Reliability Optimization of Distributed Access Networks With Constrained Total Cost

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***Abstract*—In this paper, we study the system reliability optimization of distributed access networks subject to a constraint on the total cost. We first formulate the cost-constrained system reliability optimization problem as a searching process in a combinatorial tree, which enumerates all the possible solutions to the problem. Because the calculation of each possible solution for the reliability problem is extremely time-consuming, a novel algorithm, the Shrinking & Searching Algorithm (SSA), is proposed to speed up the searching process. SSA jointly considers the upper bound of the system reliability for each branch in the combinatorial tree, and the cost constraint on the possible solutions. It avoids most of the redundant calculations in the searching process by gradually *shrinking* the difference between lower & upper bounds of the length of a path in the corresponding combinatorial tree, which represents a feasible solution. Case study & simulation results are presented to demonstrate the performance of the SSA.**

***Index Terms*—Branch and bound, distributed access networks, optimization, system reliability.**

ACRONYMS[1]

AR	Access router
TS	Transceiver set
POP	Point-of-presence (location of an access point to the internet)
Mux	Multiplexer
Sys-	System

NOTATION

N_i	Node set in the i-th access network with $\lvert N_i\rvert$ nodes, where $\lvert N_i\rvert$ is the number of nodes defined in set N_i
L_i	Link set in the i-th access network with $\lvert L_i\rvert$ links, where $\lvert L_i\rvert$ is the number of links defined in intra-link set L_i
p, q	Link reliability, and unreliability, respectively, for all links: $p + q = 1$
$G_i(N_i, L_i, p)$	The i-th access network with node set N_i, intra-link set L_i, and link probability p $(i = 1, 2, \ldots, m)$
$m, n, \bar{l}$	m is the number of access networks, $n = \sum_{i=1}^{m} \lvert N_i\rvert$, $\bar{l} = \sum_{i=1}^{m} \lvert L_i\rvert$
$E_{i,j}$	The set of candidate links between G_i & G_j
E	The set of candidate links available to connect all the access networks, i.e., $E = \bigcup_{i=1}^{m-1} \bigcup_{j=i+1}^{m} E_{i,j}$ & $E = \{e_1, e_2, \ldots, e_{\lvert E\rvert}\}$, where $e_1, e_2, \ldots, e_{\lvert E\rvert}$ are in an ascending order in terms of their cost
G	Network with node set $N = \bigcup_{i=1}^{m} N_i$, link set $L = \bigcup_{i=1}^{m} L_i$, and some candidate links in E
$e_{i,j}$	The link connecting nodes i & j in G
$c_{i,j}, c(e)$	The cost of $e_{i,j}$ or link e; $c_{i,j} = 0$ if link $e_{i,j}$ is not in E
$R(G)$	Sys-reliability of G
$H(G)$	The upper bound of sys-reliability of G
$\Omega(n, k)$	A class of networks with n nodes & k links
$h(k)$	The upper bound of sys-reliability for $\Omega(n, k)$
$x_{i,j}$	A binary digit indicating whether nodes in network G are connected or not; $x_{i,j} = 1$ if nodes i & j are connected, otherwise $x_{i,j} = 0$
x	The node connection vector of $x_{i,j}$, $x = \{x_{1,2}, x_{1,3}, \ldots, x_{1,n}, x_{2,3}, x_{2,4}, \ldots, x_{n-1,n}\}$
$c(x)$	The cost function for vector x, $c(x) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{i,j} x_{i,j}$
$f(x)$	The value of sys-reliability with an interconnection pattern x
$g(v)$	The minimum cost of all l-hop paths passing v in the combinatorial tree

NOMENCLATURE

- A complete path: A path of a combinatorial tree from the root to a leaf node, or to an inspected node.
- A node group: A group of nodes representing an access network $G_i(N_i, L_i, p)$. It is assumed that the interconnection has been defined in each access network, or the links in each access network have been determined.

I. INTRODUCTION

THE INTERCONNECTION of distributed access networks with the core network plays an important role in the infrastructure development of the next-generation Internet & Metropolitan Area Networks (MAN), to ensure the system-level reliability & versatile Quality-of-Service (QoS) requirements by different commercial applications [1], [2]. Due to heterogeneous characteristics of service-oriented networks, joint consideration of multiple objectives & constraints in the development of the network infrastructure is necessary for

Manuscript received April 6, 2004; revised November 21, 2004. This work was supported by a research grant from the Natural Science and Engineering Research Council (NSERC) of Canada. Associate Editor M. J. Zuo.

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Digital Object Identifier 10.1109/TR.2005.853440

[1]The singular and plural of an acronym are always spelled the same.

achieving cost-effectiveness & capacity-efficiency. In general, a core network interconnects a group of distributed access networks, each composed of multiple bandwidth add-drop units, such as wireless base stations, campuses, private sectors, and government buildings. One of the nodes in an access network serves as the gateway that aggregates the traffic from the nodes in the access network, and distributes the traffic from the Internet to the nodes in the access network.

Because the network components (which are limited to nodes & links in the following context) are not always reliable [1], [3]–[6], *system reliability* (*sys-reliability*) on network connectivity serves as an important index for evaluating the survivability of the network infrastructure exposed to component failures. Sys-reliability is defined as the probability that each pair of nodes in the network topology can be connected by at least one physical path (or termed one-connected) subject to a failure event, which is largely determined by the network topology & link reliability (or the probability that a link functions correctly) [10]. Because modern communication networks must support multi-class services & applications with various QoS requirements, network survivability in terms of topology reliability becomes an interesting design issue. Different from the research regarding spare capacity allocation in the service layer, where the network topology is assumed to be fixed, the deployment of the network topology is envisioned to address fundamental influences upon the resultant network survivability in the service layer. Thus, this paper tackles the problem of optimizing sys-reliability in the deployment of network infrastructure, with a constraint on the total cost.

Approaches of improving sys-reliability in network topology design have been extensively reported in the past a few years [8]–[19], and can be generally divided into two categories. The first category focuses on minimizing the total link cost subject to a constraint on the sys-reliability. By using a Branch & Bound algorithm, the optimization problem can be decomposed into multiple sub-problems for an efficient search of the exact optimal solution [12]. Similar to the study in [12], the problem of network topology optimization & network expansion with a reliability constraint is solved by [16]. In the second category, sys-reliability is maximized while a constraint on the total cost is imposed. A genetic algorithm-based scheme is thus introduced to find an optimal solution [10], [15]. However, the parameters taken in the generic algorithm-based approach for approximating the optimal solutions are difficult to determine, which can dramatically slow down the solving process.

Different from the previous studies, this paper solves the cost-constrained sys-reliability optimization problem for distributed network topology deployment, where the gateway assignment for each distributed access network is jointly taken into account. The problem is considered more practical, yet more complicated, than that of the reported studies. Obviously, in the case that every access network contains a single node serving as the gateway, the problem is degraded to the same as that in [10].

To reduce the design space, a group of candidate links connecting between gateways is pre-defined, and the constrained optimization problem becomes to select some links from the candidate links to form the network topology, such that the sys-reliability is maximized while the total cost of the selected links is constrained. The constrained optimization problem is first formulated as a searching Because the calculation of each possible solution for sys-reliability is extremely time-consuming, a novel algorithm, called Shrinking & Searching Algorithm (SSA), is proposed to speed up the searching process. SSA jointly considers the upper bound on sys-reliability for a branch in the combinatorial tree, and the cost constraint on some possible solutions. It avoids most otherwise-redundant calculation in the searching process by *shrinking* the difference between the lower & upper bounds of the length of a path, which represents a feasible solution in the combinatorial tree. Case study & simulation results are given to demonstrate the effectiveness & efficiency of the SSA.

The rest of the paper is organized as follows. In Section II, the cost-constrained sys-reliability optimization problem is formulated as a searching process in combinatorial trees. Section III presents the proposed SSA for searching for the optimal solution. Case study & simulation results are given in Section IV, followed by the conclusions in Section V.

II. Problem Formulation

In this section, assumptions are first introduced; then the targeted constrained optimization problem is formulated.

Assumptions:

Assumption 1: for an access network, the links in the intra-link set L_i $(i = 1, 2, 3, \ldots, m)$ & the links in the candidate inter-link set $E_{i,j}$ between different node groups are bi-directional, and the probability of failure for any link in an access network is fixed.

Assumption 2: the cost of a candidate link is known, and a candidate link only exists between two nodes that come from different node groups.

With the two assumptions, the sys-reliability optimization problem can be formulated as m distributed access networks (node groups) interconnected such that the sys-reliability is maximized with the total cost (denoted as C_0) being bounded.

$$R(G) = \max_x \big(f(x)\big)$$
$$\text{subject to} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{i,j} x_{i,j} \le C_0 \tag{1}$$

where x has $2^{|E|}$ possible values. In general, with m node groups, at least $m-1$ links are required to interconnect them, although this may not be a good solution because the resultant sys-reliability is very small. The sys-reliability can be certainly improved by adding more links to the node groups; however, the cost may be out of bound. Our objective is to explore the topology with the maximum sys-reliability subject to the total cost constrained in the interconnection of the distributed node groups. To enumerate all possible solutions regardless of the cost constraint, the cases with i links are considered, where $i = m-1, m, \ldots, |E|$, and $|E|$ is the number of total candidate links. A combinatorial tree represents all the possible patterns of interconnection under a specific value of i, in which most complete paths from the root to a leaf in the combinatorial tree with a depth i stands for a possible solution with i candidate links in the developed topology. Because computing the

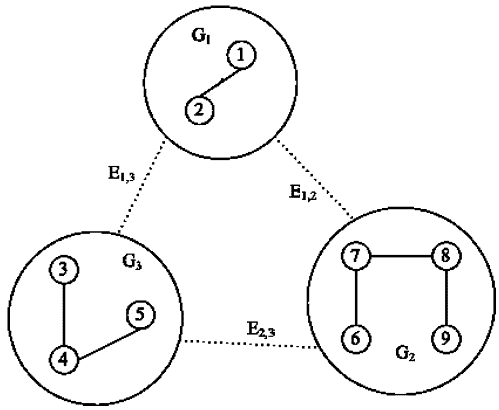


Fig. 1. Interconnection of three node groups.

sys-reliability $f(x)$ in (1) is an NP-complete problem, and is very time-consuming [7], it is critical to avoid any unnecessary inspection of the complete paths in the combinatorial trees. The design objective can be further illustrated by the following example. Fig. 1 shows three node groups $(m = 3)$, G_1, G_2, and G_3.

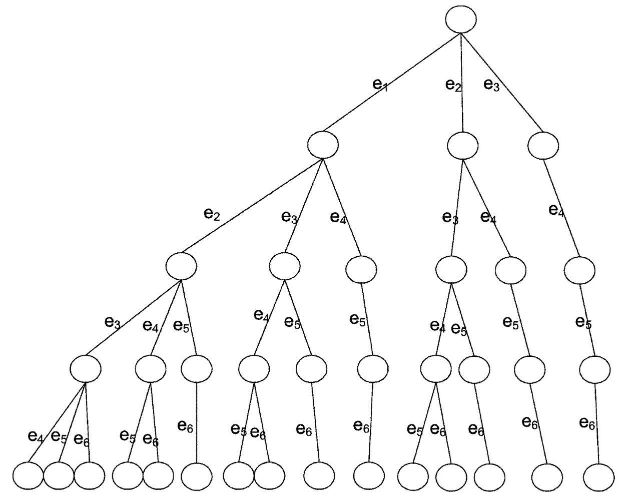
Suppose $E_{1,2} = \{e_{1,6}, e_{2,9}\}$, $E_{1,3} = \{e_{1,3}, e_{2,5}\}$, $E_{2,3} = \{e_{4,7}, e_{5,6}\}$; and the cost of links $c_{1,6} = 2$, $c_{2,9} = 4$, $c_{1,3} = 3$, $c_{2,5} = 6$, $c_{4,7} = 5$, $c_{5,6} = 4$. The links can be arranged according to their cost $c_{i,j}$ in an ascending order: $\mathrm{e}_1 = e_{1,6}$, $\mathrm{e}_2 = e_{1,3}$, $\mathrm{e}_3 = e_{2,9}$, $e_4 = e_{5,6}$, $e_5 = e_{4,7}$, $e_6 = e_{2,5}$. Let $E = \{\mathrm{e}_1, \mathrm{e}_2, \ldots, \mathrm{e}_6\} = \bigcup_{i=1}^{2} \bigcup_{j=i+1}^{3} E_{i,j}$, $C_0 = 14$. The objective is to connect the three node groups such that the sys-reliability is maximized, subject to the cost constraint C_0. The combinatorial trees representing all the possible interconnections with $i = 4$, 3, & 2 for the network topology, respectively, are shown in Fig. 2(a), (b), (c), respectively, where a complete path from the root to any leaf represents a solution. Note that the sum of the cost along a complete path is the total cost required by the corresponding interconnection pattern. In Fig. 1, if each node group G_{i} has only one node, the cost-constrained sys-reliability optimization problem is degraded to the one studied in [10], which can be solved by using the proposed algorithm.

Intuitively, a *Brute Force* algorithm [7] can be applied to inspect every complete path in each of the combinatorial trees with a possible value of i, where $i = m-1, m, \ldots, |E|$ according to the optimization objective & pre-defined constraint. However, the Brute Force method is far from efficient because it may be unnecessary to inspect some complete paths in a combinatorial tree, or the whole combinatorial tree. The proposed algorithm is a novel approach of saving those unnecessary inspections during the searching process by imposing upper & lower bounds on the length of the feasible complete paths (which represents the sys-reliability) along with the cost constraint.

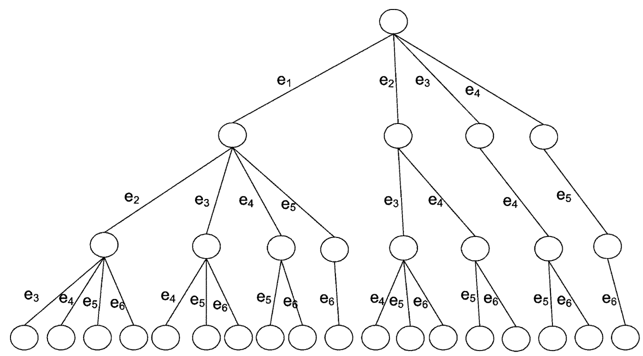
The upper bound of the length of a complete path in a combinatorial tree is l' which satisfies

$$\sum_{j=1}^{l'} c(e_j) < C_0, \quad \sum_{j=1}^{l'+1} c(e_j) > C_0. \tag{2}$$

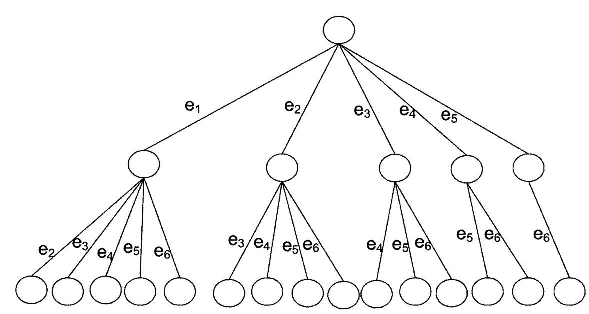
On the other hand, the lower bound of the length of a complete path (denoted by l^*) should be $m-1$, in order for all the node



(a)



(b)



(c)

Fig. 2. The combinatorial tree representing all the possible interconnections with (a) $i = 4$, (b) $i = 3$, and (c) $i = 2$, respectively.

groups to be interconnected. Thus, we need to only consider the combinatorial trees with a depth i, where $m-1 = l^* \leq i \leq l'$. In Fig. 1, the lower, and upper bounds are $l^* = 2$, and $l' = 4$, respectively; and the corresponding combinatorial trees are shown in Fig. 2(a), 2(b), and 2(c) with $i = 2$, 3, and 4, respectively. The total number of leaf nodes in Fig. 2 is $\binom{6}{4} + \binom{6}{3} + \binom{6}{2} = 50$. To investigate the redundancy between trees with different depths, it is noticed that a complete path from a combinatorial tree with a larger depth may contain a complete path from a combinatorial tree with a smaller depth. For instance, the complete path $\mathrm{e}_1 - \mathrm{e}_2 - \mathrm{e}_3 - \mathrm{e}_5$ in Fig. 2(a) contains a path $\mathrm{e}_1 - \mathrm{e}_2 - \mathrm{e}_5$, which is a complete path in Fig. 2(b). If $\mathrm{e}_1 - \mathrm{e}_2 - \mathrm{e}_3 - \mathrm{e}_5$ yields a feasible solution, the sys-reliability of a network connected by $\mathrm{e}_1 - \mathrm{e}_2 - \mathrm{e}_5$ is not necessary to be calculated because the former

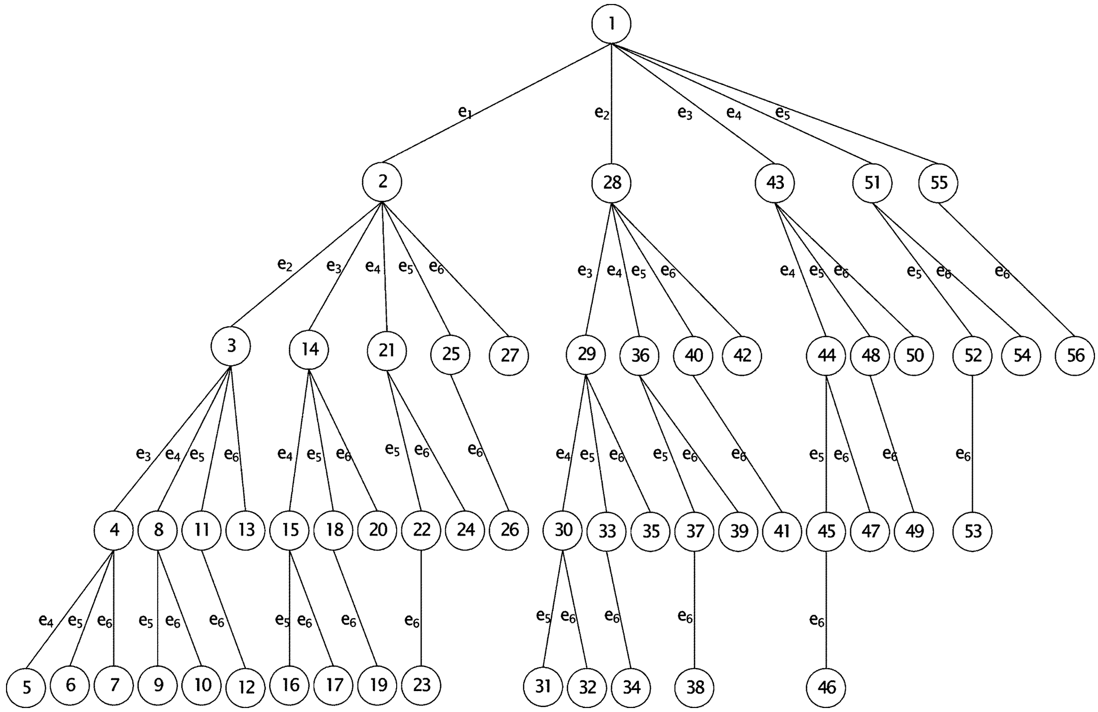


Fig. 3. The summarized combinatorial tree for the three trees in Fig. 2.

network path gives a better reliability. Therefore, we can construct a new combinatorial tree that combines all the complete paths from the combinatorial trees with depth limitation l^* & l'. The newly constructed combinatorial tree is named *Summarized Combinatorial Tree*. Fig. 3 shows the Summarized Combinatorial Tree from the three combinatorial trees in Fig. 2.

Although each complete path from the newly constructed combinatorial tree may yield a feasible solution, it is still not necessary to calculate the sys-reliability for all the complete paths to obtain the optimal solution. In order to further reduce the redundancy in the searching process, an efficient, effective algorithm named the Shrinking & Searching Algorithm (SSA) is developed for searching the optimal solution. SSA jointly considers the upper bound of sys-reliability for a branch in the summarized combinatorial tree along with the cost constraint. It avoids most of the redundant calculation in the searching process by *shrinking* the difference between the lower & upper bounds of the length of a path which represents a feasible solution in the summarized combinatorial tree.

III. Shrinking and Searching Algorithm

To reduce the computation complexity, the upper bound on the sys-reliability, and the cost constraint along with the predefined cost function, can be used to remove those unnecessary branches.

A. The Upper Bound of Sys-Reliability

The upper bound of sys-reliability can be applied toward the following two goals: (1) to avoid calculating sys-reliability for some unnecessary complete paths in the summarized combinatorial tree; and (2) to determine the length of a complete path.

TABLE I
The Upper Bound of the Sys Reliability for a Class of Networks

L	Upper bound of the sys-reliability for $\Omega(n, k)$
$k = n\text{-}1$	p^{n-1}
$k = n$	$p^{n-1}(1+(n-1)q)$
$k = n+1$	$p^{n-1}(1+(n-1)q+(\frac{n^2-n+1}{3})q^2)$
$k > n+1$	$1-\sum_{j=1}^{n} q^{d_j} \prod_{k=1}^{m_j}(1-q^{d_k-1}) \prod_{k=m_j+1}^{j-1}(1-q^{d_k})$

This can be attributed as the *degree sequence* method in [13], and is composed of the following two parts.

Part 1: The upper bound of the sys-reliability for a class of networks $\Omega(n,k)$ is denoted as a function of $h(k)$ for $k = n - 1, \mathrm{n}, \ldots, n(n-1)/2$, which is listed in Table I.

In Table I, $m_j = min(d_j, j-1)$, where d_i is the degree of node i, representing the number of incident links to node i. $d = (d_1, d_2, \ldots, d_n)$, $d_1 \leq d_2 \leq \ldots \leq d_n$, is the degree sequence, which must satisfy $|d_i - d_j| \leq 1$ $(i \neq j)$.

Part 2: The upper bound of sys-reliability $H(G)$ for network topology G can be determined by referring Table I. For $k > n+1$, it is not necessary that $|d_\mathrm{i} - d_\mathrm{j}| \leq 1$ $(i \neq j)$.

B. Two Cost Functions

The cost function $c(x)$ is defined as the cost of a complete path

$$c(x) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} c_{i,j} x_{i,j} = \sum_{i=1}^{l} c(e_{j_i}) \tag{3}$$

where $c(e_{j_i})$ is the cost of e_{j_i}, and l is the number of links along the complete paths in the summarized combinatorial tree. The $\{e_{j_i}\}_{i=1}^{l}$ are links in the complete path from the root to node v.

An additional cost function $g(v)$ is defined to evaluate the least cost of a l-hop branch from the root passing node v to any leaf node in a group of branches in the summarized combinatorial tree. Thus, to calculate the entire cost of a complete path with l hops passing v, we have

$$\begin{aligned} g(v) &= \min\left\{\sum_{i=1}^{l} c\left(e_{j_i}\right) + \sum_{e\in U} c(e)\ for\ all\ U\right\} \\ &= \sum_{i=1}^{l} c\left(e_{j_i}\right) + \sum_{i=1}^{l^*-l} c\left(e_{j_k+i}\right) \end{aligned} \quad (4)$$

where l^* is the lower bound of the length of feasible complete path, and U is the set of $l^* - l$ links chosen from the remaining link set $\{\{e_{j_i}\}_{i=j_k+1}^{|E|}$. The selection of e_{j_i} is from the new order of $e_i (i = 1, 2, \ldots, |E|)$. e_{j_k+i} is the i-th link after the j_k-th link in the new order of e_i. $g(v)$ can be used to inspect node v in the summarized combinatorial tree whether the complete path traversing through v is feasible or not. If $g(v) > C_0$, the branches traversing through v are certainly not feasible solutions. For instance, in Fig. 3, $l^* = 4$, and the l^*-length branch traversing through node 28 with a minimum cost is $e_2 - e_4 - e_5 - e_6$. If $g(28) > C_0$, any other complete path traversing through node 28 can never be a feasible solution.

C. The Upper Bound and Lower Bound of the Feasible Complete Paths

The initial lower bound of the length of the feasible complete paths is $m - 1$. It is updated when the reliability $f(x)$ is given. This lower bound can also be determined by $h(i)$. For $h(k-1) < f(x) \le h(k)$, the lower bound on the length of feasible complete paths is $l^* = k - \bar{l}$ because the sys-reliability of any network with $l^* - 1$ selected links is less than $h(\bar{l} + l^* - 1)$, where $h(\bar{l} + l^* - 1) < f(x)$. In other words, it is impossible to find the feasible solution with a larger value of sys-reliability than $f(x)$ in the summarized combinatorial tree whose branch length is equal to or less than $l^* - 1$. The upper bound of the length of the feasible complete paths is l' iff the inequality in (2) holds.

Because the length of the optimal solution must be between the lower bound (l^*) & the upper bound (l') of the length of the feasible complete paths, the upper bound defines the size of the summarized combinatorial tree, while the lower bound is used to exclude unnecessary inspections upon nodes in the summarized combinatorial tree. Hence, the larger the lower bound is, the faster the searching procedure can be. In addition, it is necessary to count the number of links selected to a branch, and this can be denoted as

$$l(x) = \sum_{i,j} x_{i,j} - \sum_{i=1}^{m} |L_i|. \quad (5)$$

The method of obtaining the updated l^* is illustrated in Fig. 4. It is obvious that l^* grows as $R = f(x)$ increases.

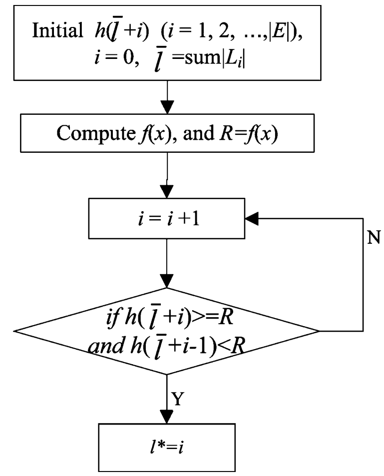


Fig. 4. A flowchart demonstrating the derivation of the updated lower bound l^*.

TABLE II
RULES OF THE PROPOSED ALGORITHM

Rule No.	The rules of the proposed algorithm
1	$R(G\backslash\{e\}) < R(G)$ if removing link e from network G
2	$R(G_1) < R(G)$ if $H(G_1) < R(G)$
3	The branch length of the feasible solutions must be $l^* \le l(x) \le l'$
4	All the branches traversing through v are infeasible solutions if $g(v) > C_0$

D. The Feasible Solutions

Based on the above discussions, the following rules given in Table II are concluded to avoid inspecting most unnecessary complete paths in the summarized combinatorial tree.

According to rule 1, the sys-reliability of network G with link e being removed is less than that of the original network G. If a branch is a feasible solution, the sys-reliability of any complete path consisting of the branch is smaller than that of this branch. In rule 2, the upper bound should be calculated before calculating the sys-reliability of G. If $H(G_1) < R(G)$ holds, $R(G_1)$ does not need to be computed again. Rule 3 can also be applied to the construction of the summarized combinatorial tree. Rule 4 can be used to inspect each node in a combinatorial tree. The cost function $c(x)$ for every node v corresponding to x in a combinatorial tree is an important parameter for inspecting whether the branch corresponding to x is feasible or not.

To summarize the rules listed above, a feasible solution must satisfy the following three conditions, which are taken as the criteria for constructing & searching in the summarized combinatorial tree: 1) $l' \ge l(x) \ge l^*$; 2) $c(x) \le C_0$, and 3) $g(v) \le C_0$.

E. Shrinking & Searching Algorithm

An efficient algorithm, the Shrinking & Searching algorithm, is shown in Fig. 5, and solves the optimization problem, in which the rules described in Section III-D are jointly applied. A flowchart for the algorithm is given in Fig. 9.

The computation of sys-reliability for a network topology has been proven to be NP-complete [7]. In the searching process of SSA, although a suite of decent approaches have been developed

```
LET K = φ;    %live-node list, K, is a priority queues storing live nodes%
Put root node v in the live-node list, K
Let g(v) = 0, R = 0, l*= m – 1; Compute the upper bound l'; Compute h(i + l̄ ), i = m-1 to |E|
While |K| ≠ 0, do the loop
BEGIN   % Filtering feasible solutions in K %
 Choose node v from K
  IF l(x) ≤ l' and g(v) ≤ C0, THEN
  Put the first child v1, next brother of v in K;
      BEGIN   % Testing nodes whether it is feasible or not %
          IF l(x) > l*, c(x) <= C0, and c(x1) > C0, THEN
              Compute the H(G) corresponding to x
                  IF H(G) > R, THEN
                   Compute f(x)
                      IF f(x) > R, THEN
                       R = f(x)
                          Begin   % Determine the lower bound l* %
                            FOR i = 1 to |E|
                             IF h(l̄ + i) ≥ R, and h(l̄ + i - 1) < R, THEN
                           l* = i, remove node v from the live-node list K
                               END IF
                           END FOR
                          END
                      ELSE
                      Remove node v from the live-node list K, and return to the loop
                      END IF
                  ELSE
                  Remove node v from the live-node list K, and return to the loop
                  END IF
          ELSE
          Remove node v from the live-node list K, and return to the loop
          END IF
      END
  ELSE
  Remove node v from the live-node list K, and return to the loop
  END IF
END
Output the optimal value R, and x
```

Fig. 5. Shrinking & searching algorithm.

TABLE III
COSTS FOR LINK $E_{i,j}$, $i = 1, 2$; $j = 2, 3$

	$E_{1,2}$		$E_{1,3}$		$E_{2,3}$	
$e_{i,j}$	(1,6)	(2,9)	(1,3)	(2,5)	(4,7)	(5,6)
$c_{i,j}$	2	4	3	6	5	4

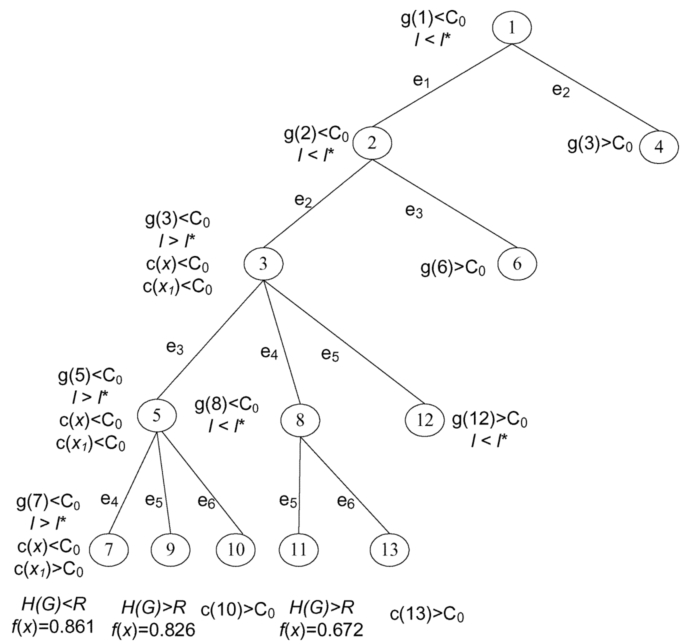


Fig. 6. Searching steps for optimal solution in Case 1.

by manipulating the upper/lower bounds on the length of complete paths in the summarized combinatorial tree so as to reduce the computation of sys-reliability as much as possible, it is impossible to completely avoid all of them. Therefore, solving the problem with SSA is undoubtedly NP-hard. In the following, we will show that the SSA contributes by providing a great capability in reducing the calculation of sys-reliability, and the resultant computation time, which will be demonstrated in the next section through two case studies & simulations.

IV. SIMULATION RESULTS

In this section, two case studies & simulation results are given to demonstrate the effectiveness of the proposed SSA. The study of Case 1 mentioned in Section II gives a complete process of searching optimal solution by using SSA. The study of Case 2 obtains an optimal solution of a practical problem for interconnecting three access networks with partially mesh topologies. Simulation results show how the link reliability affects the upper bound of the network reliability, and are summarized in Table V.

Case 1: The network topology G is the same as that in Fig. 1.

Suppose the probability of each link $p = 0.9$, and the cost threshold $C_0 = 14$.

$$G_1: \quad N_1 = \{1,2\}; \quad L_1 = \{(1,2)\}$$
$$G_2: \quad N_2 = \{6,7,8,9\}; \quad L_2 = \{(6,7),(7,8),(8,9)\}$$
$$G_3: \quad N_3 = \{3,4,5\}; \quad L_3 = \{(3,4),(4,5)\}.$$

The cost for each link $E_{i,j}$ is provided in Table III.

The constrained optimization problem is defined as:

$$R(G) = \max_x \left(f(x)\right)$$
$$\text{Subject to} \sum_{i=1}^{8} \sum_{j=i+1}^{9} c_{i,j} x_{i,j} \le C_0$$

The computing procedure is as follows:

1. $K = \phi$;
2. $g(x) = 1$, $R = 0$, $l^* = m - 1 = 2$, $l' = 4$, $h(2) = 0.4305$, $h(3) = 0.7748$, $h(4) = 0.8796$;
3. Put node 1 of the combinatorial tree (see Fig. 3) into K;
4. $g(1) = 5 < C_0$, put node 2 into K;
5. $l < l^* = 2$, remove node 1;
6. $g(2) = 5 < C_0$, put nodes 3 & 4 into K;
7. $l' > l > l^* = 2$, $c(x) < C_0$, $c(x_1) < C_0$, remove node 2;
8. $g(3) = 5 < C_0$, put nodes 5 & 6 into K;
9. $l' > l > l^* = 2$, $c(x) < C_0$, $c(x_1) < C_0$, remove node 3;
10. $g(5) = 5 < C_0$, put nodes 7 & 8 into K;
11. $l' > l > l^* = 2$, $c(x) < C_0$, $c(x_1) < C_0$, remove node 5;
12. $g(7) = 5 < C_0$, put node 9 into K;
13. $l' = l > l^* = 2$, $c(x) < C_0$, $c(x_1) > C_0$;
14. *Compute* $H(G) = 0.8796$, $f(x) = 0.861$;
15. $R = f(x) = 0.861$, remove node 7;
16. $h(3) < R$, $h(4) > R$, $l^* = 4$;

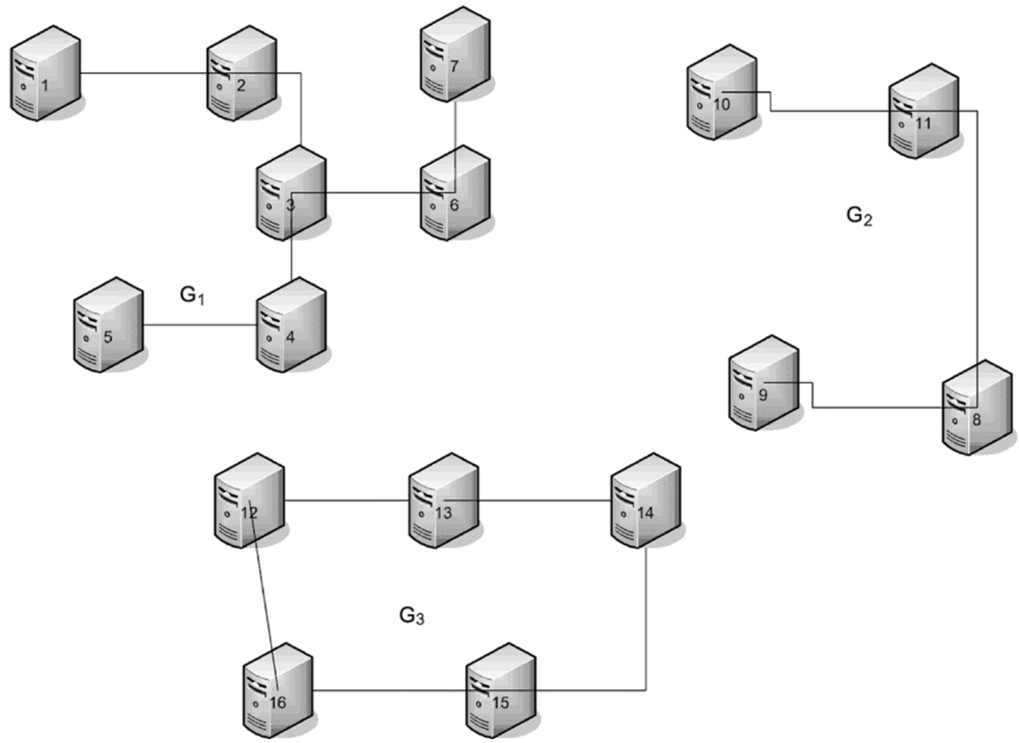


Fig. 7. Three wireless access networks with partial mesh topology.

TABLE IV
THE COSTS OF LINK $E_{\mathrm{i,j}}$, $i = 1, 2$; $j = 2, 3$ (THE UNIT OF COST IS $\$10^5$)

	$E_{1,2}$				$E_{1,3}$				$E_{2,3}$			
$e_{\mathrm{i,j}}$	(1,10)	(4,11)	(5,9)	(7,10)	(1,12)	(5,15)	(7,15)	(5,16)	(9,16)	(10,12)	(11,16)	(9,15)
$c_{\mathrm{i,j}}$	8	6.5	8.3	2.5	5.8	5	7.5	4	3	8.1	6	9

17. $g(4) = 16 > C_0$, $g(6) = 15 > C_0$, remove nodes 4 & 6;
18. $g(8) = 14 <= C_0$, put nodes 11 & 12 into K;
19. $l \leq l^*$, remove node 8;
20. $g(9) = 14 <= C_0$, put node 10 into K;
21. $l' = l > l^*$; $c(x) < C_0$, $c(x_1) > C_0$;
22. *Compute* $H(G) = 0.8796$, $f(x) = 0.826 < R$, remove node 9;
23. $c(10) = 15 > C_0$, remove node 10;
24. $g(11) = 14 <= C_0$, put node 13 into K;
25. $l' = l > l^*$; $c(x) < C_0$, $c(x_1) > C_0$;
26. *Compute* $H(G) = 0.8796$, $f(x) = 0.672 < R$, remove node 11;
27. $g(12) = 16 > C_0$, remove node 12;
28. $c(13) > C_0$, remove node 13;
29. The optimal sys-reliability is $R^* = 0.861$, and selected links are (1,6), (1,3), (5,6), (2,9). And the node connection vector $x = \{x_{1,2}, x_{1,3}, \ldots, x_{1,9}, x_{2,3}, x_{2,4}, \ldots, x_{8,9}\}$ $= \{1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,1,0,0,0,0,1,0,0,0,1,0,0,1,0,1\}$

In the simulation, the reliability $f(x)$ is obtained by executing SSA three times in Fig. 6, and $\binom{6}{4} + \binom{6}{3} + \binom{6}{2} - 3 = 47$ leaf nodes in Fig. 2 are excluded.

Case 2: The network topology G is the same as that in Fig. 7.

$$G_1: \quad N_1 = \{1,2,3,4,5,6,7\}$$
$$L_1 = \{(1,2),(2,3),(3,4),(3,6),(4,5),(6,7)\}$$
$$G_2: \quad N_2 = \{8,9,10,11\}$$
$$L_2 = \{(8,9),(10,11),(11,8)\}$$
$$G_3: \quad N_3 = \{12,13,14,15,16\}$$
$$L_3 = \{(12,13),(13,14),(14,15),(15,16),(16,12)\}$$

Each node is either a group of computers representing users, or a processing node representing a router. A link is deployed either in a point-to-point, or point-to-multipoint manner. The costs of AR-to-AR or AR-to-POP, AR, transceiver, and the 40 Mbps to 2 Mbps Mux are \$50,000, \$100,000, \$10,000, and \$10 000 respectively. To satisfy the design requirements, both the link capacity & cost should be jointly considered. The connection from the AR to the POP has at least a capacity of 40 Mbps. The maximum number of the 40 Mbps Mux, and the AR-to-AR

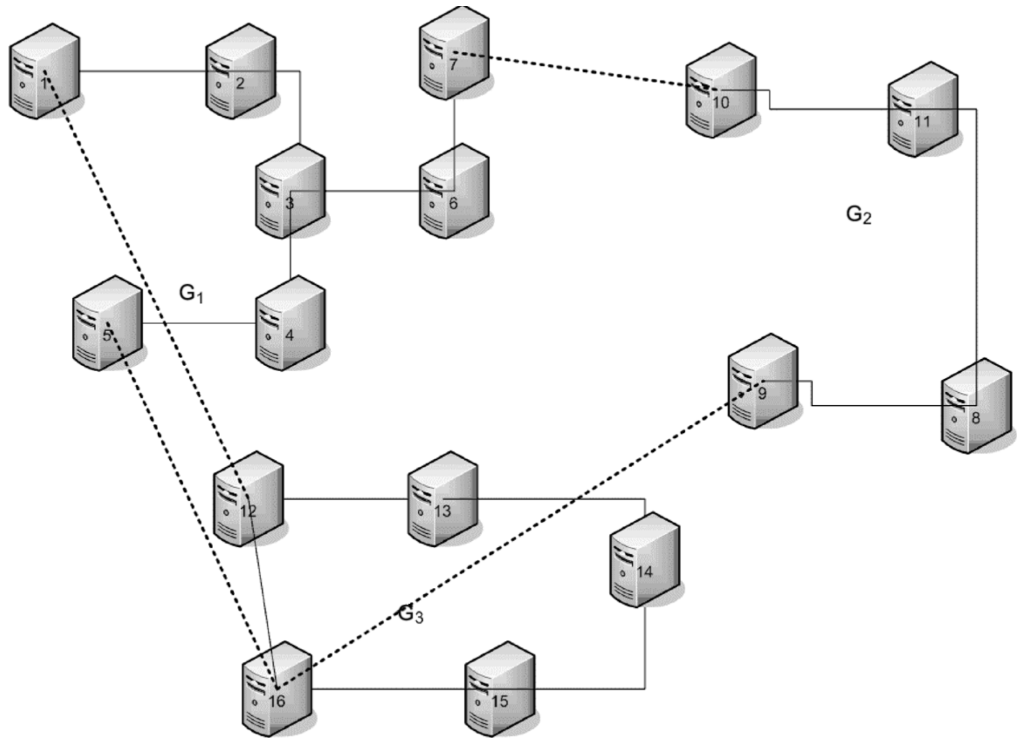


Fig. 8. The interconnection with the maximum reliability for the access networks.

TABLE V
UPPER BOUNDS OF NETWORK RELIABILITY FOR DIFFERENT p

p	$h(\bar{l}+2)$	$h(\bar{l}+3)$	R(G)	T
0.9000000000	0.5147278302	0.68012703978	0.7333842125	4
0.9999000000	0.9999988011	0.9999996032	0.9999995999	55

interfaces are determined by the capacities of the AR-to-AR (or AR-to-POP), and AR (AXI 540) links. The AXI 540 has a throughput of 20 Gbps. The initial cost depends on the topology designed to meet the capacity constraint, and the requirements from the users. Each transceiver can support no more than 100 users. If the Mux serves more than 100 users, an additional TS is required. The maximum number of transceivers on a Mux is determined by the number of users with a 2 Mbps peak rate, each of which multiplexes on a 40 Mbps link. The cost for each link $E_{\mathrm{i,j}}$ is provided in Table IV.

Suppose the reliability of each link $p = 0.9999$, and the cost threshold $C_0 = \$16 \times 10^5$.

The constrained optimization problem is defined as

$$R(G) = \max_x \big(f(x)\big)$$

$$\text{Subject to: } \sum_{i=1}^{15} \sum_{j=i+1}^{16} c_{i,j} x_{i,j} \leq C_0.$$

By SSA, the optimal sys-reliability is $R^* = 0.999\,999\,599\,9$; and the selected links are (7,10), (9,16), (5,16), (1, 12). Fig. 8 shows the resultant interconnection with the maximum reliability for the access networks. And the node connection vector $x = \{x_{1,2}, x_{1,3}, \ldots, x_{1,16}, x_{2,3}, x_{2,4}, \ldots, x_{15,16}\}$

$= \{1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0, 0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0, 0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,1,0, 0,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0, 0,0,0,0,0,1,0,1,1,1,0,0,0,1\}$

Simulation Results: We further study how the link reliability, p, affects the upper bound of network reliability. Table V gives the upper bounds of network reliability in Case 2 for $p = 0.9$, and 0.9999, respectively. In Table V, T is the maximum time unit to compute network reliability $f(x)$, R is the sys-reliability, and $h(\bar{l}+2)$ & $h(\bar{l}+3)$ are the upper bounds of network reliability for $l = 2$ & 3, respectively.

Table VI summarizes simulation results, where some input values, such as the cost of links, are generated randomly. Different values of (m, n, p) in Table IV are considered in simulations so that the effectiveness of SSA can be illustrated. α is the number of sys-reliability computations at most by using SSA. β is the number of generated leaf nodes of Summarized Combinatorial Trees. The ratio α/β gives the efficiency of the proposed algorithm, because it represents the number of the leaf nodes out of all generated leaf nodes which should be checked.

In the simulation, most wired links are assumed to be available with a link reliability of up to 0.9999, which indicates

TABLE VI
COMPUTATION RESULTS FOR OPTIMIZATION PROBLEMS

m	n	$\bar{l}$	$\|E\|$	P	l'	α	β	α/β
3	12	11	8	0.90	4	7	154	0.045455
					5	54	210	0.257143
				0.99	4	35	154	0.227273
					6	88	238	0.369748
			9	0.90	4	32	246	0.130081
					5	63	372	0.169355
				0.99	4	64	246	0.260163
					6	120	456	0.263158
4	13	12	10	0.90	5	84	582	0.14433
					7	155	912	0.169956
				0.99	5	120	582	0.206186
					6	250	792	0.315657
			11	0.90	5	66	957	0.068966
					7	201	1749	0.114923
				0.99	5	234	957	0.244514
					6	432	1419	0.30444
5	15	14	12	0.90	6	553	2211	0.250113
					8	934	3498	0.26701
				0.99	6	942	2211	0.426052
					9	1100	3718	0.295858
			13	0.90	6	1142	3718	0.307154
					8	1534	6721	0.22824
				0.99	6	1521	3718	0.409091
					9	2311	7436	0.310785

outage time less than 53 minutes a year. With a threshold level of 0.9999 on sys-reliability, the total number of leaf nodes are $\binom{12}{2}+\binom{12}{3}+\ldots+\binom{12}{12}=4083$. By using the lower bound l^* & the upper bound l' in the SSA, the maximum number of leaf nodes subject to inspection should be $\binom{12}{2}+\binom{12}{3}+\binom{12}{4}=781$. As $p < 0.9$, by using the updated lower bound l^*, at most $\binom{12}{4}=495$ leaf nodes need to be inspected. Furthermore, by using the cost function, only 4 out of 495 leaf nodes need to be inspected. From the analysis, it can be seen that the optimization problem can be solved efficiently by using the proposed SSA when a proper value of p is taken, and the upper bound is small enough. Table V gives the relationship between the upper bound & sys-reliability for different values of p. As p is 0.9, the upper bound is smaller than the sys-reliability; when $p \geq 0.99$, the upper bound is larger than the sys-reliability, which indicates that more leaf nodes need to be inspected. Some optimization problems can not be solved in short time, if the $|E|$ is much larger than $m+2$, by using SSA in Table VI; and the complexity of topology of G_i & the value of p are important factors to the efficiency of SSA. The computation efficiency can be determined by how small the upper bound can be found.

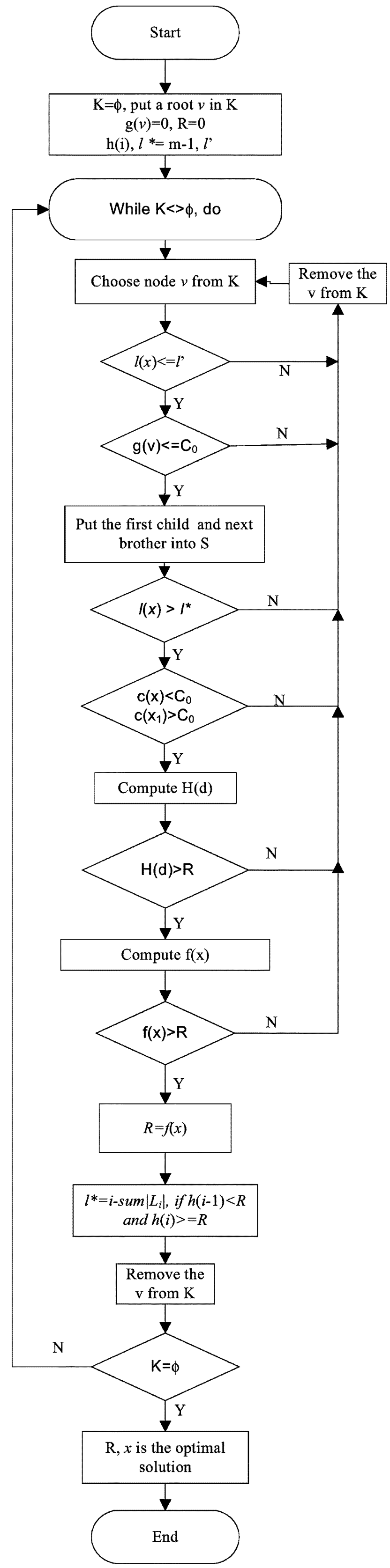


Fig. 9. The flowchart of shrinking & searching algorithm.

V. CONCLUSIONS

In this paper, a novel algorithm called the Shrinking & Searching Algorithm (SSA), was developed for solving the cost-constrained sys-reliability optimization problem. SSA jointly considers the upper bound of system reliability for each branch in the combinatorial tree, and the cost constraint on the possible solutions, such that most of the redundant calculation in the searching process can be avoided by gradually *shrinking* the difference between the lower & upper bounds of the length of a path in the corresponding combinatorial tree (which represents a feasible solution). We described in detail how the problem is modeled, and how the constrained optimization problem is solved, where the effectiveness & efficiency of the proposed algorithm have been verified through two case studies & simulation results. It is concluded that the proposed approach contributes to the future development of distributed access networks supporting versatile commercial applications, where the requirement on system reliability of the network topology is high.

Acknowledgment

The authors wish to thank the anonymous reviewers for their helpful comments & suggestions.

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