaluate the performance of the proposed relay selection method, we consider a single source-destination network

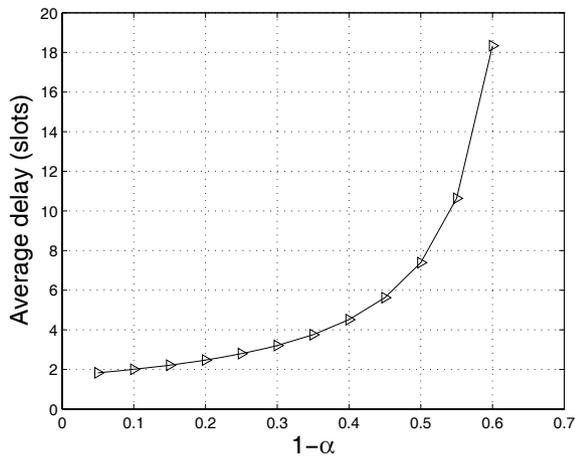


Fig. 4. Average transmission delay in Case 3.

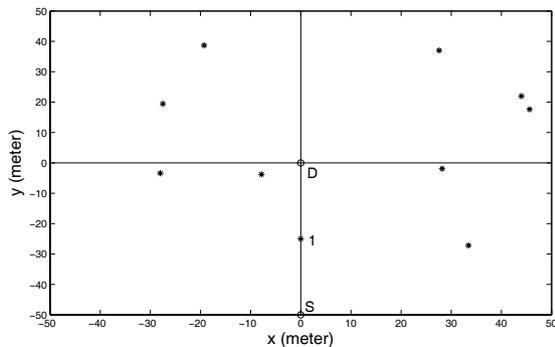
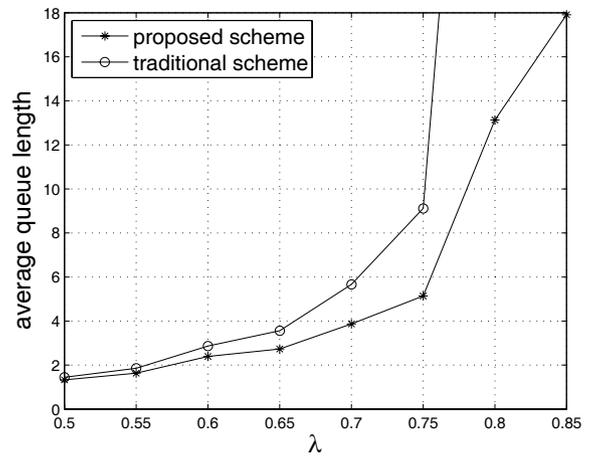


Fig. 5. Network topology for simulation.

covering a 50-meter square area, as shown in Fig. 5. The coordinates of the source node and the destination node are $(0, -50)$ and $(0, 0)$, respectively. There are 10 available relay nodes. Node 1 is located at the middle point between the source and the destination, while all other nodes are uniformly distributed in the covered area. One example of the relay node locations are shown in Fig. 5 as stars. Obviously, node 1 has minimum propagation loss to both source and destination nodes so that it has the highest probability of being selected as the relay node. The channel gain between any pair of nodes include propagation path loss with exponent equal to 2 and Rayleigh fading with unit variance. Each relay node maintains its own buffer, and the packet arrival obeys the Poisson distribution with previously defined average arrival rate λ . We use Case 3 as an example. For comparison, the relay node selection algorithm based on physical layer parameters, called the traditional scheme, is also simulated. For the traditional scheme, the selection criterion is based on a maximization of the utility function

$$U' = \beta_{sr}\beta_{rd}. \quad (36)$$

Fig. 6 shows the average queue length of node 1 with respect to two selection schemes. Obviously, with the increment of the arrival rate λ , both curves increase correspondingly. The figure clearly indicates the performance improvement of the proposed scheme over the traditional scheme, especially

Fig. 6. Average queue length with respect to arrival rate λ .

when λ is large or the traffic load is heavy at node 1. Since the traffic status is not considered in the traditional scheme, the average queue length increases dramatically and node 1 becomes unstable when λ is larger than 0.75 in the simulation. Compared to the traditional scheme, since the relay requests are distributed among multiple good nodes, the proposed one exhibits a gradual increase in average queue length and provides a wider arrival rate range for stability (λ can reach 0.85 before becoming unstable.)

VI. CONCLUSIONS

In this paper, the effects of participating in user relaying on the performance of the relay node has been studied by modelling the behavior of the relay node as an M/G/1-type Markov chain. In order to simplify the calculation, the original M/G/1-type Markov chain is represented as a QBD process and matrix-analytic methods are applied. Analytical results indicate that participating in user relaying may significantly degrade the transmission performance of the relay node especially when the relay node has been already under heavy traffic load. As an application of the analytical results, a utility function based relay node selection scheme is proposed, adhering to the cross-layer design principle. It is demonstrated that the proposed relay node selection scheme can significantly improve the network stability. Our future work will include performance analysis by considering the arrival process of the source node, the optimality of the relay node selection, and the design of relay node selection algorithms for multi-hop networks.

APPENDIX A

DERIVATION OF BLOCK ELEMENTS OF MARKOV CHAIN IN CASE 1

Let

$$A_k = \begin{bmatrix} \phi_{11}^k & \phi_{12}^k \\ \phi_{21}^k & \phi_{22}^k \end{bmatrix} \text{ and } C_k = \begin{bmatrix} \varphi_{11}^k & \varphi_{12}^k \\ \varphi_{21}^k & \varphi_{22}^k \end{bmatrix}.$$

Since two slots happen iteratively, we must have

$$\phi_{11}^k = \phi_{22}^k = 0, \quad (37)$$

$$\varphi_{11}^k = \varphi_{22}^k = 0. \quad (38)$$

If $k = 0$, the number of packets in the system reduces by one after one slot. Since the relay node can only transmit its own packets at time slot $(n, 2)$, we have

$$\phi_{21}^0 = 0 \quad (39)$$

$$\phi_{12}^0 = \alpha a_0. \quad (40)$$

Equation (40) is the probability of the event that one transmission happens in time slot $(n, 2)$ and there are no arrivals.

If $1 \leq k \leq K$, after one slot, the number of packets in the system increases by $k - 1$. It means there are $k - 1$ arrivals at time slot $(n, 1)$, or at time slot $(n, 2)$, there are k arrivals after one transmission or $k - 1$ arrivals without transmission. Therefore, we have

$$\phi_{12}^k = \alpha a_k + (1 - \alpha)a_{k-1} \quad (41)$$

$$\phi_{21}^k = a_{k-1}. \quad (42)$$

If $k = K + 1$, since the maximum number of arrivals is K , we have

$$\phi_{12}^k = (1 - \alpha)a_K \quad (43)$$

$$\phi_{21}^k = a_K. \quad (44)$$

C_k represents the event that the number of packets in the system increases by k from zero. Since there is no packet at the beginning of the slot, no transmission happens and the packet number increment results from the packet arrivals only. Then,

$$\varphi_{12}^k = a_k \quad (45)$$

$$\varphi_{21}^k = a_k. \quad (46)$$

In summary, the block elements of Markov chain in Case 1 can be represented by (23).

APPENDIX B

ITERATIVE ALGORITHM FOR CALCULATING $[\mathbf{x}_0, \mathbf{x}_1]$

Let $Z = [\mathbf{x}_0, \mathbf{x}_1]$. According to (15), we have

$$Z = ZB[R].$$

Notice that $B[R]$ is stochastic, i.e.,

$$B[R] \cdot \mathbf{1} = \mathbf{1}$$

where $\mathbf{1}$ denotes a column vector of ones with suitable dimension, and the matrix Z satisfies

$$Z \cdot \mathbf{1} = \mathbf{1}. \quad (47)$$

Based on (47), we can have the following simple iterative algorithm for calculating Z :

- Let ϵ be a very small positive value, for example, $\epsilon = 10^{-12}$.
- At the initial stage, let $Z^0 = [1, 0, 0, \dots, 0]$.
- At any step k , update

$$Z^{k+1} = Z^k \cdot B[R].$$

In order to avoid machine precision problem, at each step k , Z^{k+1} is normalized by

$$Z^{k+1} = Z^{k+1} / (Z^{k+1} \cdot \mathbf{1}).$$

- Repeat the previous step until

$$|Z^{k+1} - Z^k|_j < \epsilon$$

for any component j of the vector. Then, $Z^* = [\mathbf{x}_0^*, \mathbf{x}_1^*] = Z^{k+1}$ is the solution of (15).

- Since Z must satisfy the normalization condition in (16), the final solution of $[\mathbf{x}_0, \mathbf{x}_1]$ is

$$[\mathbf{x}_0, \mathbf{x}_1] = \frac{[\mathbf{x}_0^*, \mathbf{x}_1^*]}{\mathbf{x}_0^* \mathbf{1} + \mathbf{x}_1^* (I - R)^{-1} \mathbf{1}}. \quad (48)$$

APPENDIX C

ITERATIVE ALGORITHM FOR CALCULATING MATRIX G_i

Let $G_i, i = 1, 2, \dots, K$, be a square matrix with appropriate dimensionality. Therefore, G_i has 2×2 and 4×4 dimensions for Case 2 and Case 3, respectively. The iterative algorithm has the following calculation procedure for G_i :

- At initial stage, let

$$G_i^0 = 0, \quad i = 1, 2, \dots, K$$

where the superscript denotes the index of iteration.

- At any step $k + 1$, calculate

$$G_1^{k+1} = A_0 + \sum_{i=1}^K A_i G_i^k + A_{K+1} G_1^k G_K^k$$

$$G_j^{k+1} = \sum_{i=0}^{K-j+1} A_i G_{i+j-1}^k + \sum_{i=0}^{j-1} A_{K-j+i+2} G_{i+1}^k G_K^k, \\ j = 2, \dots, K.$$

- The sequence G_i^{k+1} converges to $G_i, i = 1, 2, \dots, K$.

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