Understanding the Scheduling Performance in Wireless Networks with Successive Interference Cancellation

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***Abstract*—Successive interference cancellation (SIC) is an effective way of multipacket reception to combat interference in wireless networks. We focus on link scheduling in wireless networks with SIC, and propose a layered protocol model and a layered physical model to characterize the impact of SIC. In both the interference models, we show that several existing scheduling schemes achieve the same order of approximation ratios, independent of whether or not SIC is available. Moreover, the capacity order in a network with SIC is the same as that without SIC. We then examine the impact of SIC from first principles. In both chain and cell topologies, SIC *does* improve the throughput with a gain between 20% and 100%. However, unless SIC is properly characterized, any scheduling scheme cannot effectively utilize the new transmission opportunities. The results indicate the challenge of designing an SIC-aware scheduling scheme, and suggest that the approximation ratio is insufficient to measure the scheduling performance when SIC is available.**

***Keywords*-Network capacity; link scheduling; successive interference cancellation**

I. INTRODUCTION

The capacity of a modern wireless communication system is interference-limited. Due to the broadcast nature, what is arriving at a receiver is a composite signal from all near-by transmissions. In general, the receiver tries to decode only one transmission by regarding all the others as interference and noise. When the arrivals of multiple transmissions overlap, collision occurs and the reception fails.

Multiple packet reception (MPR) is a promising technique at the physical layer to combat the interference. When the links interfering with each other transmit simultaneously, a receiver node can separate the collided signals with the MPR capability. It is shown in [1–4] that MPR can significantly increase the capacity of a wireless network.

SIC is an effective way of MPR to resolve the transmission collisions [5]. With SIC, the receiver tries to detect multiple received signals using an iterative approach. In each iteration, the strongest signal is decoded, by treating the remaining signals as interference. If a required SINR (signal to interference and noise ratio) is satisfied, this signal can be decoded and removed from the received composite signal. In the subsequent iteration, the next strongest signal is decoded, and the process continues until either all the signals are decoded or a point is reached where an iteration fails.

Though significant progress has been made in MPR techniques at the physical layer, little attention has been paid to the design of support protocols at high layers. As not all composite signals are decodable, it is indispensable to avoid harmful collisions (i.e., when the involved signals cannot be separated). In particular, as there are specific requirements to ensure the feasibility of an MPR method, it is necessary to coordinate the transmissions carefully to meet the requirements.

Dealing with interference is one of the primary challenges in wireless communication system design. In the literature, there are two major interference models: the protocol model and the physical model. Though several extensions have been introduced to the models to deal with MPR, the unique feature of SIC is not captured accurately. For example, in [1], the protocol model is extended by increasing the number of permitted interferers from zero to N ($N \geq 1$). The extension ignores the constraint in the received signal strength imposed by the sequential detection in SIC. To better understand scheduling performance, here we introduce an *layered* physical model and a layered protocol model, i.e., M-protocol model, where M is a pre-defined system parameter, to characterize the impact of SIC.

We take successive interference cancellation (SIC) [5] as an example of MPR to study scheduling performance in a network with SIC. The protocol design has been considered only recently, e.g., link scheduling [6, 7] and topology control [8], in a network with SIC. However, it is lacking in understanding the generic behavior of a network with SIC. To completely understand the effect of SIC, in this paper, we study the scheduling performance from three different aspects.

First, given a scheduling scheme that is unaware of SIC, we analyze the effect of SIC on the approximation performance of the scheme. In our recent work [6, 7], we show that link scheduling with SIC is NP-hard in both the M-protocol model and the layered physical model. As there is no optimal solution with polynomial time complexity for any NP-hard problem, we resort to an approximation scheme to perform the scheduling. We demonstrate that, in both the M-protocol model and the layered physical model, the same order of the approximation ratio is achieved for several SIC-unaware scheduling schemes, no matter whether or not SIC is available. A key insight is that the number of simultaneous transmissions increases at most by a limited factor after SIC is applied.

The second contribution is the derivation of the capacity of a network with SIC and the finding that it has the same order as that without SIC. In the M-protocol model, the capacity is $O(\sqrt{n})$ where n is the total number of nodes. In the layered physical model, if the transmission power can be set arbitrarily, $O(n)$ capacity is achievable; otherwise, the capacity falls down to $O(n^{(\eta-1/\eta)})$, where η is the path loss exponent. In comparison with the result in [9], the capacity order is not changed when SIC is applied. As a result, any scheduling scheme can achieve the same order of the approximation ratio in a network without SIC as that with SIC.

The third contribution is the study of the impact of SIC from first principles. In both chain and cell network topologies, SIC improves the performance significantly. The optimal throughput with SIC is 20% to 100% higher than that without SIC. However, unless SIC is properly characterized and exploited, any scheduling scheme cannot effectively utilize the new transmission opportunities. Moreover, there is an essential correlation between the scheduling performance and the usage of the transmission opportunities from SIC. Therefore, to accurately measure the performance of a scheduling scheme, in addition to the approximation ratio, new metrics are required to explicitly characterize the SIC capability.

All in all, the results indicate the importance of designing an SIC-aware scheduling scheme, and suggest that: first, SIC can significantly improve the network capacity, and characterizing the impact of SIC is indispensable to exploit the new transmission opportunities; second, the approximation ratio is not sufficient to measure the performance of a scheduling scheme when SIC is available. The findings of this work should shed some light on the protocol design in a network of SIC and the impact of other similar MPR techniques on scheduling.

The rest of this paper is organized as follows. Section II overviews the related work and Section III describes the system model. Section IV derives the approximation ratio of two scheduling schemes when SIC is available. Section V analyzes the network capacity and Section VI examines the impact of SIC. We conclude the research in Section VII. The proofs are given in Section VIII.

II. RELATED WORK

In the literature, there are two major interference models: the protocol model and the physical model [9]. To deal with the MPR, the protocol model is extended by increasing the number of permitted interferers from zero to N ($N \geq 1$) [10], while the physical model is enhanced by allowing reception with a lower SINR threshold [11]. The model used in [12] correlates the reception probability with the number of concurrent transmissions, while neglecting any difference among transmissions.

Scheduling packet transmission in a network without SIC has been considered in [13, 14] based on the protocol model, and in [15, 16] based on the physical model. In [17], a scheduling scheme is proposed to achieve a constant approximation ratio in the protocol model. Also, efficient approximation algorithms in the physical model are given in [16] under the assumption that transmitters can either broadcast at full power or not at all, and in [15, 18] by choosing different transmission powers for different transmitter nodes.

The capacity of a random network in both the physical and the protocol models is examined in [9]. The capacity of an ad hoc network is studied in [19] under different topologies and traffic patterns. Also, SIC is shown to improve the performance significantly in various wireless networks [20].

To realize the potential of MPR, network protocols must be designed accordingly. There are some studies to support MPR in a centralized network [12] and in a distributed scenario, e.g., distributed MAC [11] and joint routing and scheduling [10]. SIC-aware protocol design in a network with SIC has only recently been considered. For example, link scheduling in a network with SIC is studied in [6, 7] based on both the protocol and physical models. Also, in [8], topology control is examined in a multi-user MIMO network with SIC.

III. SYSTEM MODEL

Consider a single-channel wireless network of n stationary nodes (i.e., $\mathcal{X} = \{X_1, \ldots, X_n\}$) and N links. A link is denoted by $L_{S_l R_l}$ or L_l ($1 \leq l \leq N$) with transmitter node $S_l \in \mathcal{X}$ and receiver node $R_l \in \mathcal{X}$, respectively. We also use X_i ($1 \leq i \leq n$) to denote the position of node X_i and $|X_i X_j|$ the distance between two nodes X_i and X_j. Assume that:

- All nodes are located in a planar area;
- The signal removal of SIC is perfect;
- The network node is homogenous. Each node has an omni-directional antenna, operates in the half duplex mode, transmits with the same transmission power over the common wireless channel, and is not able to transmit multiple packets simultaneously.
- The transmission rate is the same for all transmitter nodes, i.e., rate adaptation is disabled.

Note that signal removal is challenging in a near-far situation. In practice, likely they will be residual interference after signal removal even without the near-far effect. We here do not consider the effect of residual interference [20] and leave it as a future work.

A. Layered protocol model

In the original protocol model, there is one transmission range and one interference range. A transmission from S_i to R_i is successful when S_i is within the transmission range of R_i and there is no other active transmitter within the interference range of R_i.

We propose a layered protocol model, i.e., the M-protocol ($M \geq 1$) model. Here, M is a pre-defined system parameter and, without loss of generality, we assume that M is a bounded constant and independent of the network size (i.e., n). Let r_k ($1 \leq k \leq M$) denote the kth transmission range, $(1 + \delta_k) r_k$

the kth interference range. In general, we assume that $r_M > r_{M-1} > \ldots > r_1 > r_0 = 0$ and $\delta_k > 0$ for all $1 \le k \le M$.

Definition 1 Link L_i is a k-level link if $r_{k-1} < |S_i R_i| \le r_k$. A signal from X_i to X_j is a k-level ($1 \le k \le M$) signal if $r_{k-1} < |X_i X_j| \le r_k$. Then a function $\mathcal{U}$ is defined as $\mathcal{U}(X_i, X_j) = k$. In particular, $\mathcal{U}(X_i, X_j) = \infty$ when $|X_i X_j| > r_M$.

Link L_j is a *correlated* link of L_i if, $\mathcal{U}(S_i, R_i) < \infty$, $k = \mathcal{U}(S_j, R_i) < \infty$, and $|S_i R_i| > (1 + \delta_k) r_k$. When the two links transmit simultaneously, in order to detect its desired signal (i.e., from S_i), R_i should first detect and remove the signal transmitted from S_j.

For a link L, suppose there are J ($J \le N - 1$) links active simultaneously with L and D ($D \le J$) of them are correlated links of L. Without loss of generality, all the links are ordered with respect to the distance to the receiver of L as $L_1, \ldots, L_{J+1}$, where L_{D+1} is the targeting link L. Suppose $|S_1 R_{D+1}| \le \ldots \le |S_{J+1} R_{D+1}|$ and the set of correlated links is $\{L_1, \ldots, L_D\}$. To successfully detect the signal of L_{D+1}, the required condition is, for any integer x ($1 \le x \le D$), we have: $u = \mathcal{U}(S_x, R_{D+1}) < \infty$, and for every $x < y < J + 1$,

$$|S_y R_{D+1}| > (1 + \delta_u) r_u. \tag{1}$$

B. Layered physical model

Let N_0 denote the noise power, P the transmission power, and $P_j^i = P/|S_j R_i|^\eta$ the received signal power at R_i from S_j, where η is the path-loss exponent and usually $2 \le \eta \le 6$. Link L_j is a *correlated* link of L_i if, at node R_i, the signal of L_j is sufficiently strong so that it can be detected in the presence of that of L_i. Afterwards, the signal of L_j is removed to reduce the interference to L_i. The required condition is

$$\frac{P_j^i}{N_0 + P_i^i} \ge \theta \tag{2}$$

where θ specifies the reception SINR threshold.

For a link L, suppose there are J ($J \le N - 1$) links active simultaneously with L and D ($D \le J$) of them are correlated links of L. Without loss of generality, all the links are ordered with respect to the distance to the receiver of L as $L_1, \ldots, L_{J+1}$, where L_{D+1} is the targeting link L. Suppose $|S_1 R_{D+1}| \le \ldots \le |S_{J+1} R_{D+1}|$ and the set of correlated links is $\{L_1, \ldots, L_D\}$. To successfully detect the signal of L_{D+1}, the required condition is,

$$\begin{aligned} &\frac{P_x^{D+1}}{N_0 + \sum_{(x+1)\le j \le J+1} P_j^{D+1}} \ge \theta, \forall x \le D \\ &\frac{P_{D+1}^{D+1}}{N_0 + \sum_{(D+2)\le j \le J+1} P_j^{D+1}} \ge \theta. \end{aligned} \tag{3}$$

It is clear that the protocol model and the physical model are a special case of the two new models, respectively. The original protocol model is the same as the M-protocol model when $M = 1$, and the original physical model is the same as the layered physical model when no iterative detection is allowed.

IV. Maintenance of the Order Optimality

For packet transmission, time is partitioned to slots of a constant duration. Each slot is for transmission of one packet. To measure the performance of a scheduling scheme, *schedule length* is defined as the total number of time slots used by the scheme. The objective of a scheduling scheme is to allocate each link at least one slot while assuring the schedule length is as short as possible. For a scheduling scheme A, *approximation ratio* is defined as the ratio of the schedule length of A to the optimal one, which is the minimum number of slots to schedule all the links. Below, for each interference model, we choose a scheduling scheme to examine its performance in a network with SIC.

A. Scheduling Based on the M-Protocol Model

The scheduling scheme shown in Algorithm 1 is similar to that presented in [6] except the definitions of the incoming and outgoing degrees. We show that, based on the M-protocol model, it achieves a constant approximation ratio no matter whether or not SIC is available.

Algorithm 1: Scheduling based on the M-protocol model

Data: A set of links located arbitrarily on the plane
Result: A feasible schedule S_{LO}

$\mathcal{U} \leftarrow$ all links;
repeat
  Find a link L in $\mathcal{U}$ that has the *maximum IN* difference and let L_{n-m+1} denote the mth chosen link;
  $\mathcal{U} \leftarrow \mathcal{U} - \{L_{n-m+1}\}$;
until $\mathcal{U} == \emptyset$
for i=1 **to** n **do**
  Schedule link L_i in the first d_i available slots such that the resulting set of scheduled links is feasible, where d_i is the number of slots required by L_i;
  If currently available slots are not sufficient to schedule d_i slots for L_i, add new slots at the end of the schedule S_{LO} and schedule link L_i in these slots;
end
return the schedule S_{LO};

We first introduce the concept of *IN difference* to order the links to be scheduled.

Definition 2 For link L_{SR}, a link that can interfere with the reception of L_{SR} is defined as the *interfering link* of L_{SR}. Based on the M-protocol model, link $L_{S'R'}$ is an interfering link of L_{SR} when $|S'R| \le (1+\delta_k) r_k$, where $k = \mathcal{U}(S, R)$. The *incoming degree* of L_{SR} is the number of all interfering links. The disk area centering at R with radius $(1 + \delta_k) r_k$ is defined as the *interference zone* of L_{SR}.

Definition 3 For link L_{SR}, a link that is interfered by L_{SR} is defined as the *interfered link* of L_{SR}. Based on the M-protocol model, link $L_{S'R'}$ is an interfered link of L_{SR} when $|SR'| \le (1+\delta_{k'}) r_{k'}$, where $k' = \mathcal{U}(S', R')$. The *outgoing degree* of L_{SR} is the number of all interfered links.

Definition 4 The *IN difference* of a link is the difference between the incoming degree and the outgoing degree.

The scheduling scheme is summarized in Algorithm 1, which has two major procedures.

- Link ordering: The first link that has the *maximum IN* difference is chosen. While not all links are scheduled, do the following: select the link with the maximum *IN* difference; and remove the chosen link. The selection process provides a particular ordering of all links.
- Slot allocation: Time slots are assigned to each link from the last one to the first. When the demand of a link is larger than one slot, multiple slots are assigned to meet the demand. If currently available slots are not enough, new slots are allocated to schedule the link. Finally, at every time slot, a feasible link set is constructed.

In [6], it is shown that, when the demand is one for every link, the schedule length of Algorithm 1 is bounded by $O(\Delta^{in})$, where Δ^{in} is the maximum incoming degree. It can be verified that this is still valid in the M-protocol model.

Lemma 1 (From [6]) Based on the M-protocol model, the schedule length reported by Algorithm 1 is at most $(2\Delta^{in}+1)$.

Now we present the approximation ratio of Algorithm 1.

Theorem 1 Based on the protocol model, Algorithm 1 has a constant approximation ratio in a network without SIC.

The basic approach to prove the theorem is to divide the interference zone of a link into several regions. Fig. 1 shows the partition of the interference zone: first draw K circles within the zone with radius $d_k = \frac{k(1+\delta)r}{K}$ $(k = 1, \ldots, K)$ and then divide the area between two consecutive circles into $\lceil \frac{2\pi}{\alpha} \rceil$ regions, where $\alpha \in (0, 2\pi)$ is a constant determined by r and δ. The shadow area in Fig. 1 shows an example of the region, which is termed as a $(k-1, k, \alpha)$ region. The endpoints of the region are denoted by $A_{k,1}$, $A_{k,2}$, $A_{k-1,1}$ and $A_{k-1,2}$, where $A_{k,1}$ and $A_{k,2}$ reside on the kth circle (i.e., with radius $\frac{k(1+\delta)r}{K}$) and $A_{k-1,1}$ and $A_{k-1,2}$ reside on the (k-1)th circle (i.e., with radius $\frac{(k-1)(1+\delta)r}{K}$). Afterwards, it can be shown that (i) the number of regions, i.e., $K\lceil \frac{2\pi}{\alpha} \rceil$, depends on r and δ only, and (ii) the incoming links whose senders are in the same region must interfere with each other. In consequence, at least $\Omega(\Delta^{in})$ slots are required for an optimal schedule.

Theorem 2 Based on the M-protocol model, Algorithm 1 has a constant approximation ratio in a network with SIC.

Theorem 1 is a special case of Theorem 2 when $M = 1$. The approach to derive the result when $M \geq 2$ (i.e., when SIC is available) is similar except three differences: (i) For a link L_i, the interference range is $(1+\delta_u)r_u$, where $u = \mathcal{U}(S_i, R_i)$; (ii) The set of incoming links is divided into M subsets such that every link in the kth subset is a k-level link; (iii) The number of regions depends on $\{r_1, \ldots, r_M, \delta_1, \ldots, \delta_M\}$.

To our best knowledge, Algorithm 1 is the first scheduling scheme that is shown to achieve a constant approximation ratio in a network with SIC. However, though Algorithm 1 may take advantage of some transmission opportunities from SIC, its design is not SIC-aware. For example, the incoming degree of link L_i counts all the links interfering L_i in the M-protocol model. When L_j is a correlated link of L_i, though the impact of the L_j interference on L_i is removable, L_j is still included in counting the incoming degree of L_i.

One may argue that the result likely attributes to the simplicity of the interference model, e.g., no accumulative effect of interference is considered. Next, we show a similar behavior in an accumulative interference model.

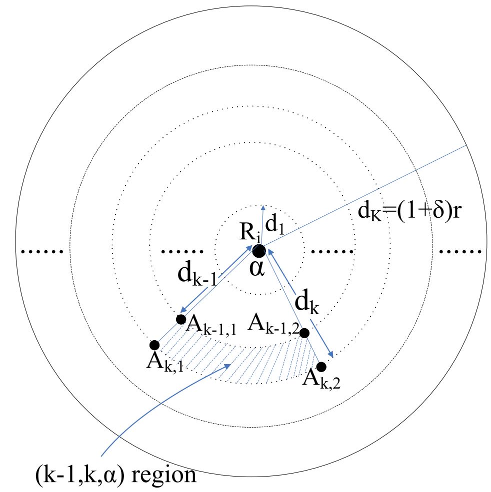

Fig. 1: Partition of the interference zone of link $L_{S_iR_i}$. The shadow area is called a $(k-1, k, \alpha)$ region with four endpoints $A_{k,1}$, $A_{k,2}$, $A_{k-1,1}$, $A_{k-1,2}$.

B. Scheduling Based on the Layered Physical Model

We study the performance of the scheduling scheme given in Algorithm 2 [16]. It consist of two steps: first, the problem instance is partitioned into disjoint link length classes; then, a feasible schedule is constructed for each length class using a greedy strategy. For more details, please refer to [16]. For a non-negative integer x, we say that, L_i is an x-class link when $2^x \leq |S_iR_i| < 2^{x+1}$.

Algorithm 2: Scheduling based on the physical model [16]

Data: A set of links located arbitrarily on the plane
Result: A feasible schedule

Let $R = R_0, \ldots, R_{\log(l_{max})}$ such that R_k is the set of links L_i of length $2^k \leq |S_iR_i| < 2^{k+1}$;
$t = 1$;
for *all* $R_k \neq \emptyset$ **do**
  Partition the plane into squares of width $\mu \cdot 2^k$;
  4-color the cells such that no two adjacent squares have the same color;
  for j=1 **to** 4 **do**
    Select color j;
    repeat
      For each square A of color j, pick one link $L_i \in R_k$ with receiver R_i in A, assign it to slot t;
      $t = t + 1$;
    until *all links of* R_k *in the selected squares are scheduled;*
  end
end
return the schedule;

Definition 5 For a link set $\mathcal{L}$, let *length diversity*, i.e., $g(\mathcal{L})$, denote the number of non-empty length classes. Let $\mathcal{N}$ be the

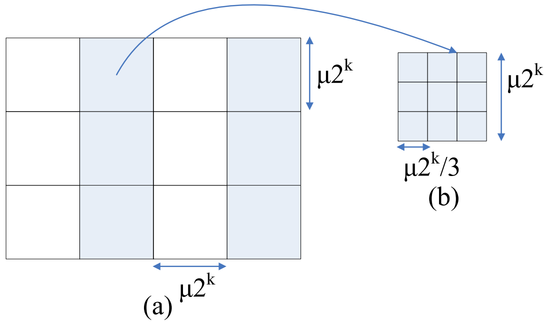

Fig. 2: (a) Partition of the plane into square grid cells of side $\mu \cdot 2^k$; (b) Partition of a cell into 9 subcells of side $\mu \cdot 2^k/3$.

set of non-negative integers, then $g(\mathcal{L})$ is given by

$$g(\mathcal{L}) = |\{m|m \in \mathcal{N}; \exists L_i, L_j \in \mathcal{L} : \lfloor \log |S_i R_i|/|S_i R_j| \rfloor = m\}|.$$

For each $1 \leq k \leq g(ALS)$, where ALS is the set of all links, the plane is partitioned into square grid cells of side $\mu \cdot 2^k$, where

$$u = 4(8\theta \cdot \frac{\eta - 1}{\eta - 2})^{1/\eta}.$$

Fig. 2 (a) shows an example of the partition. Let C be the number of cells and $\mathcal{L}_y^x$ be the set of links L_i whose receiver is located in the yth cell and $2^x \leq |S_i R_i| < 2^{x+1}$. Then we choose a special set $\mathcal{L}_m^k$ such that $|\mathcal{L}_m^k| = \max_{1 \leq x \leq g(ALS), 1 \leq y \leq C}\{|\mathcal{L}_y^x|\}$ and let $\Delta_m^k = |\mathcal{L}_m^k|$.

Lemma 2 (From [16]) Based on the physical model, the schedule length of Algorithm 2 is at most $O(g(ALS) \cdot \Delta_m^k)$ in a network without SIC.

When SIC is available, the performance of the scheduling scheme is stated as follows.

Theorem 3 Based on the layered physical model with uniform transmission power, the approximation ratio of Algorithm 2 is $O(g(ALS))$ in a wireless network with SIC.

When SIC is applied, the schedule computed by Algorithm 2 is still feasible. Therefore, with Lemma 2, we need to show that the optimal schedule requires at least $\Omega(\Delta_m^k)$ slots. We further partition a cell of side $\mu \cdot 2^k$ into 9 sub-cells of side $\frac{\mu}{3} \cdot 2^k$, as shown in Fig. 2 (b). Afterwards, we bound the number of links in $\mathcal{L}_m^k$ such that (i) they transmit simultaneously; (ii) R_i is located in the same sub-cell; and (iii) $2^k \leq |S_i R_i| < 2^{k+1}$. The upper bound q depends only on the path loss exponent η and the SINR reception threshold θ. As a result, an optimal schedule requires at least $\Delta_m^k/(9q)$ slots.

It can be verified that, if the transmission power is non-uniform, the length of the optimal schedule is decreased by a factor at most σ, where σ is the ratio of the maximum transmission power to the minimum one. Therefore, the approximation ratio is still $O(g(ALS))$ when σ is a small constant. If the transmission power can be set arbitrarily, a higher gain can be expected when SIC is available. At this time, the result of Theorem 3 no longer holds. The joint design of power control and scheduling is beyond the scope of this paper, and we leave it for future work.

It is shown in [16] that the approximation ratio of Algorithm 2 is $O(g(ALS))$ in a network without SIC. Theorem 3 shows that the same order of approximation ratio is achieved when SIC is available. Note that the scheme is unaware of SIC and does not exploit any transmission opportunity from SIC. On the other hand, it is shown that the capacity is significantly increased when SIC is applied [20]. To explore why a SIC-unaware scheduling scheme can maintain its order optimality in a network with SIC, we are interested in understanding (i) the impact of SIC on the network capacity and (ii) the scheduling performance in practice when SIC is applied.

V. Capacity Analysis

To explore the generic behavior of the scheduling schemes, we analyze the capacity in a network with SIC.

Definition 6 ([9]) The network transports one *bit-meter* when one bit has been transported a distance of one meter toward its destination. The sum of products of bits and the distances over which they are carried is defined as the *transport capacity*.

To analyze the capacity of a wireless network, we scale the network coverage area and consider that n nodes are located arbitrarily in a disk of area A m^2 on the plane. The transmission rate over the channel is W bits per second. Each node can transmit λ bits per second on average and the network transports λnT bits over T seconds. The average distance of a link is $\overline{B}$, which implies that a transport capacity of $\lambda n\overline{B}$ bit-meters per second is achieved.

To simplify the analysis, we relax the M-protocol model, i.e., replacing (1) by

$$|S_{i_y} R_d| > (1 + \delta_{u_x})|S_{i_x} R_d|. \quad (4)$$

As $|S_{i_x} R_d| \leq r_{u_x}$, the new model is more optimistic in the sense that a feasible link set in the "old" M-protocol model is still feasible in the new model.

Theorem 4 Based on the M-protocol model, the transport capacity of a network with SIC is bounded as follows

$$\lambda n\overline{B} \leq \frac{\sqrt{8}}{\sqrt{\pi}} \frac{\sqrt{A}W}{\delta} \sqrt{n} \quad (5)$$

where $\delta = \min\{\delta_1, \ldots, \delta_M\}$.

In the M-protocol model, the minimum in $\{\delta_1, \ldots, \delta_M\}$ helps us to bound the transport capacity of a network with SIC. Note that $\{\delta_1, \ldots, \delta_M\}$ is determined by the system capability (e.g., the decoding policy) and independent of the network size (e.g., n). As shown in [9], the capacity of a network without SIC is characterized by δ_M. Hence, a slightly higher bound can be expected when SIC is available. However, the order of the capacity with SIC is the same as that without SIC.

Now turn to the accumulative interference model. At first, if arbitrary transmission power is allowed, $O(n)$ capacity can be achieved. Consider a unique receiver node R at the center with transmitter nodes $S_1, \ldots, S_{n-1}$ at distances as $d_1, \ldots, d_{n-1}$ to

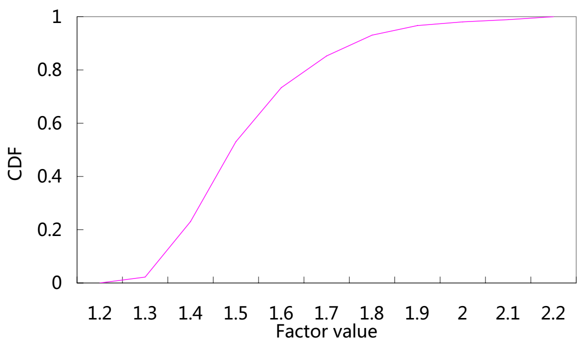

Fig. 3: Cumulative distribution function of the factor $(1+D)^{1/\eta}$.

R, respectively, where $d_1 \geq d_2 \geq \ldots \geq d_{n-1}$. The transmission power levels (e.g., P_i for node S_i, $1 \leq i \leq n-1$) are given by

$$P_1 = \theta \cdot d_1^{\eta} \cdot N_0$$
$$P_i = \theta \cdot d_i^{\eta} \cdot (N_0 + \sum_{1 \leq k < i} P_k \cdot d_k^{\eta}), 1 < i \leq n-1.$$

When all the (n-1) nodes transmit simultaneously, node R can detect and remove the signal from S_{n-1} to that from S_2 in sequence. Finally, node R detects the signal from the furthest node S_1. As all the $(n-1)$ nodes can transmit simultaneously, the capacity is $O(n)$.

If the transmission power cannot be arbitrarily chosen, less simultaneous transmissions can be supported. In particular, when the transmission power is the same at all the transmitter nodes, the transport capacity falls down to $O(n^{(\eta-1)/\eta})$.

Theorem 5 In the layered physical model with uniform transmission power, the transport capacity of a network with SIC is bounded as follows

$$\lambda n \overline{B} \leq (\frac{2\theta + 2}{\theta})^{1/\eta} \frac{\sqrt{A} W}{\sqrt{\pi}} (1 + D)^{1/\eta} n^{(\eta-1)/\eta} \quad (6)$$

where $D \leq 1 + \frac{\eta \log \frac{2\sqrt{A}}{\sqrt{\pi}} - \log \theta}{\log(1+\theta)}$.

Compared to the capacity of a network without SIC [9], the difference is the factor $(1 + D)^{1/\eta}$. Such factor is independent of the network size. Fig. 3 shows the cumulative distribution function (CDF) of the factor when the area A is between 1 and 100, the loss exponent η is between 2 and 6, and the reception threshold θ is between 3 and 13. The fact that the factor is always larger than 1 demonstrates the advantage of SIC. Nevertheless, even when the area is as large as 100, the maximum of the factor is less than 2.2.

It can be verified that, for non-uniform transmission power, the result in Theorem 5 is still valid except that the upper bound of D is scaled by a constant factor when the ratio of the maximum transmission power to the minimum one is a small constant. The proof is given in the appendix.

It is shown in [9] that $O(\sqrt{n})$ is also a lower bound of the capacity. Therefore, the maintenance of the order optimality shown in Section IV is not an exceptional behavior of the chosen scheduling schemes, but inherently imposed by the fact that no meaningful gain is provided by SIC in terms of capacity scaling. For any scheduling scheme, the same order of the approximation ratio can be achieved independent of whether or not SIC is used.

Comparison with the previous results: Franceschetti et al show that [21], by distributing uniformly an order of n users inside a two-dimensional domain of size of the order of n, the number of independent information channels is only of the order of $\sqrt{n}$, so the per-user information capacity must follow an inverse square-root of n law. Recently, Ozgur et al indicate that [22], the spatial degrees of freedom limitation found by Franceschetti et al is actually dictated by the diameter of the network, or more precisely, $\sqrt{A}/\phi$, where A is the area of the network and ϕ is the carrier frequency. This number can be heuristically thought of as an upper bound to the total degrees of freedom in the network and puts a limitation on the network capacity. The conclusion that the capacity scales like $\sqrt{n}$ comes from the assumption that the density of nodes is fixed as the number of nodes n grows, so that $\sqrt{A}/\phi$ is proportional to $\sqrt{n}$. Therefore, when the order of $\sqrt{A}/\phi$ is larger than $\sqrt{n}$, a higher capacity can still be achieved.

Our result is independent of the diameter of the network and different from that in [1, 4], where it is shown that the capability of MPR provides a higher order of capacity. The difference stems from the adopted interference model. In our model, we take into account a practical constraint on the received signal strength required by the sequential detection of SIC. Therefore, the number of transmissions in a composite signal is strictly limited. In comparison, in the interference model used in [1], an arbitrary number of transmitter nodes can transmit simultaneously to a receiver node R as long as they are within a radius of r from R and all the other transmitter nodes have a distance larger than $(1 + \delta)r$ to the receiver R. Based on the model, when there are a unique receiver node and $(n-1)$ transmitter nodes, $O(n)$ capacity is always achieved if all the $(n-1)$ nodes are within a radius of r to the receiver.

Our results provide a deeper understanding of SIC. In fact, the results in the previous work (e.g., [2–4]) indicate that, to obtain a higher order of capacity, the number (i.e., k) of simultaneous transmissions resolved by a receiver node should be at some orders of the network size. For example, Guo et al [4] show that, when $k = \Omega(\sqrt{\log n})$, the capacity gain is at least $\Theta(\sqrt{\log n})$. When the network size is large, a receiver node is required to resolve the collisions among a huge number of transmissions. Obviously, the available techniques such as SIC cannot meet the requirement. This explains in part why SIC cannot achieve the capacity as expected.

Relation with rate adaptation: Rate adaptation (RA) is deployed widely to effectively utilize the dynamic channel in wireless networks [23, 24]. To understand the interplay of RA and SIC, consider a three-node network scenario with two transmitters, S_1 and S_2, and one receiver R_1. Without loss of generality, assume $P_{11} > P_{12}$. Suppose that S_1 and S_2 can transmit to R_1 separately, i.e., $P_{11}/N_0 \geq \theta$ and $P_{12}/N_0 \geq \theta$.

Consider the effect of RA on SIC first. With SIC, the two nodes can transmit simultaneously when $P_{11}/(N_0 + P_{12}) \geq \theta$. Otherwise, a harmful collision occurs and no signal can be detected. The relation between S_1 and S_2 is binary: either they can transmit simultaneously, or not. In comparison, with the help of RA, simultaneous transmissions can always be sup-

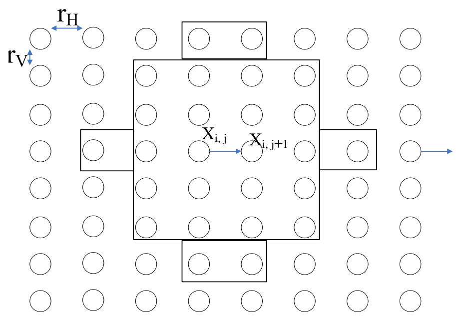

Fig. 4: Illustration of a network consisting of multiple chains. The nodes within the rectangles are affected by the transmission from $X_{i,j}$ to $X_{i,j+1}$.

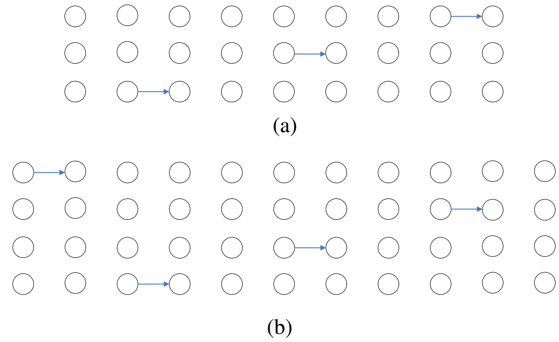

Fig. 5: A snapshot of the optimal schedule at one slot in a network without SIC: (a) three chains and (b) four chains.

ported. To combat the interference, however, the transmission rate should be adjusted accordingly. When $P_{11}/(N_0+P_{12}) \geq \theta$, as the signal from S_1 can be detected and removed first, there is no need for S_1 and S_2 to change the transmission rate. Otherwise, both S_1 and S_2 must use a lower transmission rate to tolerate the mutual interference. Next, consider the effect of SIC on RA. With SIC, RA can utilize the channel more efficiently. Take S_2 as an example. Without SIC, the chosen transmission rate must tolerate both noise and the interference from S_1. On the other hand, with the help of SIC, when $P_{11}/(N_0 + P_{12}) \geq \theta$, the signal from S_1 can be removed first so that it is enough to consider the effect of noise only. Eventually, a higher transmission rate can be chosen by S_2.

In summary, to achieve the optimal network performance, RA and SIC should be deployed jointly. It is thus important to extend the analysis to investigate the joint effect of SIC and RA, which is one of our ongoing works.

VI. Scheduling Performance in Practical Networks

The scheduling performance is examined in a network with SIC from first principles. For simplicity, we limit the discussion to the M-protocol model with $M = 2$. Note that when M is larger, a higher performance gain can be expected. Hence, the result for $M = 2$ is a lower bound of the gain from SIC. Let $r_1 = \frac{3}{5}r_2$, $\delta_2 = 1$, and $\delta_1 = 1/2$. We investigate the scheduling performance in two scenarios.

- Chain topology: Each chain contains a sufficiently large number of nodes located on a line. The network comprises one or more chains.
- Cell topology: In each cell, there is a receiver node at the center of a circle area and one or more transmitter nodes uniformly located within the area. The network comprises one or more cells.

A. Chain Topology

Fig. 4 illustrates a network consisting of multiple chains, where $r_H = r_2$ and $r_V = r_1$. We assume that the number of nodes in a chain is sufficiently large and denote the node at the ith ($i \geq 1$) chain by $X_{i,1}, X_{i,2}, \ldots$. At the ith ($i \geq 1$) chain, every $X_{i,j}$ ($j \geq 1$) transmits at 1pkt/slot to $X_{i,j+1}$.

We first derive the optimal average throughput in a network without SIC. As the transmission distance is r_2, the interference range is $2r_2$. Thus, a node can communicate directly with its neighbor nodes. In Fig. 4, the five rectangles cover the nodes that cannot transmit or receive simultaneously with the ongoing transmission (i.e., $X_{i,j} \rightarrow X_{i,j+1}$).

One chain: When node $X_{1,j}$ transmits to $X_{1,j+1}$, there are two interfered nodes ($X_{1,j-2}$ and $X_{1,j-1}$) that cannot receive packets from a node other than $X_{1,j}$, and two interfering nodes ($X_{1,j+2}$ and $X_{1,j+3}$) that cannot transmit simultaneously. The distance between two active transmitters is at least three hops. Hence, the average optimal throughput is $\frac{1}{4}$ pkt/s.

Two chains: The transmission at one chain can affect that at the other. For example, when node $X_{1,j}$ transmits to $X_{1,j+1}$, in addition to the four nodes in the first chain (i.e., $\{X_{1,j-2}, X_{1,j-1}, X_{1,j+2}, X_{1,j+3}\}$), there are two interfered nodes ($X_{2,j-1}$ and $X_{2,j}$), and two interfering nodes ($X_{2,j+1}$ and $X_{2,j+2}$) in the second chain. There is no spatial reuse among any four consecutive nodes in one chain and the four neighbors in the other. The average optimal throughput is $\frac{1}{8}$ pkt/s without SIC.

Three chains: The distance between $X_{1,j}$ ($j \geq 1$) and $X_{3,j}$ is $2 \times \frac{3}{5}r_2 < 2r_2$. The distance between $X_{1,j}$ and $X_{3,j-1}$ (or $X_{3,j+1}$) is $\frac{\sqrt{61}}{5}r_2 < 2r_2$. Hence, there is no spatial reuse among twelve nodes at the three chains (i.e., $X_{1,j}$ to $X_{1,j+3}$, $X_{2,j}$ to $X_{2,j+3}$ and $X_{3,j}$ to $X_{3,j+3}$, for $j \geq 1$). A snapshot of the optimal schedule at one slot is shown in Fig. 5 (a). The optimal average throughput is $\frac{1}{9}$ pkt/s.

Four or more chains: The distance between $X_{1,j}$ ($j \geq 1$) and $X_{4,j}$ is $3 \times \frac{3}{5}r_2 < 2r_2$. The distance between $X_{1,j}$ and $X_{4,j-1}$ (or $X_{4,j+1}$) is $\frac{\sqrt{106}}{5}r_2 > 2r_2$. Spatial reuse is feasible between a node at the jth chain and that at the (j+3)th chain. Thus, an optimal schedule is to schedule the nodes in the first and fourth chains together and the nodes in the second and third chains in separate slots. A snapshot of the optimal schedule at one slot is shown in Fig. 5 (b). At each slot, four packets are transmitted among every 44 nodes. Therefore, the optimal average throughput is $\frac{4}{44} = \frac{1}{11}$ pkt/s.

When there are more than four chains, note that the transmission at the first chain does not affect that at the fifth chain. The same throughput can be achieved as that in a network with four chains.

Now consider the impact of SIC. When there is only one

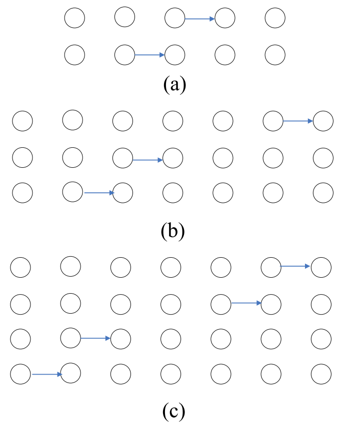

Fig. 6: A snapshot of the optimal schedule at one slot in a network with SIC: (a) two chains; (b) three chains; and (c) four chains.

chain, as the distance between any two nodes is at least r_2, SIC cannot be applied. Thus, the optimal average throughput with SIC is the same as that without SIC.

Consider a network of two chains. When node $X_{1,j}$ transmits to $X_{1,j+1}$, as the distance between $X_{1,j}$ and $X_{2,j}$ is r_1, $X_{2,j}$ can detect the signal from $X_{1,j}$ in the presence of a signal from $X_{2,j-1}$. This leads to a new transmission opportunity, i.e., $X_{1,j}$ and $X_{2,j-1}$ can transmit simultaneously. Finally, a snapshot of the optimal schedule at one slot is shown in Fig. 6 (a). The optimal average throughput is $\frac{1}{4}$ pkt/s. For a network of three or more chains, due to the interference among different chains, a fewer number of simultaneous transmissions can be supported. The optimal schedules for three and four chains are shown in Fig. 6 (b) and (c), respectively.

Table I summarizes the optimal average throughput for various network sizes. The *SIC gain* is defined as $(T_w - T_{wo})/T_{wo}$, where T_{wo} and T_w refer to the optimal average throughput in a network without and with SIC, respectively. When there is no SIC, spatial reuse is possible only after the signal is sufficiently attenuated. Therefore, with an increase of the number of chains, the throughput decreases from $\frac{1}{4}$ to $\frac{1}{11}$. In comparison, when SIC is available, simultaneously transmission is feasible even when the transmitter nodes are close to each other. Hence, SIC helps to obtain more spatial reuse and a much higher network throughput. The performance gain ranges from 29% to 100%.

TABLE I: Throughput comparison in chain topology.

Scenario	Single chain	Two chains	Three chains	Four or more chains
without SIC	1/4	1/8	1/9	1/11
with SIC	1/4	1/4	1/7	1/7
SIC Gain	n/a	100%	29%	57%

A scheduling scheme unaware of SIC cannot exploit the transmission opportunity from SIC. For example, the simultaneous transmissions of $X_{k,j+1}$ to $X_{k,j+2}$ and $X_{k+1,j}$ to $X_{k+1,j+1}$ ($k, j \geq 1$) are prohibited without the capability of SIC. Therefore, unless the unique feature of SIC is characterized, any scheduling scheme will fail to recognize the new transmission

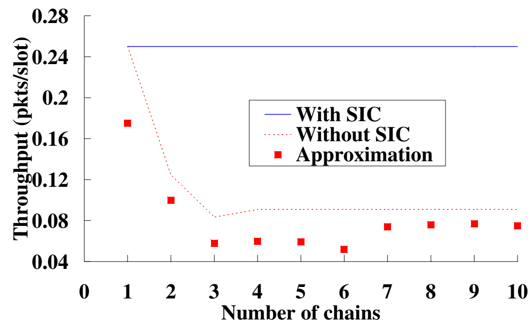

Fig. 7: Average throughput versus the number of chains.

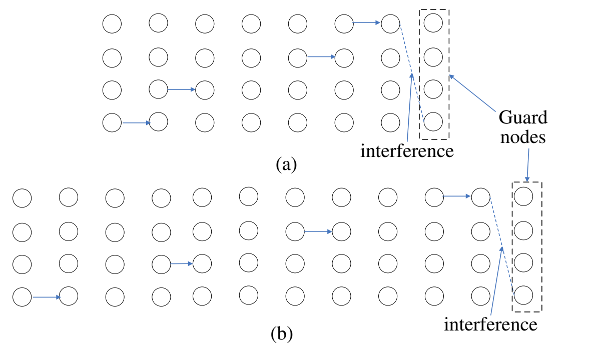

Fig. 8: A snapshot of the optimal schedule at one slot in a network of four chains when $r_V = 2r_2/5$: (a) with SIC; (b) without SIC.

opportunities. To verify the analysis, we conduct simulation to investigate the performance of Algorithm 1.

Fig. 7 compares the throughput of the approximation scheme with the optimal ones. Though the design of the approximation scheme is not SIC-aware, the scheduling scheme can exploit some transmission opportunities from SIC when allocating time slot for a link. As an approximation scheduling scheme, it is naturally sub-optimal. However, the throughput of the scheduling scheme is close to the optimal one without SIC. This means that the sub-optimality is compensated to a large extent by the usage of the SIC capability. Nevertheless, the throughput is much lower than the optimal one with SIC, which indicates that it is challenging to exploit all transmission opportunities from SIC.

With different node distances, the degree spatial reuse and the opportunities of simultaneous transmissions are also different. For example, when r_V changes from $\frac{3}{5}r_2$ to $\frac{2}{5}r_2$, the distance between $X_{1,j}$ and $X_{4,j-1}$ (or $X_{4,j+1}$) is $\frac{\sqrt{61}}{5}r_2 < 2r_2$. As a result, there is interference between the nodes at the jth chain and those at the $(j+4)$th chain. The optimal schedule in a network of four chains are shown in Fig. 8. In comparison with Fig. 5 (b) and Fig. 6 (c), a column of nodes are required as guard nodes to avoid the interference. The throughput without SIC decreases to $\frac{1}{12}$ and that with SIC is $\frac{1}{8}$. The gain provided by SIC is 50%.

We also perform another group of experiments: both r_V and r_H are chosen randomly between $\frac{1}{2}r_1$ and r_2; the number of chains ranges from 4 to 10. Finally, about 120 different

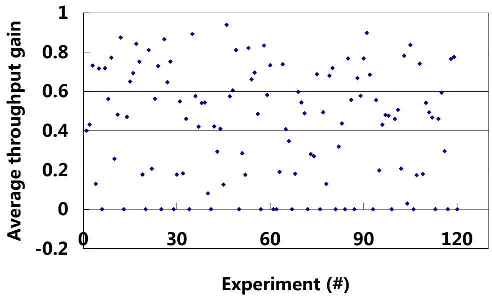

Fig. 9: Average throughput gain of SIC in the experiments with different chain topologies.

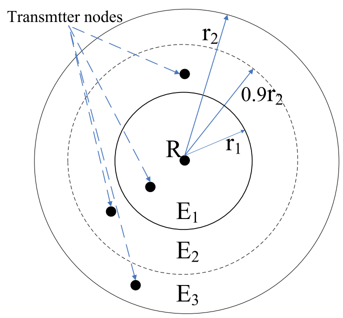

Fig. 10: Illustration of a single cell network.

topologies are generated. In addition, for every node $X_{i,j}$, a transmission probability (0.6 ∼ 0.8 in the experiments) is adopted to determine whether or not it transmits to $X_{i,j+1}$. Fig. 9 shows the average throughput gain of SIC in different networks. There is no gain provided by SIC in about 20 topologies, where every two links do not satisfy the constraints (i.e., (1)). In the remaining 100 networks, the throughput gain provided by SIC is on average 50% and up to 100%.

B. Cell Topology

Now we consider the cell topology. A cell is a disk area with radius r_2. In each cell, there is a unique receiver node at the center and several transmitter nodes located uniformly within the cell. Let ρ denote the node density, i.e., the number of the transmitter nodes per unit area.

Note that the 1-level interference range is $(1+\frac{1}{2})\frac{3}{5}r_2 = \frac{9}{10}r_2$. Consider a network of a single cell, as shown in Fig. 10. Let E_1 denote the area with distance to the receiver no more than r_1, E_2 between r_1 and $\frac{9}{10}r_2$, and E_3 between $\frac{9}{10}r_2$ and r_2. We use $\mathcal{E}_1$, $\mathcal{E}_2$ and $\mathcal{E}_3$ to denote the three sets of transmitter nodes in the three areas, respectively. With SIC, the nodes in $\mathcal{E}_1$ can transmit simultaneously with those in $\mathcal{E}_3$. The optimal schedule length is $|\mathcal{E}_2| + \max\{|\mathcal{E}_1|, |\mathcal{E}_3|\}$. Without SIC, as no concurrent transmissions are permitted, the optimal schedule length is $|\mathcal{E}_1| + |\mathcal{E}_2| + |\mathcal{E}_3|$.

The areas of E_1, E_2 and E_3 are $\frac{9\pi}{25}r_2^2$, $\frac{9\pi}{20}r_2^2$ and $\frac{19\pi}{100}r_2^2$, respectively. Then, the optimal schedule length with SIC is $\rho(\frac{9\pi}{20}r_2^2 + \frac{36\pi}{100}r_2^2) = \frac{81\pi}{100}r_2^2\rho$. In comparison, the optimal schedule length without SIC is $\pi r_2^2\rho$. The performance gain is 19%.

When a network comprises two or more cells, the performance depends on the intersection among different cells.

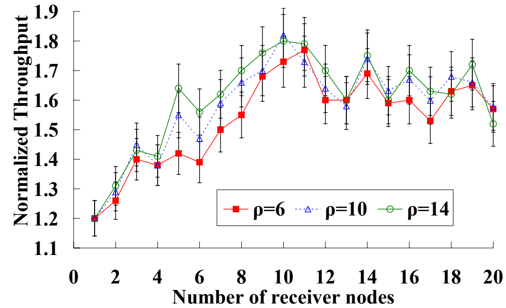

Fig. 11: Normalized throughput with SIC versus the number of receiver nodes with different densities of the transmitter nodes.

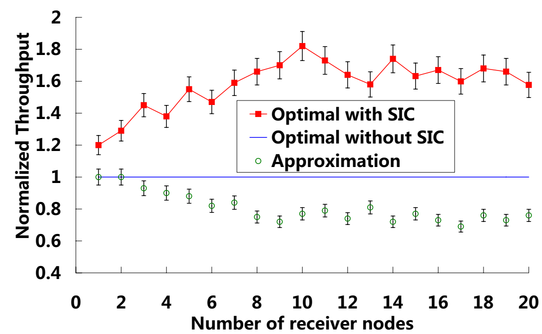

Fig. 12: Normalized throughput versus the number of receiver nodes in the cell topology.

With SIC, the simultaneous transmissions in a single cell are always feasible in a network of multiple cells. The gain in a single cell is thus a lower bound of that in a larger network. As it is impossible to accurately derive the optimal schedule length in a large network, we use simulation to investigate the performance. We set $r_2 = 1/\sqrt{\pi}$ and randomly chooses the positions of the receiver nodes in a $3r_2 \times 3r_2$ plane. For each receiver node R, in the area centering at R with radius r_2, the transmitter nodes are generated uniformly with density ρ.

Fig. 11 shows the normalized throughput with SIC versus the number of receiver nodes with different ρ. The 95% confidence interval is also shown. The optimal average throughput without SIC is normalized to 1. When the density is larger, a slightly higher throughput is obtained but the difference is not significant. Fig. 12 shows the normalized throughput versus the number of receiver nodes when $\rho = 10$. The throughput of the scheduling scheme is close to the optimal one without SIC but much lower than that with SIC. As a significant gain as large as 80% is obtained when SIC is available, it is important to explore how to exploit all the new transmission opportunities.

Finally, we investigate the correlation between the scheduling performance and the usage of the SIC capability. The simulation settings are the same except that: (i) the network plane is $5r_2 \times 5r_2$, and (ii) the maximum number of receiver nodes is 50 and $\rho = 10$. Fig. 13 shows the *throughput percentage* versus the *SIC utilization ratio* when the number of receiver nodes ranges from 30 to 50. The throughput percentage is defined as the ratio of the throughput of the

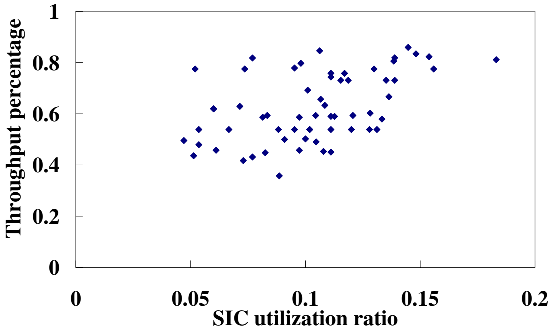

Fig. 13: Throughput percentage versus the SIC utilization ratio in the cell topology.

scheduling scheme to the optimal one with SIC. The SIC utilization ratio is defined as the ratio of the number of *used* correlated links to the total number of correlated links. Let L_1 be a correlated link of L_2, L_1 is used when the same time slot is assigned to L_1 and L_2.

The correlation coefficient between the throughput percentage and the SIC utilization ratio is given in Table II with different numbers of receiver nodes. For each number, the experiments are repeated 500 times with different random seeds. It is clear that with a higher utilization ratio, a higher throughput can be expected. This is not surprising since a higher utilization ratio means that a larger number of transmission opportunities from SIC have been exploited. In addition, the fact that the correlation coefficients in Table II are close to or larger than 0.5 indicates the essential correlation between the scheduling performance and the usage of the SIC capability. The relatively low utilization ratio points out that there is still a large room for future work to fully exploit the SIC capability in the design of protocols such as a link scheduling scheme.

TABLE II: Correlation coefficient between the throughput percentage and the SIC utilization ratio.

Number of receiver nodes	10	20	30	40	50
Correlation coefficient	0.537	0.508	0.485	0.552	0.488

As a common metric for all approximation algorithms, approximation ratio fails to carry sufficient information about the usage of the transmission opportunities from SIC. As a result, to accurately measure the performance of a scheduling scheme, in addition to the approximation ratio, new metrics are required to explicitly characterize the effect of SIC.

In summary, two important observations are obtained. First, though there is no improvement in the capacity order, the performance gain obtained from the new transmission opportunities due to SIC is significant, i.e., between 20% and 100%. Second, the approximation ratio is not a sufficient indicator of the scheduling performance in a network with SIC. Even for a scheduling scheme with a constant approximation ratio, there are still many transmission opportunities not yet exploited.

VII. CONCLUSIONS AND FUTURE WORK

This paper investigates the scheduling performance in wireless networks with successive interference cancellation. After introducing two interference models to capture the impact of SIC, we show that the capacity in a network with SIC has the same order as that without SIC. It is therefore not surprising that a scheduling scheme unaware of SIC maintains its order optimality when SIC is available. We examine the impact of SIC from first principles and find out that a significant throughput gain between 20% and 100% is obtained from SIC. Moreover, the performance gain of a scheduling scheme is essentially correlated with the usage of the transmission opportunities from SIC. This work demonstrates the importance of designing an SIC-aware scheduling scheme, and suggests that the approximation ratio is not a sufficient indicator of the scheduling performance when SIC is available.

There are several directions to extend the work. First, it is one of our ongoing works to define a performance metric to properly evaluate the usage of the SIC capability for a scheduling scheme. Second, it is important to consider the joint design of link scheduling and power control in a network with SIC. Third, it is necessary to consider the effect of imperfect signal removal, especially in a near-far situation. Finally, it is interesting to study link scheduling in a network with both SIC and rata adaptation. Currently, we do not consider the effect of rate adaptation and the present studies should be revisited carefully when the transmission rate is adjusted adaptively.

VIII. SUMMARY OF THE PROOFS

Proof of Theorem 1: With Lemma 1, it is sufficient to show that the optimal schedule length is at least $\Omega(\Delta^{in})$.

Suppose the incoming degree of L_{SR} is Δ^{in}. For any incoming link $L_{S'R'}$ of L_{SR}, we have $|S'R| \le (1+\delta)r$. Using R as the center, we can draw K circles with radius $d_k = (1+\delta)kr/K$ ($k = 1, \ldots, K$) and divide the interference zone of L_{SR} into several regions (cf. Fig. 1). The number of the regions is at most $K\lceil 2\pi/\alpha \rceil$. Both K and α are constants determined by r and δ. The values of them will be specified later. For a ($k-1$, k, α) region, letting $D(k-1, k, \alpha)$ be the maximum distance between any two points in the region, we have

$$D(k-1, k, \alpha) = \max\{|A_{k,1}A_{k-1,1}|, |A_{k,1}A_{k-1,2}|, |A_{k,1}A_{k,2}|\}.$$

It is clear that $|A_{k,1}A_{k-1,1}| = (1+\delta)r/K$, and

$$|A_{k,1}A_{k-1,2}| = \sqrt{d_k^2 + d_{k-1}^2 - 2d_kd_{k-1}\cos\alpha}$$
$$|A_{k,1}A_{k,2}| = d_k\sqrt{2 - 2\cos\alpha}.$$

Now, we show that, if

$$D(k-1, k, \alpha) \le \delta \cdot r \tag{7}$$

then any two incoming links whose senders are in the same region must interfere with each other. Considering two such incoming links, e.g., $L_{S_1R_1}$ and $L_{S_2R_2}$, then

$$|S_1R_2| \le |S_1S_2| + |S_2R_2| \le r + D(k-1, k, \alpha) \le (1+\delta)r.$$

Next, we show how to choose K and α to satisfy (7), which is equivalent to the following three inequalities

$$(1+\delta)r/K \leq \delta \cdot r \tag{8}$$

$$\sqrt{d_k^2 + d_{k-1}^2 - 2d_k d_{k-1}\cos\alpha} \leq \delta \cdot r \tag{9}$$

$$d_k\sqrt{2-2\cos\alpha} \leq \delta \cdot r. \tag{10}$$

Substituting $d_{k-1} = d_k - (1+\delta)r/K$ in (9), we have

$$\begin{aligned} f(d_k) =& 2(1-\cos\alpha)\cdot d_k^2 \\ & - 2(1-\cos\alpha)\cdot\frac{1+\delta}{K}\cdot r\cdot d_k + \frac{(1+\delta)^2}{K^2}\cdot r^2 \\ & \leq \delta^2\cdot r^2. \end{aligned} \tag{11}$$

It is clear that $f(d_k)$ monotonically increases with d_k when $d_k > (1+\delta)r/K$ (which is true when $k > 1$). Thus, $f(d_k)$ is maximized when $d_k = (1+\delta)r$ (i.e., $k = K$). Substituting $d_k = (1+\delta)r$ in (11) and rearranging, we obtain

$$\frac{(1-x\mathcal{A})^2 + 1 - \mathcal{A}^2}{2(1-x\mathcal{A})} \leq \cos\alpha \tag{12}$$

where $\mathcal{A} = \delta/(1+\delta)$ and $x = 1/(K\cdot\mathcal{A})$.

Choosing $K = (1+\delta)^2/\delta^2$ and substituting it in (12) yield

$$\cos\alpha \geq 1 - \frac{\delta^2}{2(1+\delta)^2}.$$

It can be verified that, when $K = (1+\delta)^2/\delta^2$ and $\alpha = \arccos(1-\frac{\delta^2}{2(1+\delta)^2})$, (8) - (10) are all satisfied.

Now we divide the set of the incoming links of L_{SR} into several subsets such that the incoming links whose senders are in the same region are grouped together. The number of the subsets is at most $K\lceil\frac{2\pi}{\alpha}\rceil$. As the links in the same group must interfere with each other, the least number of slot required by an optimal solution is

$$\frac{\Delta^{in}}{K\lceil 2\pi/\alpha\rceil} = \Omega(\Delta^{in}).$$

Proof of Theorem 2: With Lemma 1, it is sufficient to show that the optimal schedule length is at least $\Omega(\Delta^{in})$.

Suppose the incoming degree of L_{SR} is Δ^{in}. First, consider the case of $M = 2$. Let $\mathcal{S}_1$ denote the set of the incoming links $L_{S'R'}$ of L_{SR} such that $|S'R'| \leq r_1$ and $\mathcal{S}_2$ the set of the remaining incoming links. Obviously, $|\mathcal{S}_1| + |\mathcal{S}_2| = \Delta^{in}$. Then,

- We can divide $\mathcal{S}_1$ into at most $K_1\lceil\frac{2\pi}{\alpha_1}\rceil$ subsets, where K_1 and α_1 are determined by $\{r_1, r_2, \delta_1, \delta_2\}$. The links in the same subset interfere with each other.
- We can divide $\mathcal{S}_2$ into at most $K_2\lceil\frac{2\pi}{\alpha_2}\rceil$ subsets, where K_2 and α_2 are determined by $\{r_1, r_2, \delta_1, \delta_2\}$. In each subset, at most two links can transmit simultaneously.

The optimal schedule length is at least

$$\begin{aligned} &\max\{\frac{|\mathcal{S}_1|}{K_1\lceil 2\pi/\alpha_1\rceil}, \frac{|\mathcal{S}_2|}{2K_2\lceil 2\pi/\alpha_2\rceil}\} \geq \min\{\frac{\Delta^{in}}{2K_1\lceil 2\pi/\alpha_1\rceil}, \frac{\Delta^{in}}{4K_2\lceil 2\pi/\alpha_2\rceil}\} \\ &= \frac{\Delta^{in}}{\max\{2K_1\lceil 2\pi/\alpha_1\rceil, 4K_2\lceil 2\pi/\alpha_2\rceil\}} = \Omega(\Delta^{in}). \end{aligned}$$

Let $r = r_{\mathcal{U}(S,R)}$, $\delta = \delta_{\mathcal{U}(S,R)}$. For $\mathcal{S}_1$, we can draw K_1 circles centering at R with radius $d_k = (1+\delta)r/K_1$ $(k = 1, \ldots, K_1)$ and then divide the interference zone of L_{SR} into $\mathcal{Z}_1$ regions, where $\mathcal{Z}_1 \leq K_1\lceil 2\pi/\alpha_1\rceil$. Then $\mathcal{S}_1$ is divided into $\mathcal{Z}_1$ subsets such that the links whose senders are in the same region are grouped together. Consider a $(k-1,\ k,\ \alpha_1)$ region, and let $D(k-1, k, \alpha_1)$ be the maximum distance between any two points within a $(k-1,\ k,\ \alpha_1)$ region. If

$$D(k-1, k, \alpha_1) \leq \delta_1 \cdot r_1 \tag{13}$$

then any two incoming links in $\mathcal{S}_1$ whose senders are in the same region must interfere with each other. For two such incoming links, e.g., $L_{S_1R_1}$ and $L_{S_2R_2}$, we have

$$|S_1R_2| \leq |S_1S_2| + |S_2R_2| \leq r_1 + D(k-1, k, \alpha_1) \leq (1+\delta_1)r_1.$$

With the same process as in the proof of Theorem 1, one can obtain a feasible setting of K_1 and α_1 to satisfy (13), i.e., $K_1 = \beta^2$, and $\alpha_1 = \arccos(1-\frac{1}{2\beta^2})$, where $\beta = \frac{(1+\delta)}{\delta_1}\cdot\frac{r}{r_1}$. As $\mathcal{U}(S,R)$ can be 1 or 2, we set

$$\beta = \max\{\frac{(1+\delta_1)}{\delta_1}, \frac{(1+\delta_2)}{\delta_1}\cdot\frac{r_2}{r_1}\}.$$

For $\mathcal{S}_2$, we can draw K_2 circles centering at R with radius $d_k = (1+\delta)r/K_2$ $(k = 1, \ldots, K_2)$. Similarly, we divide the interference zone into $\mathcal{Z}_2$ regions, where $\mathcal{Z}_2 \leq K_2\lceil 2\pi/\alpha_2\rceil$, and $\mathcal{S}_2$ into $\mathcal{Z}_2$ subsets such that the links whose senders are in the same region are grouped together. Consider a $(k-1,\ k,\ \alpha_2)$ region, and let $D(k-1, k, \alpha_2)$ be the maximum distance between any two points within a $(k-1,\ k,\ \alpha_2)$ region. If

$$D(k-1, k, \alpha_2) \leq \delta_2 \cdot r_2 \tag{14}$$

then at most two incoming links in $\mathcal{S}_2$ whose senders are in the same region can transmit simultaneously. For two such incoming links, e.g., $L_{S_1R_1}$ and $L_{S_2R_2}$, we have

$$|S_1R_2| \leq |S_2R_2| + |S_1S_2| \leq r_2 + D(k-1, k, \alpha_2) \leq (1+\delta_2)r_2.$$

As $M = 2$, in a composite signal, at most one signal can be removed by SIC. Hence, when three or more links transmit simultaneously, at any receiver node, at least one interfering signal that can interfere the two-level signal is retained. As every link in $\mathcal{S}_2$ is a two-level link, a detection failure must occur when three or more links in the same subset of $\mathcal{S}_2$ transmit simultaneously.

With the same process as in the proof of Theorem 1, a feasible setting of K_2 and α_2 can be derived to satisfy (14): $K_2 = \beta'^2$, and $\alpha_2 = \arccos(1-\frac{1}{2\beta'^2})$, where

$$\beta' = \max\{\frac{(1+\delta_2)}{\delta_2}, \frac{(1+\delta_1)}{\delta_2}\cdot\frac{r_1}{r_2}\}.$$

Now turn to the case of $M \geq 3$. We divide the incoming links of L_{SR} into M groups: for $1 \leq j \leq M$, $\mathcal{S}_j$ contains the incoming link $L_{S'R'}$ such that $r_{j-1} < |S'R'| \leq r_j$. Then,

- We can divide $\mathcal{S}_1$ into $K_1\lceil\frac{2\pi}{\alpha_1}\rceil$ subsets, where K_1 and α_1 are determined by $\{r_1, \ldots, r_M, \delta_1, \ldots, \delta_M\}$. The links in the same subset interfere with each other.
- We can divide $\mathcal{S}_j$ $(2 \leq j \leq M)$ into $K_j\lceil\frac{2\pi}{\alpha_j}\rceil$ subsets, where K_j and α_j are determined by $\{r_1, \ldots, r_M, \delta_1, \ldots, \delta_M\}$. In each subset, at most two links can transmit simultaneously.

The optimal schedule length is at least

$$\max\{\frac{|\mathcal{S}_1|}{K_1\lceil 2\pi/\alpha_1\rceil}, \frac{|\mathcal{S}_2|}{2K_2\lceil 2\pi/\alpha_2\rceil}, \dots, \frac{|\mathcal{S}_M|}{2K_M\lceil 2\pi/\alpha_M\rceil}\}$$
$$\geq \min\{\frac{\Delta^{in}}{MK_1\lceil \frac{2\pi}{\alpha_1}\rceil}, \frac{\Delta^{in}}{2MK_2\lceil 2\pi/\alpha_2\rceil}, \dots, \frac{\Delta^{in}}{2MK_M\lceil 2\pi/\alpha_M\rceil}\}$$
$$= \frac{\Delta^{in}}{M\cdot\max\{K_1\lceil 2\pi/\alpha_1\rceil, 2K_2\lceil 2\pi/\alpha_2\rceil, \dots, 2K_M\lceil 2\pi/\alpha_M\rceil\}}$$
$$= \Omega(\Delta^{in}).$$

First, for $1 \leq i \leq M$, $3 \leq j \leq M$, define

$$\beta_i = \max\{\frac{(1+\delta_1)}{\delta_i}\cdot\frac{r_1}{r_i}, \dots, \frac{(1+\delta_M)}{\delta_i}\cdot\frac{r_M}{r_i}\}$$
$$\beta_{j(2)} = \max\{\frac{(1+\delta_1)r_1}{r_{j-1}-(1+\xi)r_{j-2}}, \dots, \frac{(1+\delta_M)r_M}{r_{j-1}-(1+\xi)r_{j-2}}\}$$

where $\xi \in (0, \min\{\frac{r_2}{r_1}, \dots, \frac{r_{M-1}}{r_{M-2}}\} - 1)$ is a small constant.

Let $r = r_{\mathcal{U}(S,R)}$, $\delta = \delta_{\mathcal{U}(S,R)}$. For each $1 \leq j \leq M$, we can draw K_j circles centering at R with radius $d_k = (1+\delta)r/K_j$ ($k = 1, \dots, K_j$), and divide the interference zone into $\mathcal{Z}_j$ regions, where $\mathcal{Z}_j \leq K_j\lceil 2\pi/\alpha_j\rceil$, and $\mathcal{S}_j$ into $\mathcal{Z}_j$ subsets such that the links whose senders are in the same region are grouped together.

The processes of both $\mathcal{S}_1$ and $\mathcal{S}_2$ are similar to that when $M = 2$. A feasible setting for $\mathcal{S}_1$ is $K_1 = 1/\beta_1^2$, and $\alpha_1 = \arccos(1 - \frac{1}{2\beta_1^2})$, while a feasible setting for $\mathcal{S}_2$ is $K_2 = 1/\beta_2^2$, and $\alpha_2 = \arccos(1 - \frac{1}{2\beta_2^2})$.

For $\mathcal{S}_j$ ($3 \leq j \leq M$), consider a ($k-1$, k, α_j) region, and let $D(k-1, k, \alpha_j)$ be the maximum distance between any two points within a ($k-1$, k, α_j) region. If

$$D(k-1, k, \alpha_j) \leq \delta_j \cdot r_j \tag{15}$$

and

$$D(k-1, k, \alpha_j) \leq r_{j-1} - (1+\xi)r_{j-2} \tag{16}$$

then at most two incoming links in $\mathcal{S}_j$ whose senders are in the same region can transmit simultaneously. For two such incoming links, e.g., $L_{S_1R_1}$ and $L_{S_2R_2}$, we have

$$|S_1R_2| \leq |S_2R_2| + |S_1S_2| \leq r_j + D(k-1, k, \alpha_j) \leq (1+\delta_j)r_j.$$

On the other hand,

$$|S_1R_2| \geq |S_2R_2| - |S_1S_2| \geq r_{j-1} - D(k-1, k, \alpha_j) > r_{j-2}.$$

Thus, at R_2, the strongest interfering signal is at most a $(j-1)$-level signal, and the weakest one can at least interfere the j-level signal. As every link in $\mathcal{S}_j$ is a j-level link, any three links in $\mathcal{S}_j$ whose senders are in the same region cannot transmit simultaneously.

With the same process as in the proof of Theorem 1, to satisfy (15), a feasible setting can be derived as $K_{j(1)} = 1/\beta_j^2$, and $\alpha_j = \arccos(1 - \frac{1}{2\beta_j^2})$. Similarly, to satisfy (16), a feasible setting is $K_{j(2)} = 1/\beta_{j(2)}^2$, and $\alpha_{j(2)} = \arccos(1 - \frac{1}{2\beta_{j(2)}^2})$.

Finally, when $3 \leq j \leq M$, a feasible setting is given by $K_j = \max\{K_{j(1)}, K_{j(2)}\}$ and $\alpha_j = \min\{\alpha_{j(1)}, \alpha_{j(2)}\}$.

Proof of Theorem 3: With Lemma 2, it is sufficient to show that the optimal schedule length is at least $\Omega(\Delta_m^k)$.

We divide a cell into 9 sub-cells of side $\frac{\mu}{3}\cdot 2^k$. For two links $L_i, L_x \in \mathcal{L}_m^k$, when R_i and R_x are in the same sub-cell, we have $2^k \leq |S_xR_x| < 2^{k+1}$ and $|R_xR_i| \leq \frac{\sqrt{2}\mu}{3}\cdot 2^k$. Then,

$$|S_xR_i| \leq |S_xR_x| + |R_xR_i| \leq 2^{k+1} + \frac{\sqrt{2}\mu}{3}\cdot 2^k = (\frac{\sqrt{2}\mu}{3}+2)\cdot 2^k$$

and

$$|S_xR_i| \geq \max\{|R_xR_i| - |S_xR_x|, |S_xR_x| - |R_xR_i|\}$$
$$\geq |\frac{\sqrt{2}\mu}{3}\cdot 2^k - 2^{k+1}| = |\frac{\sqrt{2}\mu}{3} - 2|2^k.$$

Now we bound the number of L_i links such that (i) they transmit simultaneously, (ii) R_i is located in the same sub-cell, and (iii) $2^k \leq |S_iR_i| < 2^{k+1}$.

To ensure the successful detection of $L_{S_iR_i}$, all stronger interfering signals must be removed and the aggregate interference of the remaining should be tolerable. Consider a link $L_{S_xR_x}$ with $|S_xR_i| \leq |S_iR_i|$, we have

$$\theta \leq \frac{\frac{P}{|S_xR_i|^\eta}}{N_0 + \sum_{|S_yR_i|\geq|S_xR_i|}\frac{P}{|S_yR_i|^\eta}} \leq \frac{\frac{P}{|\sqrt{2}\mu/3-2|^\eta 2^{\eta k}}}{N_0 + \sum_{|S_yR_i|\geq|S_xR_i|}\frac{P}{|S_yR_i|^\eta}}$$
$$\leq \frac{\frac{P}{|\sqrt{2}\mu/3-2|^\eta 2^{\eta k}}}{q_1\cdot\frac{P}{(\sqrt{2}\mu/3+2)^\eta\cdot 2^{\eta k}}} = \frac{(\sqrt{2}\mu/3+2)^\eta}{q_1|\sqrt{2}\mu/3-2|^\eta}.$$

Thus, $q_1 \leq \frac{1}{\theta}(\frac{\sqrt{2}\mu/3+2}{|\sqrt{2}\mu/3-2|})^\eta$. On the other hand,

$$\theta \leq \frac{\frac{P}{|S_iR_i|^\eta}}{N_0 + \sum_{|S_yR_i|\geq|S_iR_i|}\frac{P}{|S_yR_i|^\eta}} \leq \frac{\frac{P}{2^{k\eta}}}{N_0 + \sum_{|S_yR_i|\geq|S_iR_i|}\frac{P}{|S_yR_i|^\eta}}$$
$$\leq \frac{\frac{P}{2^{k\eta}}}{q_2\cdot\frac{P}{(\sqrt{2}\mu/3+2)^\eta\cdot 2^{\eta k}}} = \frac{\cdot(\sqrt{2}\mu/3+2)^\eta}{q_2}.$$

Thus, $q_2 \leq (\sqrt{2}\mu/3+2)^\eta/\theta$. Therefore, if there are more than (q_1+q_2) links satisfying constraints (i)-(iii), for an active link L_i, either the signal of L_i or that of a correlated link of L_i cannot be detected. The least number of time slots required to schedule the links in Δ_m^k is

$$\frac{\Delta_m^k}{9\cdot(q_1+q_2)} = \Omega(\Delta_m^k).$$

Proof of Theorem 4: The proof is mainly based on [9]. Considering the bth bit ($1 \leq b \leq \lambda nT$), suppose that it moves from its origin to its destination in a sequence of $h(b)$ hops, where the hth hop traverses a distance of r_b^h. Then,

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} r_b^h \geq \lambda nT\overline{B}. \tag{17}$$

Note that in any slot at most $n/2$ nodes can transmit. Hence,

$$H := \sum_{b=1}^{\lambda nT} h(b) \leq \frac{WTn}{2}. \tag{18}$$

Consider a k-level link L_{SR} and a k'-level link $L_{S'R'}$ ($1 \leq k, k' \leq M$). If they can transmit simultaneously, we have

$$|RR'| \geq |SR'| - |SR| \geq (1+\delta_{k'})|S'R'| - |SR|.$$

$$|RR'| \geq |S'R| - |S'R'| \geq (1+\delta_k)|SR| - |S'R'|.$$

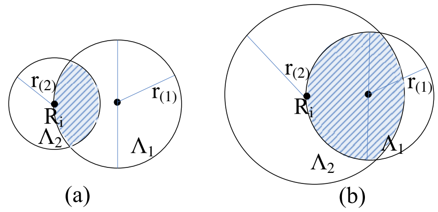

Fig. 14: Illustration of the common area between Λ_1 and Λ_2: (a) $r_{(2)} < \sqrt{2}r_{(1)}$; (b) $\sqrt{2}r_{(1)} \leq r_{(2)} < 2r_{(1)}$.

Adding the two inequalities, we obtain

$$|RR'| \geq \frac{1}{2}(\delta_{k'}|S'R'| + \delta_k|SR|) \geq \frac{\delta}{2}(|S'R'| + |SR|)$$

where $\delta = \min\{\delta_1, \ldots, \delta_M\}$.

Hence disks of radius $\frac{\delta}{2}$ times the lengths of hops centered at the receivers are essentially disjoint. Let $r_{(2)} = \frac{\delta}{2}r_b^h$ and $r_{(1)} = \sqrt{A/\pi}$, then $r_{(2)} < 2r_{(1)}$. Let Λ_1 denote the network coverage area and Λ_2 the disk centering at a receiver node R_i with radius $r_{(2)}$. The common area between Λ_1 and Λ_2 is minimized when R_i is near the periphery of Λ_1. The shadow regions in Fig. 14 illustrate the common area, for (a) $r_{(2)} < \sqrt{2}r_{(1)}$ and (b) $\sqrt{2}r_{(1)} \leq r_{(2)} < 2r_{(1)}$. It can be verified that the common area is at least a quarter of Λ_2. As at most $W\tau$ bits can be carried in a slot of length τ from a transmitter to a receiver, at any tth slot ($t \geq 1$), we have

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \mathcal{D}(b,h,t)\frac{\pi(\frac{\delta}{2}r_b^h)^2}{4} \leq AW\tau$$

where $\mathcal{D}(b,h,t)$ is one when the bth bit is transmitted over the hth hop at the tth slot, and zero otherwise. As T comprises one or more slots, summing over the slots gives

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{\pi\delta^2}{16A}(r_b^h)^2 \leq WT. \tag{19}$$

This can be rewritten as

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \leq \frac{16AWT}{\pi\delta^2 H}. \tag{20}$$

As the quadratic function is convex, we have

$$(\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h))^2 \leq \sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)^2 \tag{21}$$

which leads to

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} \frac{1}{H}(r_b^h)) \leq \sqrt{\frac{16AWT}{\pi\delta^2 H}}. \tag{22}$$

Substituting (17) and (18) in (22) gives

$$\lambda n\overline{B} \leq \sqrt{\frac{8}{\pi}}\frac{\sqrt{A}W}{\delta}\sqrt{n}.$$

Proof of Theorem 5: The proof is similar to that of Theorem 4 except the difference stemming from the need to replace (19) by a new expression.

Consider the reception of link L, suppose there are J ($J \leq N-1$) links active simultaneously with L and D ($D \leq J$) of them are correlated links of L. Without loss of generality, all the links are ordered with respect to the distance to the receiver of L as $L_1, \ldots, L_{J+1}$, where L_{D+1} is the targeting link L. Suppose $|S_1R_{D+1}| \leq \ldots \leq |S_{J+1}R_{D+1}|$ and the set of correlated links is $\{L_1, \ldots, L_D\}$. We have

$$\begin{aligned}
&P_D^{D+1} \geq \theta \cdot \sum_{D+1\leq x\leq J+1} P_x^{D+1}\\
&P_{D-1}^{D+1} \geq \theta \cdot (P_D^{D+1} + \sum_{D+1\leq x\leq J+1} P_x^{D+1}) \geq \theta(1+\theta)\sum_{D+1\leq x\leq J} P_x^{D+1}\\
&\ldots\\
&P_1^{D+1} \geq \theta(1+\theta)^{D-1}\sum_{D+1\leq x\leq J+1} P_x^{D+1}.
\end{aligned} \tag{23}$$

Note that $P_1^{D+1} \leq P$ and $\sum_{D+1\leq x\leq J+1} P_x^{D+1} \geq (J-D+1)\cdot\frac{P}{(2\sqrt{A}/\sqrt{\pi})^\eta}$. Substituting the last inequality in (23), we obtain

$$P \geq \theta(1+\theta)^{D-1}\cdot(J-D+1)\cdot\frac{P}{(2\sqrt{A}/\sqrt{\pi})^\eta}. \tag{24}$$

As $J-D+1 \geq 1$, we obtain

$$D \leq 1 + \frac{\eta\log\frac{2\sqrt{A}}{\sqrt{\pi}} - \log\theta}{\log(1+\theta)}. \tag{25}$$

For the detection of L_{D+1}, we have

$$\frac{P/|S_{D+1}R_{D+1}|^\eta}{N_0 + \sum_{(D+1)\leq j\leq J+1, j\neq k} P/|S_jR_{D+1}|^\eta} \geq \theta.$$

Rewriting the inequality, we obtain

$$\frac{P/|S_{D+1}R_{D+1}|^\eta}{N_0 + \sum_{(D+1)\leq j\leq J+1} P/|S_jR_{D+1}|^\eta} \geq \frac{\theta}{1+\theta}. \tag{26}$$

Hence,

$$\begin{aligned}
|S_{D+1}R_{D+1}|^\eta &\leq \frac{1+\theta}{\theta}\frac{P}{N_0 + \sum_{(D+1)\leq j\leq J+1} P/|S_jR_{D+1}|^\eta}\\
&\leq \frac{1+\theta}{\theta}\frac{P}{N_0 + (\frac{\pi}{4A})^{\eta/2}(J-D+1)P}
\end{aligned} \tag{27}$$

which leads to

$$\begin{aligned}
\sum_{1\leq k\leq J+1} |S_kR_k|^\eta &\leq \frac{1+\theta}{\theta}\frac{(J+1)P}{N_0 + (\frac{\pi}{4A})^{\eta/2}(J-D+1)P}\\
&\leq \frac{1+\theta}{\theta}\cdot(\frac{4A}{\pi})^{\eta/2}\cdot\frac{J+1}{J-D+1}\\
&\leq \frac{1+\theta}{\theta}\cdot(\frac{4A}{\pi})^{\eta/2}\cdot(1+D).
\end{aligned} \tag{28}$$

As a result, we have

$$\sum_{b=1}^{\lambda nT}\sum_{h=1}^{h(b)} (r_b^h)^\eta \leq \frac{1+\theta}{\theta}\cdot 2^\eta\cdot(A/\pi)^{-\eta/2}\cdot(1+D)WT. \tag{29}$$

The rest of the proof proceeds along lines similar to those of Theorem 4, invoking the convexity of r^η instead of r^2. Finally, we obtain

$$\lambda n\overline{B} \leq (\frac{2+2\theta}{\theta})^{1/\eta}\cdot\frac{\sqrt{A}W}{\sqrt{\pi}}\cdot(1+D)^{1/\eta}\cdot n^{(\eta-1)/\eta}. \tag{30}$$

For non-uniform transmission power, let $\sigma = P_{\max}/P_{\min}$, where $P_{\max}$ is the maximum transmission power and $P_{\min}$ the minimum one. The above statement is still valid when σ is a small constant except a new expression for the upper bound of D. It is easy to see that (23) still holds. Afterwards, we have $P_1^{D+1} \le P_{\max}$ and $\sum_{D+1\le x\le J+1} P_x^{D+1} \ge (J-D+1)\cdot \frac{P_{\min}}{(2\sqrt{A}/\sqrt{\pi})^\eta}$. Then, we obtain

$$P_{\max} \ge \theta(1+\theta)^{D-1}\cdot(J-D+1)\cdot\frac{P_{\min}}{(2\sqrt{A}/\sqrt{\pi})^\eta}. \quad (31)$$

As $J-D+1\ge 1$, we have

$$\sigma \ge P_{\max}/P_{\min} \ge \theta(1+\theta)^{D-1}\cdot\frac{1}{(2\sqrt{A}/\sqrt{\pi})^\eta}. \quad (32)$$

Rewriting the inequality, we obtain

$$D \le 1+\frac{\eta\log\frac{2\sqrt{A}}{\sqrt{\pi}}+\log\sigma-\log\theta}{\log(1+\theta)}. \quad (33)$$

Following the same process, we can obtain the capacity finally with the same expression as (30).

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