Calculation of Loss Probability in a Finite Size Partitioned Buffer for Quantitative Assured Service

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***Abstract*—This paper proposes an approximate yet accurate approach to calculate the loss probabilities in a finite size partitioned buffer system for the achievement of a quantitative assured service in differentiated services networks. The input is modeled as a fractional Brownian motion (FBM) process including J classes of traffic with different packet loss requirements. A first-in first-out buffer partitioned with $J-1$ thresholds is used to provide J loss priorities. Heuristic expressions of the loss probabilities for all the J classes are derived, and validated by computer simulations. The proposed loss calculation technique is then extended to a general input process by using the recently proposed traffic substitution technique, where both long-range dependent and short-range dependent input sources are equivalent to a properly parameterized FBM. We also apply the loss calculation to admission control, where the partition thresholds are optimally configured for quality of service guarantee and maximal resource utilization. Computer simulation results demonstrate that resource allocation based on the accurate finite buffer loss analysis results in much more efficient resource utilization than that based on the classic large-buffer overflow approximation.**

***Index Terms*—Fractional Brownian motion (FBM), long-range dependence (LRD), loss probability, quantitative assured service, traffic substitution.**

I. Introduction

IN THE differentiated services (DiffServ or DS) model [1] for Internet quality of service (QoS), traffic flows having similar QoS requirements are aggregated into a common service class and experience the same per hop behavior (PHB) within the network. Two PHBs have been standardized, which are the *expedited forwarding* (EF) PHB [2] and the *assured forwarding* (AF) PHB [3]. The EF PHB is intended to achieve a low loss, low delay, and low jitter *premium service* with peak rate bandwidth allocation. However, the premium service results in low bandwidth utilization and is to be charged at a high price. The AF PHB aims at achieving a predictable throughput by using an in-class loss differentiation scheme [3], which characterizes an *assured service*. Although the assured service achieves higher resource utilization, it does not support *quantitative* QoS guarantee [4], [5], which considerably limits its applications. For example, it is defined in International Telecommunication Union—Telecommunication Standardization Sector (ITU-T) G.114 [6] that, for real-time services, the tolerable packet loss rate is 1%–3%. The throughput guarantee for the Transmission Control Protocol (TCP) traffic is also quantitatively related to the packet loss probability (PLP) control [7], [8].

In this paper, we study the loss probability calculation and admission control for a *partitioned buffer* [9], in order to achieve a quantitative AF (QAF) PHB. Consider that the AF traffic is marked for $J (J \geq 2)$ levels of packet dropping precedences. Hereafter, we limit the discussions within the AF context and refer to the J dropping precedences of the assured service as J service classes for convenience. A first-in first-out (FIFO) buffer of size B partitioned with $J-1$ thresholds is used to provide the J loss priorities within the service, while preserving the delivery order of all the AF packets. When the queue level reaches or exceeds the partition threshold $B_j (1 \leq j \leq J-1)$, newly arrived packets of class 1 to class j are dropped, and those of class $(j+1)$ to class J are accepted. In the QAF PHB, traffic for each class can have a unique PLP requirement; sufficient bandwidth should be allocated to serve the partitioned buffer under admission control [10] to guarantee the packet loss requirements of all the J classes. While the AF traffic classes are normally determined by a traffic conditioner to differentiate the *in* profile and *out* profile traffic [3], the content-aware traffic classification and service differentiation are also meaningful and important. For example, the layer-coded video traffic, where a *base layer* contains the most important features of the video and some *enhancement layers* contain data refining the reconstructed video quality [11], can be well supported by the proposed QAF PHB with different layers marked to different classes for different levels of loss protection. It is noteworthy that our focus is on provisioning QoS to User Datagram Protocol (UDP) traffic, while the interaction between the TCP (in the transport layer) and the AF PHB (in the network layer) [8] is out of the scope of this paper.

Packet loss calculation in the partitioned buffer system with multiclass Markovian modulated fluid traffic sources has been studied in [9] and [12]–[14]. However, all the previous studies have the following limitations: 1) the *loss probability* is approximated by the *overflow probability* in an infinite buffer system. It is well known that the overflow probability is normally a very conservative approximation of the loss probability [15]; 2) in the Markov modeling, when a large number of Markovian sources are multiplexed, the large state space of the aggregate arrival process results in computational infeasibility. Our

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study in [9] avoids the complexity in dealing with the aggregate traffic by using the *effective bandwidth* of a single multiclass Markovian source derived in the large-buffer asymptotic regime. However, the effective bandwidth approach cannot fully exploit the statistical multiplexing gain in admission control; 3) extensive network traffic measurement/analysis studies suggest that Internet traffic exhibits self-similar property or long-range dependence (LRD) [16]–[18], which cannot be captured by the short-range dependent (SRD) Markovian model.

This paper proposes an approximate yet accurate approach to calculate the PLP in a finite size partitioned buffer system, which has not been considered in the open literature, to the best of our knowledge. The input traffic is modeled as a fractional Brownian motion (FBM) process [19] for three reasons. First, in current high-speed networks, a large number of sources are expected to be multiplexed at a link. According to the *central limit theorem*, the aggregate traffic can be modeled as a Gaussian process [20]. The FBM is a self-similar Gaussian process, which is suitable for modeling the traffic aggregate and is able to capture the LRD within the traffic. Second, Kim and Shroff [15] recently proposed an approximation and the associated asymptotic analysis for the loss probability in a nonpartitioned finite buffer system with a Gaussian (including the FBM) input, which serves as the theoretical foundation of the loss analysis in this paper. Last, a recent study [21] shows that a properly parameterized FBM process can be used as a traffic equivalence or substitute to both LRD and SRD sources. The loss calculation technique proposed for the FBM model is then applicable to a general traffic input via the traffic substitution.

The study in [15] shows that the loss probability $P_L(x)$ in a finite buffer of size x can be approximately mapped from the overflow probability $\mathbb{P}\{Q > x\}$ of an infinite buffer system. Such a mapping relationship is extended to the partitioned buffer system in this study. The queueing analysis in [15] is based on the Lindley's equation, and assumes that the queueing process in both the infinite buffer and the finite buffer systems converges to a stationary and ergodic process when the input Gaussian process has stationary and ergodic increments.[1] The ergodicity of the queueing process validates the practical measurements of loss/overflow probability by *time averaging*. In the partitioned buffer system, we also assume that the multiclass input process has stationary and ergodic increments (which is modeled as an aggregate of J independent FBM processes), and the queueing process in both the infinite and the finite buffer systems can converge to a stationary and ergodic process as time $t \to \infty$. However, due to the differentiated dropping each time the partition thresholds $B_j (1 \leq j \leq J-1)$ are being crossed, it is difficult, if not impossible, to carry out the queueing analysis based on the Lindley's equation. For mathematical tractability of loss probabilities in the partitioned buffer system, we resort to a stronger *localized steady state* (LSS) assumption regarding the queueing behavior in a *partition region* (confined by two neighboring partition thresholds) that the steady-state overflow probability in a partition region can be determined by the initial status entering the region, the localized queueing behavior in the region (as in a separate buffer with the corresponding input), and the correlation within the input process characterized by the Hurst parameter H. Under the LSS assumption, we develop an iterative algorithm to calculate the loss probabilities for all the J classes by investigating the "loss versus overflow" mapping relationship between a finite size partitioned buffer and an infinite partitioned buffer. The LRD input process further complicates the overflow/loss analysis of the partitioned buffer system, compared to the system with an SRD input where the queueing behavior in each partition region can be considered approximately independent of each other in the large-buffer asymptotic regime [9], [14]. In this paper, we propose a novel approach to capture the correlations within the partitioned buffer system by including the *Hurst parameter* in the iterative loss calculation.

Due to the theoretical difficulty, various approximations are integrated for the loss estimation; some of the approximations are supported by large-buffer asymptotic analysis[2] and some are from intuitive conjectures. We resort to computer simulations to evaluate the performance. The simulation results presented in [15] demonstrate that the accuracy of the loss estimations by the mapping technique is within the range of one order. As our proposed loss calculations for a partitioned buffer system is based on [15], we aim at achieving the same level of accuracy as that in the nonpartitioned system. Achievement of the target accuracy in the loss estimation for various buffer sizes is demonstrated by extensive computer simulation results presented in Section VIII.

The proposed loss calculation technique is also extended to deal with a general multiclass input process by applying the traffic substitution technique [21]. Under the LSS assumption, the traffic substitution is first applied in each partition region, respectively; the iterative loss analysis is then implemented upon the equivalent FBM substitutions. We apply the FBM-equivalent-based loss analysis to the multiplexed SRD ON/OFF sources that are studied in [9], and obtain a much better loss estimate than that under the large-buffer overflow approximation, which is demonstrated via simulations. Based on the accurate loss analysis, efficient bandwidth allocation and admission control can be achieved to guarantee QoS. Moreover, our former study [9] shows that the optimal configuration of the $J-1$ partition thresholds can minimize the bandwidth required to simultaneously satisfy the packet loss requirements of all the J classes. In this paper, the optimal buffer partitioning based on

[1] If the input process has stationary and ergodic increments (i.e., the increment process is stationary and ergodic), it has been shown that the queueing process in an infinite buffer converges to a stationary and ergodic process [22]. However, the stationarity and ergodicity of the queueing process in a finite buffer can only be proved for a few specific queueing systems, e.g., GI/GI/m/x and G/M/m/x, referring to [15] and references therein.

[2] In this paper, when we mention the approximation associated with an asymptotic relationship, we mean that the dominant item in the asymptotic expression is applied to estimate the overflow/loss probability with the buffer size or partition thresholds set at *practical* values, notwithstanding the asymptotic relationship requires *infinite* buffer size or partition regions. The deviation of the approximation from the true value is evaluated by simulations. This way of tackling the asymptotic analysis and the associated overflow/loss approximation is extensively adopted in the literature [14], [15], [20], [31], [40].

the finite buffer loss analysis is also investigated for maximal resource utilization.

The remainder of this paper is organized as follows. In Section II, we give an additional review of related literature. The system model is then described in Section III. Section IV summarizes the loss probability calculation for finite buffer multiplexers. In Section V, we develop the finite buffer loss calculation technique for the partitioned buffer case. Section VI discusses how to extend the proposed loss calculation to a general input process via the traffic substitution. Admission control and optimal buffer partitioning is investigated in Section VII. Section VIII presents the simulation results to demonstrate the accuracy of the proposed loss calculation technique and the resource utilization improvement over that based on the large-buffer overflow analysis. Finally, in Section IX, we provide some concluding remarks.

II. Related Work

It is commonly thought that the traffic aggregate at a high-speed link can be well approximated by a Gaussian process according to the central limit theorem. However, traffic measurements in some cases [23], [24] do not show an agreement with the Gaussian marginal distribution. It is pointed out in [23] that if the statistical multiplexing of a large number of sources cannot smooth out the overall traffic when some of the individual sources are busty, the *generalized central limit theorem* [25] should be applied, resulting in an α-stable marginal distribution that models well both the burstiness and self-similarity. The Gaussian distribution is a special case of the α-stable distribution family. In this paper, we focus on the Gaussian model, especially the FBM, due to its popularity and mathematical tractability [15], [16], [19], [20], [26]–[31].

In performance analysis, the overflow probability $\mathbb{P}\{Q > x\}$ is averaged over time while the loss probability $P_L(x)$ is averaged over the input traffic; it is difficult to reveal a general relationship between these two quantities. For a discrete-time fluid queue with a Gaussian input, it is shown in [15] that asymptotically the loss probability and the overflow probability curves are quite similar, and if they diverge, they do so slowly. Based on such a finding, a loss approximation is obtained by a simple mapping from the overflow probability. Such a mapping relationship also exists in several other queueing systems, referring to the references given in [15]. The loss mapping technique proposed in [15] inspires our work in this paper.

By the mapping technique, the precondition for an accurate approximation of $P_L(x)$ is to find a good estimate of $\mathbb{P}\{Q > x\}$. While the overflow analysis has been widely studied in the large-buffer or many-source asymptotic regime and can be found in many papers (e.g., [20], [28], [30] and references therein), the maximum variance asymptotic (MVA) approach [20], [26], [31] based on *extreme value theory* is able to provide an accurate estimate of the overflow probability when x varies from a very small value to a very large value. The MVA approach is adopted for overflow analysis in [15] and in this paper.

In addition to the studies on the partitioned buffer system [9], [12]–[14] for quantitative DiffServ, a simple Markovian model is described and solved in [32] to quantify the QoS measures of the premium service and the assured service with the assumption of Poisson arrivals. However, applicability of the results to practical Internet traffic is doubtful due to the well-known fact that the Poisson model fails at the packet level [33]. The effect of transporting MPEG traffic over the assured service is studied in [34], where the simulation results show that the content-blind in/out profile marking may drastically deteriorate the delivery of the important packets. A content-aware mapping scheme that distributes importance-graded video packets to different DS classes or loss priorities (within one class) is proposed in [35]. The mapping scheme is able to achieve consistent service differentiation and to enhance end-to-end video quality. It is possible for the QAF PHB proposed in this paper to be seamlessly integrated to the aforementioned mapping scheme for a loss-guaranteed PHB in addition to the relative service differentiation.

In the definition of AF PHB, active queue management (e.g., the random early detection (RED) [36] with in/out (RIO) [37]) instead of the tail-dropping mechanism is suggested. The RIO scheme is not considered in this paper mainly for mathematic tractability. Another reason is that active queue management schemes primarily benefit the TCP traffic from avoiding global synchronization during congestion; this paper focuses on the UDP traffic for which the partitioned buffer (without active queue management) under admission control is sufficient to provision in-flow loss differentiation and quantitative loss probability guarantee. In addition, the QAF PHB defined in this paper is different from that defined in [4] and [38], where the quantitative delay/loss guarantee is provisioned in the context without in-flow loss differentiation; however, in-flow loss differentiation is the main characteristic of the assured service that cannot be omitted.

III. System Model

A. Partitioned Buffer

Consider a partitioned buffer of size B served by a channel of constant capacity c. The input traffic includes J ($\geq$2) classes that have different loss probability requirements. The traffic admission policy is based upon a space reservation scheme, using the buffer partition vector $\boldsymbol{B}_t = (B_1, B_2, \ldots, B_{J-1})$ to provide J loss priorities, where $0 < B_1 < B_2 < \cdots < B_{J-1} < B$. For convenience, set $B_0 = 0$ and $B_J = B$. Let $Q(t)$ be the amount of traffic queued in the buffer at time t. When $B_{j-1} \leq Q(t) < B_j$ $(1 \leq j \leq J)$, only traffic of classes $\{j,\ j+1, \ldots, J\}$ is admitted into the buffer; traffic in the buffer is served according to the FIFO rule. With the partitioned buffer, the class J traffic is served with the highest priority and the smallest loss probability, while the class 1 traffic the lowest priority and the largest loss probability.

The *loss probability* P_L is defined as the long-term ratio of the amount of the lost traffic to the amount of the total input traffic, assuming ergodicity. Let P_L^j denote the loss probability of class j traffic. If each class j traffic has a loss probability requirement $\epsilon_j (1 \leq j \leq J)$ for a QAF PHB, the resource manager or the admission controller should guarantee enough bandwidth to serve the accepted traffic so that the QoS of all the traffic classes are satisfied, i.e.,

$$P_L^j \le \epsilon_j, \qquad 1 \le j \le J \tag{1}$$

where $\epsilon_1 > \epsilon_2 > \cdots > \epsilon_J > 0$.[3]

B. Multiclass FBM Process

The FBM is used to model the input traffic $A(t)$. The standard (normalized) FBM process $\{Z(t) : t \ge 0\}$ with Hurst parameter $H \in [0.5, 1)$ is a centered Gaussian process with stationary and ergodic increments that possesses the following properties [19]: 1) $Z(0) = 0$, 2) $\mathrm{Var}\{Z(t)\} = t^{2H}$, and 3) $Z(t)$ is sample continuous path. The self-similar FBM input $\{A(t) : t \ge 0\}$ can be represented by

$$A(t) = \lambda t + \sigma Z(t) \tag{2}$$

where the mean arrival rate $E\{A(t)/t\} = \lambda$ and the variance $\mathrm{Var}\{A(t)\} = \sigma^2 t^{2H}$. Note that σ^2 is the variance of traffic in a time unit. When $0.5 < H < 1$, the FBM is both self-similar and LRD.

We consider the input process including J classes of traffic. Assume that all the traffic classes are independent and have the same Hurst parameter H. The class j input $A_j(t)$ is an FBM process characterized by λ_j, σ_j^2, and H. The number of class j packets arrived during $[0, t]$ is

$$A_j(t) = \lambda_j t + \sigma_j Z_j(t), \quad 1 \le j \le J. \tag{3}$$

It is easy to prove that the total input process $A(t) = \sum_{j=1}^{J} A_j(t)$ is an FBM process characterized by the parameters of $\lambda = \sum_{j=1}^{J} \lambda_j$, $\sigma^2 = \sum_{j=1}^{J} \sigma_j^2$, and H. When $B_{j-1} \le Q(t) < B_j$ $(1 \le j \le J)$, the input is an FBM process with parameters $\sum_{r=j}^{J} \lambda_r$, $\sum_{r=j}^{J} \sigma_r^2$, and H. Note that when the Hurst parameter of $A_j(t)$ changes with j, the aggregate $A(t)$ is not an FBM but an *asymptotically second-order self-similar* process [39]. In this case, we can resort to the traffic substitution technique presented in Section VI to find an equivalent FBM.

For the convenience of mathematical analysis, the buffer system is modeled as a discrete-time fluid queue as that used in [15] and [20], where the time index t takes only integer values. In the remainder of this paper, all the queueing analysis is based on the discrete-time fluid model, except that the traffic substitution technique presented in Section VI-A is derived for the continuous-time case.

IV. Loss Calculation for a Finite Buffer

In this section, we briefly review the loss calculation technique presented in [15], where the loss probability $P_L(x)$ of a Gaussian input with stationary and ergodic increments in a finite buffer system with size x is estimated from the tail of the queue length distribution (termed tail probability or overflow probability) $\mathbb{P}\{Q_I > x\}$ of an infinite buffer system. Hereafter, we use Q_I to denote the steady-state queue length in the infinite buffer and reserve Q to denote the steady-state queue length in the finite buffer system. The *overflow probability* $\mathbb{P}\{Q_I > x\}$ is defined as the ratio of the period that the queue length in the infinite buffer system spends above the level x to the total time, assuming ergodicity.

In [15], it is proposed that, for a Gaussian input process with stationary and ergodic increments, the loss probability can be approximated as

$$P_L(x) \approx \alpha \mathbb{P}\{Q_I > x\} \tag{4}$$

where α is a constant being heuristically calculated as $\alpha = P_L(0)/\mathbb{P}\{Q_I > 0\}$ and $P_L(0)$ denotes the loss probability in a bufferless system. The loss approximation (4) is theoretically supported by a strong asymptotic relationship that

$$\ln \mathbb{P}\{Q_I > x\} - \ln P_L(x) = O(\ln x) \tag{5}$$

where $g(x) = O(u(x))$ means that $\limsup_{x \to \infty} |g(x)/u(x)| < \infty$. Equation (5) tells that the divergence between the approximation $\alpha \mathbb{P}\{Q_I > x\}$ and the loss probability is slow, if at all [15].

The overflow probability $\mathbb{P}\{Q_I > x\}$ in (4) can be accurately estimated by the MVA approach [20], [26], [31]. Consider the Gaussian input process $A(t)$ with the parameters λ and σ, and the buffer is served with constant rate c. Let $\kappa = c - \lambda$ and $X_t = A(t) - ct$, and define m_x to be the reciprocal of the maximum of $\sigma_{x,t}^2 = \mathrm{Var}\{X_t\}/(x + \kappa t)^2$ for a given x,[4] i.e.,

$$m_x = \frac{1}{\max_{t \ge 1} \sigma_{x,t}^2} = \min_{t \ge 1} \frac{(x + \kappa t)^2}{\mathrm{Var}\{X_t\}}. \tag{6}$$

The MVA approximation of the overflow probability is then given by

$$\mathbb{P}\{Q_I > x\} \approx \exp\left(-\frac{m_x}{2}\right). \tag{7}$$

Although the MVA approximation is theoretically shown to be only an asymptotic upper bound with the relationship

$$\ln \mathbb{P}\{Q_I > x\} + \frac{m_x}{2} = O(\ln x) \tag{8}$$

simulation studies show that it is an accurate approximation even for small values of x [20], [31]. Define t_x to be the time at which $\sigma_{x,t}^2$ attains its maximum. t_x is called the *dominant time scale* (DTS) in the literature, and it is shown in [20] and [31] that, if overflow happens at x, it is most likely to happen over the DTS t_x.

For the FBM input characterized by λ, σ^2, and H, m_x can be explicitly computed by [31]

$$m_x = \frac{4\kappa^\beta x^{2-\beta}}{S\beta^\beta (2-\beta)^{2-\beta}} \tag{9}$$

[3]The relationship that a higher service priority results in a smaller loss probability is theoretically true under the assumption that the queueing process of the partitioned buffer converges to a stationary and ergodic process. Letting t_a denote an arrival moment, in the steady state, the relationship $P_L^j = \mathbb{P}\{Q(t_a) \ge B_j\} = \mathbb{P}\{Q \ge B_j\}$ exists due to the stationarity. Therefore, $P_L^j < P_L^{j-1}$ as $\mathbb{P}\{Q \ge B_j\} < \mathbb{P}\{Q \ge B_{j-1}\}$ with $B_j > B_{j-1}$, for $1 \le j \le J$. The ergodicity ensures that the practical loss estimation based on time averaging can also observe the service differentiation. In our experiments, we never observe an abnormal case violating the loss differentiation behavior under buffer partitioning.

[4]The MVA approach is first named in [20], with the consideration that $\sigma_{x,t}^2$ is the variance of the centered Gaussian process $X_t^{(x)} = (X_t + \kappa t)/(x + \kappa t)$, and the maximum of $\sigma_{x,t}^2$ is closely related to the overflow probability.

where $\beta = 2H$ and $S = \sigma^2$. With a Gaussian input, the constant α can also be explicitly calculated by [15]

$$\alpha = \frac{1}{\lambda\sqrt{2\pi}\sigma} \exp\left(\frac{\kappa^2}{2\sigma^2}\right) \int_c^\infty (r-c) \exp\left[-\frac{(r-\lambda)^2}{2\sigma^2}\right] dr. \tag{10}$$

After some mathematical manipulation to (10), we find a simplified expression of α, given by

$$\alpha = a\left[1 - \sqrt{\pi} b \exp(b^2) \operatorname{erfc}(b)\right] \tag{11}$$

where $a = \sigma/\sqrt{2\pi}\lambda$, $b = \kappa/\sqrt{2}\sigma$, and $\operatorname{erfc}(x) = (2/\sqrt{\pi}) \int_x^\infty \exp(-t^2)\, dt$ is the complementary error function of the standard Gaussian distribution. With the results given in (4), (7), and (9)–(11), the PLP for a finite buffer with an FBM input can be explicitly calculated. Extensive simulation results have been presented in [15], showing that such techniques give an accurate estimate of the loss probability over a wide range of queue lengths, not limited only to large values of x.

V. Loss Calculation for a Partitioned Buffer

In this section, we extend the loss calculation technique presented in Section IV to the partitioned buffer case. Similarly, the basic philosophy is to estimate the loss probability from the overflow probability of an infinite buffer system, and we make the assumption that $Q(t)$ and $Q_I(t)$ can converge to a stationary and ergodic process in a partitioned buffer system with the multiclass FBM input. In the following, we first study the overflow probabilities for an infinite partitioned buffer, based on which we obtain the loss probabilities.

A. Overflow Probabilities in an Infinite Partitioned Buffer

The *infinite partitioned buffer* corresponding to the finite size partitioned buffer as defined in Section III-A is a system where the buffer space larger than $B_J = B$ and up to ∞ is available, and the class-J traffic will still be accepted into the queue instead of being dropped when the queue level reaches or exceeds B_J.

In the partitioned buffer, the input process will change due to class differentiation when the queue level crosses a certain partition threshold $B_j (1 \le j \le J-1)$, which makes the queueing analysis difficult when dealing with the Lindley's equation, the approach used in [15] and [31]. We, therefore, resort to an iterative calculation, considering that the overflow probabilities at adjacent thresholds are related by

$$P_V(B_j) = P_V(B_{j-1}) \mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\}, \quad 2 \le j \le J. \tag{12}$$

The overflow analysis is then transformed to the task of finding an expression of the conditional probability $\mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\}$ by analyzing the queueing behavior in the infinite partitioned buffer.

When the input to the infinite partitioned buffer is multiclass Markov-modulated fluid sources that generate J classes of traffic at each state, it is proved in [13] that the overflow probabilities in the asymptotic case as all the partition regions $B_j - B_{j-1} \to \infty$ (by setting $B_j - B_{j-1} = b_j B_J,\ B_J \to \infty$ with $b_j > 0$ and $\sum_{j=1}^J b_j = 1$) are given by

$$P_V(B_j) = C \exp\left(\sum_{r=1}^j \eta^{r*}(B_r - B_{r-1})\right) + o\left(\exp\left(\sum_{r=1}^j \eta^{r*}(B_r - B_{r-1})\right)\right), \quad 1 \le j \le J \tag{13}$$

where $C < 1$ is a positive constant, η^{j*} denotes the dominant eigenvalue characterizing the queueing behavior in the jth partition region, and $o(u(x))$ denotes the set of functions $g(x)$ such that $\lim_{x\to\infty} |g(x)/u(x)| = 0$. It is not difficult to derive the following relationships from (13):

$$P_V(B_1) = C \exp\left(\eta^{1*} B_1\right) + o\left(\exp\left(\eta^{1*} B_1\right)\right) \tag{14}$$

$$P_V(B_j) = P_V(B_{j-1}) \exp\left(\eta^{j*}(B_j - B_{j-1})\right) + o\left(\exp\left(\sum_{r=1}^j \eta^{r*}(B_r - B_{r-1})\right)\right), \quad 2 \le j \le J. \tag{15}$$

Comparing (15) with (12), we can derive an expression of the conditional probability as

$$\mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\} = \exp\left(\eta^{j*}(B_j - B_{j-1})\right) + o\left(\exp\left(\eta^{j*}(B_j - B_{j-1})\right)\right), \quad 2 \le j \le J. \tag{16}$$

For the jth $(1 \le j \le J)$ partition region $[B_{j-1}, B_j)$, we consider a separate nonpartitioned infinite buffer with the service rate c and the input same as that observed in the jth partition region (including class j to class J), which is defined as the *virtual buffer* associated with the jth region. Using Q_I^j to denote the queue length of the virtual buffer, the overflow probability in the separate virtual buffer is known as [40]

$$\mathbb{P}\{Q_I^j > B_j - B_{j-1}\} = C' \exp\left(\eta^{j*}(B_j - B_{j-1})\right) + o\left(\exp\left(\eta^{j*}(B_j - B_{j-1})\right)\right) \tag{17}$$

where $C' < 1$ is a positive constant. Comparing (17) with (16) and (15), we can obtain

$$\mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\} \overset{\ln}{\sim} \mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, \quad 2 \le j \le J \tag{18}$$

$$P_V(B_j) \overset{\ln}{\sim} P_V(B_{j-1}) \mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, \quad 2 \le j \le J \tag{19}$$

where $g(x) \overset{\ln}{\sim} u(x)$ if $\ln g(x) \sim \ln u(x)$, and $g(x) \sim u(x)$ means $\lim_{x\to\infty} g(x)/u(x) = 1$.

The relationship (18) tells us that the queueing behavior in a partition region asymptotically approximates to that in a separate nonpartitioned buffer with the corresponding input. The relationship (19) further indicates that the overflow at B_j happens in two steps: first the queue level goes into the jth partition region, and then, overflow happens in that region; the two-step events are approximately independent in the large-buffer regime, where the queueing behavior in each partition region can be separately considered as indicated by (18). We conjecture that the relationships of (18) and (19) can be extended to a general SRD input process by the following intuitive reasoning: 1) if each partition region is large enough that the localized DTS (by which the queue level enters the next partition region) is larger than the time scale over which the net input process is correlated, the queueing steady state in a partition region can be approximately considered as independent of the queueing history in other regions, i.e., as behaving in a separate virtual buffer; 2) regarding the overflow in a nonpartitioned infinite buffer, the asymptotic log-similarity relationship $\mathbb{P}\{Q > x\} \overset{\ln}{\sim} \exp(\eta x)$ has been obtained for a large class of stochastic processes based on the *large deviation theory* [41].

In practice, when each partition region is reasonably large, the aforementioned asymptotic analysis suggests an independent approximation of the localized queueing behavior

$$\mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\} \approx \mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, \quad 2 \le j \le J \tag{20}$$

and a simple iterative approximate calculation of the overflow probabilities as

$$P_V(B_j) \approx P_V(B_{j-1})\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, \qquad 1 \le j \le J \tag{21}$$

where $P_V(0) = 1$ for convenience. The accuracy of the approximation is examined and validated in [9] by extensive simulation results. However, when the input traffic is LRD, it is inefficient to decouple the correlation between the adjacent partition regions as in (18) by increasing the buffer size. Therefore, we must investigate the correlation between partition regions and take it into account in the overflow analysis for an LRD input.

B. Overflow Analysis With an FBM Input

1) Single-Class FBM Input: To identify the impact of the long-range correlation on the loss analysis, we begin from the simple case of a single-class FBM input. A single-class FBM process is a special instance of the multiclass FBM process having J classes of traffic, where $A_j(t) = 0$ for $1 \le j \le J-1$ and $A_J(t) = \lambda_J + \sigma_J Z(t)$. The total FBM traffic $A(t) = A_J(t)$, with $\lambda = \lambda_J$ and $\sigma^2 = \sigma_J^2$. In this case, the exceeding of a partition threshold $B_j (1 \le j \le J)$ will not lead to traffic dropping as only the highest priority class J exists, and the infinite partitioned buffer performs as an infinite nonpartitioned FIFO buffer with the input $A(t) = A_J(t)$.

Proposition 1: In the partitioned buffer system with a single-class FBM input, the overflow probabilities at different thresholds $P_V(B_j)(1 \le j \le J)$ are approximately related by

$$\begin{aligned}[][-\ln(P_V(B_j))]^{1/(2-\beta)} &\approx [-\ln(P_V(B_{j-1}))]^{1/(2-\beta)} \\ &+ \big[-\ln\big(\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}\big)\big]^{1/(2-\beta)}, \quad 1 \le j \le J \end{aligned} \tag{22}$$

where $P_V(B_0) = P_V(0) = 1$,[5] based on the result that the MVA approximation can give an accurate estimate of the overflow probability over a wide range of the queue length [20], [31].

Proof: As the system can be equivalently considered as a nonpartitioned buffer with the class J input, we have the MVA approximation

$$\mathbb{P}\{Q_I > B_j\} \approx \exp\left(-\frac{2\kappa^\beta B_j^{2-\beta}}{S\beta^\beta(2-\beta)^{2-\beta}}\right) \tag{23}$$

based on (7) and (9). Letting $z = 2\kappa^\beta/[S\beta^\beta(2-\beta)^{2-\beta}]$, (23) can be rewritten as

$$[-\ln(\mathbb{P}\{Q_I > B_j\})]^{1/(2-\beta)} \approx z^{1/(2-\beta)} B_j. \tag{24}$$

Similarly, by considering the overflow at B_{j-1}, we have

$$[-\ln(\mathbb{P}\{Q_I > B_{j-1}\})]^{1/(2-\beta)} \approx z^{1/(2-\beta)} B_{j-1}. \tag{25}$$

By considering the overflow at $B_j - B_{j-1}$ in the virtual buffer associated with the partition region $[B_{j-1}, B_j)$, we also have

$$\big[-\ln\big(\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}\big)\big]^{1/(2-\beta)} \approx z^{1/(2-\beta)}(B_j - B_{j-1}). \tag{26}$$

Combining (24)–(26) and letting P_V denote the overflow probability, we obtain (22).[6] ■

When compared with (21), (22) indicates that the correlation between adjacent partition regions cannot be omitted in the loss estimation when the input process is LRD. An interesting observation is that, when $H = 0.5$ for an SRD input, $\beta = 2H = 1$ and (22) reduces to (21), which is what we expect from a more general relationship. Moreover, as the approximation (22) is a direct derivation of the MVA overflow approximation, it also inherits the MVA's application context, i.e., applicable over a wide range of the queue length.

According to the relationship of (12), we can obtain the conditional probability from (22)

$$\begin{aligned}\mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\} &\approx f\big(P_V(B_{j-1}) \\ &\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, H\big), \quad 1 \le j \le J\end{aligned} \tag{27}$$

where the expression of the function $f(\cdot)$ can be easily derived from (22). Expression (27) suggests an interpretation of the queueing behavior in a partition region that the queueing history till B_{j-1} being reached is all encapsulated in the probability $P_V(B_{j-1})$, which serves as the initial status of the queueing process in the jth region; the steady-state overflow in the jth region can then be determined by the initial status, the localized queueing behavior in the region (considered as a virtual

[5]It is indicated in [42] that for a stable queueing system with the FBM input, $P_V(0) = 1$ in a *continuous-time* storage model, but $P_V(0) < 1$ in the teletraffic reality being modeled. In the remainder of this paper, we always set $P_V(0) = 1$ for the convenience of iterative calculation and for a conservative loss approximation to guarantee QoS.

[6]It is noteworthy that Proposition 1 is not a strict theoretical statement, as at the current stage we could not give a theoretical evaluation on the accuracy of the approximation (22). However, the theoretical imperfection does not sway the main point of this paper on loss approximation, and the extensive computer simulation results presented in Section VIII do validate a satisfactory accuracy of the loss approximation developed upon (22). We still express the approximation (22) as a proposition for convenience.

buffer) $\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}$, and the correlation within the input process characterized by the Hurst parameter H. Due to the difficulty in theoretical proof, the interpretation is defined as an *LSS* assumption, applicable to a general input process with stationary and ergodic increments, to facilitate our loss approximation.

2) Multiclass FBM Input: When the input FBM includes multiple classes, the relationship of (22) or (27) cannot be directly applied, due to the input process variations upon the partition thresholds being crossed. However, given $Q_I > B_{j-1}$, the applicability of (27) (which is in the form of a conditional probability) only requires that $[B_{j-1}, \infty)$ is a nonpartitioned region and the queueing behavior therein follows the MVA approximation.

In order to exploit (27) for overflow analysis with the multiclass input, we define a *promoted system*. In an infinite partitioned buffer system with $J-1$ partition thresholds, $J-1$ promoted systems can be generated. If the jth $(1 \le j \le J-1)$ partition region $[B_{j-1}, B_j)$ is under consideration, the associated *level j promoted system* is a partitioned buffer system where the class j to class $(J-1)$ inputs to the original infinite partitioned buffer are all re-marked as class J (i.e., the service priorities of class j to class $(J-1)$ are promoted), and therefore, the input process remains unchanged while the queue length is within $[B_{j-1}, \infty)$ (i.e., $[B_{j-1}, \infty)$ is a nonpartitioned FIFO region). We use $Q_I^{m,j}$ to denote the queue length of the level j promoted system and $P_V^{m,j}(B_j) = \mathbb{P}\{Q_I^{m,j} > B_j\}(1 \le j \le J)$ the overflow probability. In the promoted system, given $P_V^{m,j}(B_{j-1})$, with the LSS assumption and the MVA approximation of the localized overflow behavior in the nonpartitioned region $[B_{j-1}, \infty)$, the relationship of (27) still holds as

$$\mathbb{P}\{Q_I^{m,j} > B_j | Q_I^{m,j} > B_{j-1}\} \approx f\big(P_V^{m,j}(B_{j-1}) \\ \mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, H\big), \quad 1 \le j \le J-1 \tag{28}$$

where

$$\mathbb{P}\{Q_I^j > B_j - B_{j-1}\} \approx \exp\left[-z_j (B_j - B_{j-1})^{2-\beta}\right] \tag{29}$$

$\kappa_j = c - \sum_{r=j}^{J} \lambda_r$, $S_j = \sum_{r=j}^{J} \sigma_r^2$, and $z_j = 2\kappa_j^\beta / [S_j \beta^\beta (2-\beta)^{2-\beta}]$.

Comparing the level j promoted system with the original partitioned buffer, the queueing processes associated with class 1 to class $(j-1)$ traffic are asymptotically the same in the two systems as $B_j - B_{j-1} \to \infty$. Therefore, we can have the approximation that $P_V^{m,j}(B_k) \approx P_V(B_k)$ for $0 \le k \le j-1$, when $B_j - B_{j-1}$ is reasonably large. Based on this reasoning and re-expressing (28) in the form similar to (22), we obtain

$$\left[-\ln\left(P_V^{m,j}(B_j)\right)\right]^{1/(2-\beta)} \approx \left[-\ln\left(P_V(B_{j-1})\right)\right]^{1/(2-\beta)} \\ + \left[-\ln\left(\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}\right)\right]^{1/(2-\beta)}, \quad 1 \le j \le J-1. \tag{30}$$

That is, when Proposition 1 is applied to the multiclass input, given $P_V(B_{j-1})$, we can only obtain the overflow $P_V^{m,j}(B_j)$ in the corresponding level j promoted system.[7] Further, the overflow probability $P_V(B_j)$ of the original partitioned buffer is related to $P_V^{m,j}(B_j)$ by the following proposition.

Proposition 2: For an infinite partitioned buffer, in the large-buffer asymptotic regime where $B_j - B_{j-1} \to \infty (1 \le j \le J)$, $P_V^{m,j}(B_j)$ and $P_V(B_j)$ are asymptotically related by

$$\ln P_V^{m,j}(B_j) - \ln P_V(B_j) = O\left(\ln(B_j - B_{j-1})\right), \\ 1 \le j \le J-1. \tag{31}$$

Proof: Details of the proof are presented in the Appendix. ■

Inspired by the mapping technique presented in [15], we also propose to approximate the overflow probabilities of an infinite partitioned buffer from those of the promoted systems as

$$P_V(B_j) \approx \alpha_j P_V^{m,j}(B_j), \quad 1 \le j \le J-1 \tag{32}$$

where α_j's (<1) are positive constants. Equation (31) tells us that the divergence between the approximation $\alpha_j P_V^{m,j}(B_j)$ and the overflow probability $P_V(B_j)$ is slow, in the sense that the logarithm scale error can at most be $O(\ln x)$ [15].

3) Iterative Overflow Calculation: Based on the earlier discussion, we can then develop an iterative algorithm to calculate the overflow probabilities in the infinite partitioned buffer system with a multiclass FBM input. The algorithm is as follows.

Step 1: set $P_V(B_0) = P_V(0) = 1$
Step 2: for $j = 1 : J$

$$\left[-\ln\left(P_V^{m,j}(B_j)\right)\right]^{1/(2-\beta)} \\ \approx \left[-\ln\left(P_V(B_{j-1})\right)\right]^{1/(2-\beta)} + z_j^{1/(2-\beta)}(B_j - B_{j-1}) \tag{33}$$

$$\text{if } j \ne J \; P_V(B_j) \approx \alpha_j P_V^{m,j}(B_j) \tag{34}$$

end
Step 3: $P_V(B_J) = P_V^{m,J}(B_J)$.

Here, (33) is obtained by combining (29) and (30). The $P_V^{m,J}(B_J)$ value does not need to be modified, as the input process remains unchanged in the buffer region $[B_{J-1}, \infty)$.

In the iterative calculation, we need to find how to calculate the mapping constants $\alpha_j (1 \le j \le J-1)$. Unfortunately, it is very difficult, if not impossible, to derive the accurate expressions for α_j. Therefore, we resort to heuristic expressions that work well both for large and small buffer sizes.

In the partitioned buffer, at thresholds $B_j (1 \le j \le J-1)$, both packet loss and overflow take place. When the upcrossing of B_j happens, class j joins the classes that experience loss, and traffic of class $(j+1)$ to class J sees an overflow at B_j. From the perspective of the aggregate traffic, the loss at threshold B_j is referred to as *part loss* (or *part overflow*). The factors $\alpha_j (1 \le j \le J-1)$ in (32) or (34) in fact reflect the mapping relationship between the overflow probability and the part loss probability, while (11) calculates the mapping factor between

[7] Note that given $P_V(B_{J-1})$, $P_V(B_J)$ can be directly obtained by the right-hand side of (30), because only class J input exists in the buffer region $[B_{J-1}, \infty)$. It can also be interpreted as $P_V(B_J) = P_V^{m,J}(B_J)$.

the overflow probability and the full loss probability. From the proof given in the Appendix, we can see that the part loss probability should be between the overflow probability (obtained in the promoted system) and the full loss probability, so the overflow probabilities only need to be partially adjusted to get the part loss probability as compared to the full loss case. We propose the heuristic calculation for α_j as

$$\alpha_j = 10a_j\left[1 - \sqrt{\pi}b_j\, \exp\left(b_j^2\right) \mathrm{erfc}(b_j)\right], \quad 1 \le j \le J-1 \tag{35}$$

where $a_j = \sqrt{\sum_{r=j}^{J} \sigma_r^2}/\sqrt{2\pi}\sum_{r=j}^{J}\lambda_r$ and $b_j = \kappa_j/\sqrt{2\sum_{r=j}^{J}\sigma_r^2}$ for $1 \le j \le J-1$.

The heuristic calculation in (35) increases the calculation in (11) by the magnitude of one order (ten times). Such a heuristic proposition stems from our objective for accurate estimate of the order of the loss probabilities. By checking against the examples given in [15] and our own numerical investigations, we find that the factors calculated by (11) (to obtain the full loss from the overflow) are normally in the order of 10^{-2}. As the part loss should fall in between the full loss and the overflow that are two orders apart, the adjustment of one order is reasonable in the sense of accurate order estimate. In Section VIII, computer simulations with a wide range of parameter settings demonstrate that such a simple heuristic approximation has robust performance in estimating the order of the overflow probabilities, and therefore, the loss probabilities in the partitioned buffer system. It is noteworthy that accurate theoretical analysis of the mapping factors is an interesting and challenging research topic for further investigation.

C. Loss Probabilities in a Finite Size Partitioned Buffer

In the following, we discuss how to obtain the loss probabilities in the finite partitioned buffer system from the overflow probabilities in the infinite partitioned buffer system.

In the finite partitioned buffer, when newly arrived class $j(1 \le j \le J-1)$ traffic is lost, the buffer content should be above B_j, and vice versa.[8] Therefore, in the discrete-time fluid queue, for $1 \le j \le J-1$, we have

$$\begin{aligned}\mathbb{P}\{Q > B_j\} &= \lim_{T\to\infty} \frac{\sum_{t=1}^{T} I(Q(t) > B_j)}{T} \\ &= \lim_{T\to\infty} \frac{\lambda_j \sum_{t=1}^{T} I(Q(t) > B_j)}{\lambda_j T} \\ &\approx \lim_{T\to\infty} \frac{\text{average class } j \text{ fluid lost during } T}{\text{average class } j \text{ fluid arrival during } T} = P_L^j\end{aligned}$$

where $I(A) = 1$ if event A is true; $I(A) = 0$ otherwise. The "$\approx$" in the previous equation is due to the slight overestimate of the traffic loss in those time units containing the time moments at which B_j is reached or crossed.

Furthermore, $\mathbb{P}\{Q > B_j\}(1 \le j \le J-1)$ can be well approximated by the overflow probability $\mathbb{P}\{Q_I > B_j\}$ of the infinite partitioned buffer system, if $\mathbb{P}\{Q_I > B_J\} \to 0$. Although a general proof of this approximation is currently not available, the approximation can be supported by the following intuitive reasoning. Consider the finite and the corresponding infinite partitioned buffer systems with exactly the same input. If $\mathbb{P}\{Q_I > B_J\}$ is sufficiently small in the infinite buffer system, within a busy cycle, the total time period when $Q_I > B_J$ is approximately equal to 0 on average. Therefore, a typical busy cycle of the infinite buffer system is statistically very similar to a busy cycle of the corresponding finite buffer system. Considering that the overflow probability can be obtained by averaging over busy cycles, we can write

$$\mathbb{P}\{Q > B_j\} \approx \mathbb{P}\{Q_I > B_j\}, \quad 1 \le j \le J-1 \tag{36}$$

and thus, $P_L^j \approx \mathbb{P}\{Q > B_j\} \approx P_V(B_j)$. Note that in a QAF PHB, the admission control is used to guarantee a very small value of $P_V(B)$ (i.e., $\mathbb{P}\{Q_I > B_J\} \to 0$) to protect the highest priority traffic; therefore, the validity of (36) can be guaranteed in practice. The admission control problem is discussed in Section VII.

To obtain the loss probability of the class J traffic, we consider the region $[B_{J-1}, \infty)$ of the infinite buffer system. As only class J input exists in this region, with the LSS assumption and the localized MVA overflow approximation, the conditional statistical behavior in the region $[B_{J-1}, \infty)$ is the same as that in a nonpartitioned buffer with the class J input. Therefore, according to [15], we can approximate P_L^J (which is due to the buffer truncating at B) by applying the mapping relationship given in (4) and the factor calculation given in (11).

Based on the earlier discussions, we propose to calculate the loss probabilities for all the J classes of traffic as

$$P_L^j \approx \mathbb{P}\{Q > B_j\} \approx P_V(B_j), \quad 1 \le j \le J-1 \tag{37}$$

$$P_L^J \approx \alpha_J P_V(B_J) \tag{38}$$

where $\alpha_J = a_J[1 - \sqrt{\pi}b_J\, \exp(b_J^2)\, \mathrm{erfc}(b_J)]$, $a_J = \sigma_J^2/\sqrt{2\pi}\,\lambda_J$, and $b_J = \kappa_J/\sqrt{2}\sigma_J$.

VI. Extension to General Input Traffic

A. Traffic Substitution

It is shown in [21] that both LRD and SRD traffic aggregates can be substituted by a properly parameterized FBM from the perspective that either the buffer overflow probability or the available capacity is preserved after the substitution. The theoretical framework for the traffic substitution is built upon the *many-source asymptotic* buffer analysis using the large deviation theory [43], [44]. The effective bandwidth of the arrival process $A(t)$ with stationary increments is defined as $w(s,t) = (1/st)\ln \mathbb{E}\left[e^{sA(t)}\right]$, where s and t are system parameters determined by the characteristics of the multiplexed sources, the QoS requirements, and the link resources (the channel capacity c and the buffer size B) [43]. According to [21], rules for the substitution are: 1) both the substitute and the original traffic have the same effective bandwidth at the *operating point* that is defined as the extremizing pair (s^*, t^*)

[8] In a strict sense, when newly arrived traffic is lost, the buffer occupancy $Q \ge B_j$. However, in the fluid model analysis, the mass probability at one point B_j can be neglected.

in $\inf_t \sup_s [s(B+ct) - stw(s,t)]$ and 2) the operating point is preserved after the substitution. The FBM is an appropriate process for constructing the equivalent source, as its analytical effective bandwidth representation allows simple parameter fitting [21].

Letting $R(t)$ denote the traffic to be replaced, parameters of the equivalent FBM can be calculated by the substitution rules as

$$\begin{cases} \lambda = w_R(s^*,t^*) - s^* \frac{\partial w_R}{\partial s}(s,t^*)\big|_{s=s^*} \\ 2H-1 = \frac{t^* \frac{\partial w_R}{\partial t}(s^*,t)\big|_{t=t^*}}{s^* \frac{\partial w_R}{\partial s}(s,t^*)\big|_{s=s^*}} \\ \sigma^2 = 2t^{*1-2H} \frac{\partial w_R}{\partial s}(s,t^*)\big|_{s=s^*}. \end{cases} \tag{39}$$

The application of the substitution relies on the knowledge of the operating point (s^*, t^*). With the FBM substitution, the operating point and the system in (39) can be solved jointly via a simple iterative algorithm [21]. When the expression of the effective bandwidth $w_R(s,t)$ is available, the system (39) can be analytically solved. Otherwise, online traffic measurement is required to estimate the effective bandwidth of $R(t)$. Traffic substitution based on measurement is also discussed in [21].

B. Loss Analysis With FBM Substitution

The traffic substitution technique greatly extends the applicability of the proposed loss analysis technique for the partitioned buffer system; the loss probabilities for a general type of input sources can be indirectly obtained by analyzing their FBM equivalence. Suppose that $A_j(t)$'s $(1 \le j \le J)$ are general J classes of independent input processes to the partitioned buffer. In the large-buffer case of $B_j - B_{j-1} \to \infty (1 \le j \le J)$, under the LSS assumption, the traffic substitution is applied to each partition region by (39). In the jth region $[B_{j-1}, B_j)$, the input traffic $\sum_{r=j}^{J} A_r(t)$ is equivalent to an FBM process characterized by parameters Λ_j, Σ_j^2, and H_j.

After applying the traffic substitution, in general, the Hurst parameter H_j of the equivalent FBM processes (in different partition regions) has different values. With the LSS assumption and the localized MVA overflow approximation, the relationship of (28) can be generalized to

$$\begin{aligned} &\mathbb{P}\{Q_I^{m,j} > B_j | Q_I^{m,j} > B_{j-1}\} \approx f\big(P_V^{m,j}(B_{j-1}) \\ &\mathbb{P}\{Q_I^j > B_j - B_{j-1}\}, H_j\big), \quad 1 \le j \le J-1. \end{aligned} \tag{40}$$

Furthermore, all the other analysis in Section V can be applied in the case of heterogeneous Hurst parameters. Therefore, a generalized loss calculation via traffic substitution is as follows.

Step 1: Find the equivalent FBM in each partition region

Step 2: Set $P_V(B_0) = P_V(0) = 1$

Step 3: for $j = 1 : J$

$$\begin{aligned} &\big[-\ln\big(P_V^{m,j}(B_j)\big)\big]^{\frac{1}{2-\beta_j}} \\ &\approx [-\ln(P_V(B_{j-1}))]^{\frac{1}{2-\beta_j}} \\ &\quad + z_j^{\frac{1}{2-\beta_j}}(B_j - B_{j-1}) \end{aligned} \tag{41}$$

$$\text{if } j \ne J, P_L^j \approx P_V(B_j) \approx \alpha_j P_V^{m,j}(B_j) \tag{42}$$

$$\text{else} \qquad P_L^J \approx \alpha_J P_V^{m,J}(B_J) \tag{43}$$

end

For the FBM equivalent traffic, the iterative calculations given in (33) and (34) are extended to (41) and (42) to take the heterogeneous Hurst parameters into account, where $\beta_j = 2H_j$. In addition, z_j and α_j are calculated with the equivalent FBM parameters Λ_j, Σ_j^2, and H_j. Note that the calculation of α_J is different from other $\alpha_j (1 \le j \le J-1)$ factors. Accuracy of the generalized loss analysis via the FBM substitution is examined through simulations in Section VIII.

VII. Admission Control and Optimal Buffer Partitioning

In a partitioned buffer, admission control[9] is necessary to satisfy the loss probability requirements. With the FBM modeling or traffic equivalence, the admission controller can directly apply loss analysis to the aggregate traffic for higher resource utilization, compared to the effective bandwidth approach [9]. In admission control with a preconfigured partition vector $\boldsymbol{B}_t$ and a given link capacity c, the aggregate of the new arrival and the already admitted traffic flows are first modeled to a properly parameterized FBM in each partition region. The loss probability for each class is then calculated from (42) and (43). If the QoS requirements of all the J classes are satisfied, the new arrival can be accepted.

One the other hand, the loss analysis can be used to guide the resource allocation so that all the loss requirements within the QAF PHB are satisfied. Let c_j be the solution of channel capacity from $P_L^j = \epsilon_j (1 \le j \le J)$; then by the loss calculation in (41) and (42), we can obtain

$$\begin{aligned} \left[-\ln\left(\frac{\epsilon_j}{\alpha_j(c_j)}\right)\right]^{1/(2-\beta_j)} &= [-\ln(P_V(B_{j-1}, c_j))]^{1/(2-\beta_j)} \\ &+ z_j(c_j)^{1/(2-\beta_j)}(B_j - B_{j-1}), \quad 1 \le j \le J \end{aligned} \tag{44}$$

where the "=" sign is used for simplicity, and the expressions $\alpha_j(c_j)$, $P_V(B_{j-1}, c_j)$, and $z_j(c_j)$ emphasize that the QoS is directly affected by the channel capacity. The previous equation can be further expressed as

$$\begin{aligned} -\ln(\epsilon_j) = -\ln(\alpha_j(c_j)) + \big\{ &[-\ln(P_V(B_{j-1}, c_j))]^{1/(2-\beta_j)} \\ &+ z_j(c_j)^{1/(2-\beta_j)}(B_j - B_{j-1})\big\}^{2-\beta_j}, \quad 1 \le j \le J. \end{aligned} \tag{45}$$

It is impossible to get a close form expression of $c_j (1 \le j \le J)$ from (45). However, c_j can be numerically solved in practice by a standard iterative root-finding technique (such as the Newton's method), with the partition thresholds given. The right-hand side

[9]Call (or connection) admission control has been implemented in circuit-switched networks and asynchronous transfer mode (ATM) networks to ensure that per-connection resource requirement, and therefore, the performance requirement are met. The classic *best effort* Internet does not support explicit connection admission control. However, as the Internet is evolving into a common communication infrastructure for all types of multimedia applications, it is widely accepted that connection admission control is necessary to provision QoS over Internet [10], [45], [46].

of (45) is a monotonic increasing function of c. The monotonicity property of $z(c)$ and $-\ln(P_V(B_{j-1}, c))$ are obvious, as $z_j(c) = [2(c - \Lambda_j)_j^{\beta_j}]/[S_j \beta_j^{\beta_j}(2-\beta_j)^{2-\beta_j}]$ and $P_V(B_{j-1}, c)$ decreases with the increase of serving capacity. By taking differentiation of (11), we can find that $\alpha(c)$ ($<$1) is a monotonic decreasing function of c, and therefore, $-\ln(\alpha_j(c))$ a monotonic increasing function of c. For QoS satisfaction of all the classes, it is required that the serving capacity for the partitioned buffer should be $c \geq \max\{c_1, c_2, \ldots, c_J\}$.

The set of equations in (45) indicates that, when other parameters are fixed, the choice of $\boldsymbol{B}_t$ determines the channel capacity required to guarantee the QoS for all the J classes. Our previous study [9] proves the following proposition for the partitioned buffer with Markovian sources.

Proposition 3: Under the QoS constraints, the required channel capacity c achieves its minimal value c^*, if $\boldsymbol{B}_t$ is adjusted to $\boldsymbol{B}_t^*$ such that

$$c_1 = c_2 = \cdots = c_J \stackrel{\triangle}{=} c^*. \tag{46}$$

The proposition is also valid for buffer partitioning with an multiclass FBM input. We omit the proof here because it can be proved easily using an approach similar to that used in [9]. The technique presented in Proposition 3 is referred to as *optimal buffer partitioning* and $\boldsymbol{B}_t^*$ the *optimal partition vector*. At $\boldsymbol{B}_t = \boldsymbol{B}_t^*$, all the J equations in (44) are satisfied simultaneously with the solution pair $(c^*, \boldsymbol{B}_t^*)$ and the QoS specification $\epsilon_j (1 \leq j \leq J)$. Substituting c^* and $\boldsymbol{B}_t^*$ into (44) and setting $\epsilon_0 = 1$, we have

$$\left[-\ln\left(\frac{\epsilon_j}{\alpha_j(c^*)}\right)\right]^{1/(2-\beta_j)} = [-\ln(\epsilon_{j-1})]^{1/(2-\beta_j)} + z_j(c^*)^{1/(2-\beta_j)}(B_j - B_{j-1}), \quad 1 \leq j \leq J. \tag{47}$$

After some manipulation, the following equation of c^* can be obtained

$$\sum_{j=1}^{J} \frac{[-\ln(\epsilon_j/\alpha_j(c^*))]^{1/(2-\beta_j)} - [-\ln(\epsilon_{j-1})]^{1/(2-\beta_j)}}{z_j(c^*)^{1/(2-\beta_j)}} = B. \tag{48}$$

Again, due to the monotonicity of $z_j(c)$ and $\ln(\alpha_j(c))(1 \leq j \leq J)$, the left-hand side of (48) is a monotonic decreasing function of c. We can solve (48) for c^* by using a standard iterative rooting-finding technique. With the c^* value, $\boldsymbol{B}_t^*$ can be easily solved from (47).

There is one hurdle when applying the optimal buffer partitioning technique via the traffic substitution. To obtain the FBM substitute, each partition-region size (and therefore, the partition thresholds) should be known in advance because the solution of operating point depends on the buffer size. But in the optimal buffer partitioning, the FBM substitute should be known beforehand. This type of loop can be broken by choosing a proper initial partition vector based on which the traffic substitution is implemented. This approach is supported by the observation from our numerical investigations that the equivalent FBM parameters are not very sensitive to the buffer size. When the original input is from multiclass Markovian sources, the initial partition vector can be obtained from the optimal buffer partitioning by the large-buffer overflow approximation [9]. If the original input cannot be explicitly modeled, we use evenly partitioned thresholds for traffic substitution.

VIII. Performance Evaluation

In this section, we examine the performance of the proposed loss calculation technique via computer simulations. The performance is evaluated from two aspects. First, the partitioned buffer is fed with the multiclass FBM traffic described in Section III-B to investigate the accuracy of the loss calculation. The FBM process is generated by the modified random midpoint displacement (RMD) algorithm [47], [48]. The loss probabilities from the proposed approximation are compared with the computer simulation results for different buffer sizes and partition thresholds. Then, the application of the proposed loss analysis to resource allocation is examined, combined with the FBM substitution and optimal buffer partitioning. The resource utilization improvement based on the finite buffer analysis is demonstrated.

All the simulation results are obtained from Monte Carlo simulations due to the difficulty in applying the *importance-sampling* technique [49] in a partitioned buffer with a self-similar input. In all the simulations, a sufficient number of arrival packets are generated to obtain accurate estimate of loss probabilities. In a run of the simulation, we count the numbers of lost/input packets for each class in each busy cycle of the queueing process, and the samples are then processed using the method of batch means [50] to compute loss estimates and the confidence intervals. When the 95% confidence interval of the estimated loss probability are within $\pm 10\%$ of the corresponding estimate for all the J classes, the simulation run stops. The error bars are not shown in the figures to not clutter the loss curves. In all the examples, the units of mean traffic arrival rate and channel capacity are packet per second, and the unit of buffer size is packet.

A. Accuracy of the Loss Analysis

First, we consider a two-class FBM input served with a partitioned buffer of size 500. The class 1 traffic is an FBM with $\lambda_1 = 10$, $\sigma_1^2 = 50$, and class 2 an FBM with $\lambda_2 = 300$, $\sigma_2^2 = 100$. The Hurst parameter H is 0.8 for both classes. Selection of the parameters is based on the consideration that high-priority traffic should be the main component of the multiclass traffic; for example, in the layer-coded video/audio traffic, the base layer always occupies the largest portion. The buffer is partitioned into two regions to differentiate the loss behaviors of class 1 and class 2. Fig. 1 shows the loss probabilities of two classes versus the partition threshold B_1, obtained from the loss analysis given in Section V and simulation, respectively. Furthermore, two scenarios with $c = 312$ and $c = 315$ are compared. Due to the difficulty in obtaining very small loss probabilities via Monte Carlo simulation, we only show simulated loss probabilities of 10^{-7} or above. It is observed that the simulation results and the calculated loss probabilities are in a close match in both scenarios and with the different partition

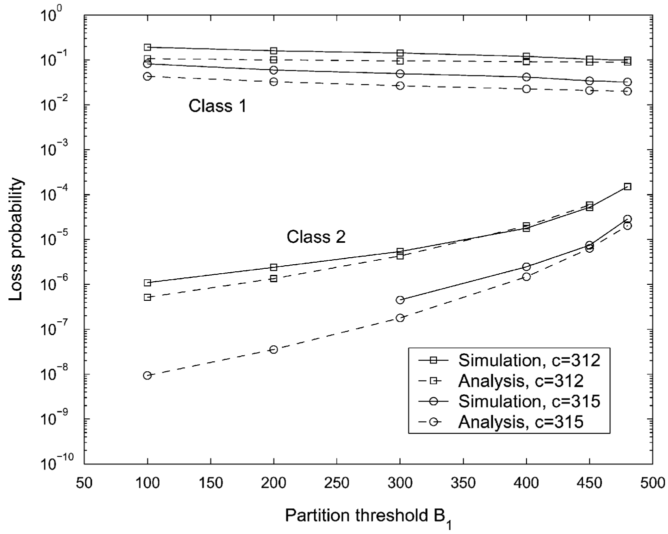

Fig. 1. Loss probabilities for the two-class FBM input, $B = 500$ packets.

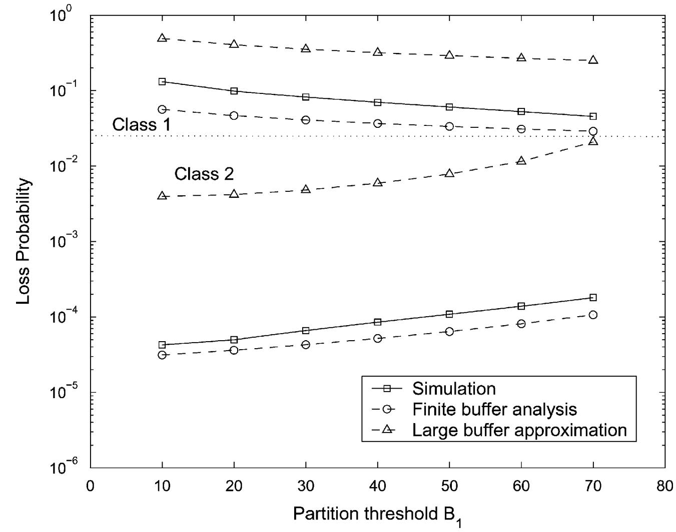

Fig. 2. Loss probabilities for the two-class FBM input, $B = 80$ packets.

configurations. Note that the increase of B_1 means a decrease of the exclusive reservation space for class 2, $B - B_1$. Therefore, the loss probability of class 2 increases with B_1. It is also observed that for the LRD input, the impact of buffer space on the loss probability is not so significant as that of the channel capacity. A small increase of the channel capacity can result in an obvious decrease of the loss probabilities.

As the theoretical foundation of the proposed loss calculation technique is built upon the large-buffer asymptotic analysis, we carry out simulations to examine the applicability of the proposed technique in small buffer cases. Here, we consider a two-class FBM input served with a partitioned buffer of size 80. The channel capacity $c = 340$. In this setting, the worst case delay is only $80/340 \approx 235$ ms, small enough for the streaming class services when the QAF is supported over a third-generation wireless system [51]. The parameters for the two FBM classes are $\lambda_1 = 20, \sigma_1^2 = 50, \lambda_2 = 300, \sigma_2^2 = 500$, and $H = 0.83$. Fig. 2 shows both the simulation and the analysis results. It is observed that the simulation and analysis results of the loss probabilities match well with each other in the small buffer case with the different partition configurations. The large-buffer overflow approximation results based on (19) are also presented in Fig. 2, which deviate far from the simulation results, especially for class 2 traffic where the finite buffer effect is more obvious. Accurate loss analysis in the small buffer case is particularly important for the real-time traffic having both delay and loss requirements.

We also examine the loss calculation technique when the input includes more than two classes. For example, the input FBM includes three classes with parameters $\lambda_1 = 12, \sigma_1^2 = 30, \lambda_2 = 10, \sigma_2^2 = 50, \lambda_3 = 300, \sigma_3^2 = 100$, and $H = 0.8$. The buffer is partitioned as $[0, 150, 290, 300]$ and served with capacity $c = 325$. The calculated loss probabilities of the three classes are $P_L^1 = 0.0917, P_L^2 = 1.9587 \times 10^{-4}$, and $P_L^3 = 6.1813 \times 10^{-7}$, respectively. On the other hand, the simulation results are $0.1408, 5.5374 \times 10^{-4}$, and 3.5654×10^{-7}, respectively. Again, the simulation results match well with the analytical results. Overall, we have run extensive simulations with different partition configurations and a variety of buffer sizes. All the simulation results are close to the analytical results, and the largest relative error is in the magnitude of one order, that is, $\log(\text{simulation result}/\text{analysis result}) \in [-1, 1]$.

The main reason for the analysis errors is the heuristic estimate of the α_j values used in (35), which directly leads to the errors in the loss calculation. The proposed heuristic given by (35) is based on the argument that the adjustment ratio of the overflow probability to obtain the *part* loss probability should be different from the ratio for the *full* loss probability. The exact ratios should be a function of the specific partition configurations and traffic characteristics. The constant adjustment factor of 10 used in this paper is a simplified treatment. However, in the sense of accurate order estimate, the proposed loss approximation has satisfactory performance.

B. FBM Substitution and Optimal Buffer Partitioning

Accuracy of the loss analysis with the FBM substitution and efficiency of the optimal buffer partitioning are demonstrated jointly in the following. We reexamine the effective bandwidth resource allocation example in [9, Sec. VIII-B], by applying the finite buffer optimal partitioning analysis to the multiclass ON/OFF sources. In [9], heterogeneous two-class ON/OFF fluid sources (type 5 and type 6 sources) are considered, with PLP requirement of 10^{-2} and 10^{-4} for class 1 and class 2, respectively. Specifically, the type 5 source with the minimal effective bandwidth of $c_5^* = 68.46$ and the type 6 source with $c_6^* = 2c_5^* = 136.92$ are multiplexed in a partitioned buffer of size $B = 1000$ and served with channel capacity $c = 20c_5^*$. K_5 (K_6) is used to denote the number of accepted type 5 (type 6) sources based on the effective bandwidth allocation. When the traffic composition (K_5, K_6) changes, the optimal partition threshold B_1^* for the aggregate traffic can be found by *dynamic buffer partitioning* [9]. Under such an optimal buffer configuration (minimal effective bandwidth allocation and dynamic buffer partitioning) from the overflow approximation analysis,

TABLE I
OPTIMAL BUFFER CONFIGURATIONS AND SIMULATED PLPS: LARGE BUFFER APPROXIMATION VERSUS FINITE BUFFER ANALYSIS, $B = 1000$, QOS SPECIFICATION $\epsilon_1 = 10^{-2}, \epsilon_2 = 10^{-4}$

(K_5, K_6)		(20, 0)	(16, 2)	(12, 4)	(8, 6)	(4, 8)	(0, 10)
Optimization with large buffer approximation and simulated PLPs	B_1^*	821	801	779	754	727	696
	c^*	1369.2	1369.2	1369.2	1369.2	1369.2	1369.2
	PLP_1	2.04×10^{-3}	1.35×10^{-3}	1.07×10^{-3}	9.97×10^{-4}	9.20×10^{-4}	8.09×10^{-4}
	PLP_2	1.10×10^{-6}	8.90×10^{-7}	8.33×10^{-7}	5.47×10^{-7}	4.67×10^{-7}	2.19×10^{-7}
Optimization with finite buffer analysis and simulated PLPs	B_1^*	997	988	967	943	919	896
	c^*	1306.9	1268.3	1232.5	1199.8	1169.9	1142.8
	PLP_1	3.40×10^{-3}	3.75×10^{-3}	3.50×10^{-3}	4.66×10^{-3}	3.56×10^{-3}	4.22×10^{-3}
	PLP_2	1.79×10^{-4}	1.97×10^{-4}	1.57×10^{-4}	1.72×10^{-4}	9.74×10^{-5}	1.31×10^{-4}

TABLE II
OPTIMAL BUFFER CONFIGURATIONS AND SIMULATED PLPS: SMALL BUFFER CONFIGURATIONS, $B = 200$, QOS SPECIFICATION $\epsilon_1 = 10^{-1}, \epsilon_2 = 10^{-4}$

(K_5, K_6)		(20, 0)	(16, 2)	(12, 4)	(8, 6)	(4, 8)	(0, 10)
Optimization with finite buffer analysis and simulated PLPs	B_1^*	133	101	78	66	58	51
	c^*	1413.0	1408.4	1403.5	1397.0	1392.0	1391.2
	PLP_1	5.33×10^{-2}	6.65×10^{-2}	7.19×10^{-2}	8.00×10^{-2}	7.93×10^{-2}	7.95×10^{-2}
	PLP_2	1.64×10^{-4}	1.99×10^{-4}	1.20×10^{-4}	1.35×10^{-4}	1.14×10^{-4}	1.42×10^{-4}
(K_5, K_6)		(40, 0)	(32, 4)	(24, 8)	(16, 12)	(8, 16)	(0, 20)
Optimization with finite buffer analysis and simulated PLPs	B_1^*	169	123	82	61	48	38
	c^*	2685.2	2621.0	2562.0	2503.8	2449.9	2403.4
	PLP_1	4.65×10^{-2}	5.78×10^{-2}	7.08×10^{-2}	7.40×10^{-2}	7.92×10^{-2}	8.18×10^{-2}
	PLP_2	1.56×10^{-4}	8.24×10^{-5}	1.26×10^{-4}	7.89×10^{-5}	1.05×10^{-4}	1.43×10^{-4}

the QoS in practice are oversatisfied, as shown by the simulation results in the upper part of Table I. This is because the additive effective bandwidth allocation cannot fully exploit the statistical multiplexing gain, and the overflow approximation does not consider the finite buffer effect.

Using the finite buffer analysis, we can obtain a more accurate bandwidth requirement for each traffic aggregate (K_5, K_6) by more accurate calculation of the loss probabilities and better exploitation of the statistical multiplexing among multiple flows. We first find the respective FBM equivalence in each partition region by applying the traffic substitution technique presented in Section VI-A. The effective bandwidth expression of the ON/OFF fluid source is given in [43]. The partition threshold from the large-buffer overflow analysis is used as the initial setting to solve the traffic substitution problem. Then, the finite buffer optimal partitioning procedure given in (47) and (48) is executed upon the FBM equivalences to obtain the optimal partition threshold and the minimal resource requirement. Packet loss simulation results (of the original ON/OFF sources) under the new optimal buffer configurations are presented in the lower part of Table I. The simulation results are very close to the QoS specification, reflecting efficient resource utilization. With the most bursty input, $(K_5, K_6) = (0, 10)$, the bandwidth utilization can be improved up to $(1369.2 - 1142.8)/1369.2 = 16.54\%$. In this example, the impact of the accurate loss analysis with the FBM substitution is demonstrated from the resource allocation perspective.

To further examine the performance of the FBM-equivalent-based loss analysis in the small-buffer system, we investigate the multiplexing of type 5 and type 6 ON/OFF sources in a two-region-partitioned buffer of size 200, with QoS specification of $\epsilon_1 = 10^{-1}$ and $\epsilon_2 = 10^{-4}$. In this case, the effective bandwidths calculated according to [9] are $c_5^* = 95.51$ and $c_6^* = 238.55$. The optimal buffer configurations are calculated upon FBM equivalences and checked against simulations for traffic aggregate with different (K_5, K_6) combinations. The results are presented in Table II. In the upper part of Table II, FBM equivalences are obtained under $c = 20c_5^*$; in the lower part, the traffic load is doubled and $c = 40c_5^*$. Again, the simulated PLPs for all the traffic combinations are very close to the QoS specifications, which demonstrates the accurate loss analysis and efficient resource allocation in small buffer cases (when $(K_5, K_6) = (40, 0)$, the delay bound is as small as $200/2685.2 \approx 74.48$ ms). Furthermore, for all the (K_5, K_6) traffic combinations considered in this example, we use the even partition with $B_1 = 100$ as the initial setting for traffic substitution, instead of using the optimal partition thresholds from the large-buffer overflow analysis, as used in Table I. The simulation results, therefore, also demonstrate the robustness of the traffic substitution with respect to buffer configurations.

It is noteworthy that for both the nonpartitioned buffer considered in [15] and the partitioned buffer in this paper, the loss approximation based on the overflow mapping technique, although accurate in estimating the order of loss probabilities, is not able to provide a conservative result. Therefore, resource allocation based on such a loss analysis may lead to slight QoS violation, which is already demonstrated in our examples. In practice, the loss approximation may need to be combined with online monitoring and resource management schemes for guaranteed QoS. The importance of accurate loss estimation lies in that it can provide a good capacity plan that serves as the operating point for the online QoS management [52].

IX. CONCLUDING REMARKS

In this paper, we study the challenging problem of loss analysis in a finite size partitioned buffer. An approximate yet accurate loss analysis technique is proposed for an FBM input, whose self-similar Gaussian property facilitates the modeling of Internet traffic aggregates. A novel approach is used to capture the correlation of the queueing behaviors in the partitioned buffer, which makes the loss analysis technique for the nonpartitioned finite buffer extendable to the partitioned buffer case. Accuracy of the proposed loss calculation technique is verified by extensive computer simulation results. Applications of the loss analysis to admission control and optimal buffer partitioning (and therefore, the optimal resource allocation) have also been investigated. Furthermore, the proposed loss analysis and optimal buffer partitioning techniques have been extended to general input sources by utilizing the recently proposed FBM traffic substitution technique. Efficiency of the traffic substitution, finite-buffer loss analysis, and optimal buffer partitioning in improving resource utilization is also demonstrated by computer simulations.

Accurate loss analysis for the partitioned buffer system makes an QAF PHB achievable in the DiffServ environment, where multiple loss levels can be provided in a buffer with quantitative QoS guarantee. The QAF PHB is particularly suitable for the layer-coded multimedia traffic. In addition, with the availability of accurate loss analysis for a small buffer system, we are even able to achieve a real-time QAF services by setting a small buffer size, where both quantitative loss and delay requirements (worst case delay bound) can be guaranteed with efficient resource utilization. For future work, we plan to have a more thorough theoretical investigation on the overflow approximation (22) given in Proposition 1 and the approximation (35) for calculating the mapping factors. We are also interested in extending the proposed loss analysis technique to RIO schemes, so that the performance of TCP traffic over QAF PHB can be evaluated quantitatively.

APPENDIX

Proof of Proposition 2: In the large-buffer asymptotic case where $B_j - B_{j-1} \to \infty (1 \leq j \leq J)$, the queueing process within the partition region $[B_{j-1},\, B_{j+1})$ $(1 \leq j \leq J{-}1)$ behaves as in a two-region infinite partitioned buffer. In the level j $(1 \leq j \leq J{-}1)$ promoted system, class j to class $J{-}1$ are all promoted to class J, and the queueing process within the partition region $[B_{j-1},\, B_{j+1})$ (in fact, within the region $[B_{j-1}, \infty)$) behaves as in a nonpartitioned infinite buffer. Let $P_V(B_j|B_{j-1}) = \mathbb{P}\{Q_I > B_j | Q_I > B_{j-1}\}$ and $P_V^{m,j}(B_j|B_{j-1}) = \mathbb{P}\{Q_I^{m,j} > B_j | Q_I^{m,j} > B_{j-1}\}$. Without loss of generality, we can find the relation between $P_V(B_j|B_{j-1})$ and $P_V^{m,j}(B_j|B_{j-1})$ by comparing the queueing behavior of a two-region infinite partitioned buffer with that of an infinite nonpartitioned buffer.

We basically follow the asymptotic analysis presented in [15]. Consider three FIFO systems, an infinite nonpartitioned buffer, a two-region infinite partitioned buffer, and a finite nonpartitioned buffer, all having the same Gaussian input traffic. Buffer overflow, differentiated loss, and tail-drop loss happen at position x in the aforementioned three systems, and the queue length in the three systems are denoted as Q_I^m (m represents that the infinite nonpartitioned buffer is from the promoted case), Q_I, and Q_F, respectively. Particularly, in the two-region partitioned system, the input traffic is differentiated into class 1 and class 2, with class 1 traffic dropped when $Q_I \geq x$. We use λ_n to denote the fluid arrival in the nth time unit, and $\lambda_{1,n}$ and $\lambda_{2,n}$ the fluid arrivals for class 1 and class 2 traffic, respectively; in the partitioned buffer case, $\lambda_{1,n} + \lambda_{2,n} = \lambda_n$ and $\overline{\lambda_1} + \overline{\lambda_2} = \overline{\lambda}$ on average. Note that the three FIFO systems are, in fact, correlated *virtual* queues used to investigate the different queueing behaviors of the same input traffic, where the queueing processes in the three systems start simultaneously (assuming at time 0) from the same initial status, based on which we have $Q_{F,n} \leq Q_{I,n} \leq Q_{I,n}^m$ at time $n > 0$. Then, the following relationship holds:

$$\mathbb{E}\{(Q_{I,n-1}+\lambda_n - c - x)^+\} \leq \mathbb{E}\left\{ \left(Q_{I,n-1}^m + \lambda_n - c - x\right)^+ \right\}$$
$$= \mathbb{E}\left\{ \left(Q_{I,n}^m - x\right)^+ \right\} = \int_x^\infty \mathbb{P}\{Q_I^m > y\}\, dy. \tag{49}$$

Furthermore, [15, Equation (32)] indicates that there are x_0, K_1, and K_2 such that

$$e^{-(m_y/2)+K_1 \ln y} \leq \mathbb{P}\{Q_I^m > y\} \leq e^{-(m_y/2)+K_2 \ln y} \quad \forall y \geq x_0. \tag{50}$$

In the infinite partitioned buffer system, letting $P_L^1(x)$ denote the loss probability of class 1 traffic, we have

$$\overline{\lambda_1} P_L^1(x) \approx \frac{\overline{\lambda_1}}{\overline{\lambda}} \mathbb{E}\left\{(Q_{I,n-1} + \lambda_n - c - x)^+\right\}. \tag{51}$$

The physical meaning of (51) is that, when the buffer content Q_I increases above the threshold x, only the portion of class 1 traffic in the total arrival is dropped. The expression of (51) is also based on the argument that the content distribution between class 1 and class 2 is independent of or very weakly related to the traffic generation rate. Combining (49)–(51), we have

$$\overline{\lambda_1} P_L^1(x) \leq \frac{\overline{\lambda_1}}{\overline{\lambda}} \int_x^\infty y^{K_2} e^{-(m_y/2)}\, dy \quad \forall x \geq x_0. \tag{52}$$

On the other hand, due to $Q_{I,n} \geq Q_{F,n} \forall n \geq 0$,

$$\overline{\lambda_1} P_L^1(x) \approx \frac{\overline{\lambda_1}}{\overline{\lambda}} \mathbb{E}\{(Q_{I,n-1} + \lambda_n - c - x)^+\}$$
$$\geq \frac{\overline{\lambda_1}}{\overline{\lambda}} \mathbb{E}\left\{(Q_{F,n-1} + \lambda_n - c - x)^+\right\} = \frac{\overline{\lambda_1}}{\overline{\lambda}} \overline{\lambda} P_L(x). \tag{53}$$

By [15, Equation (33)] and (53), there exists $K_3 > 0$ and $r \geq 1$ such that

$$\overline{\lambda_1} P_L^1(x) \geq \frac{\overline{\lambda_1}}{\overline{\lambda}} \int_x^\infty \frac{1}{K_3 y^r} y^{K_1} e^{-(m_y/2)}\, dy \quad \forall x \geq x_0. \tag{54}$$

According to [15, Lemma 6], there exist $x_1 \geq x_0, K_4 > 0$, and $K_5 > 0$ such that

$$\frac{\overline{\lambda_1}}{\overline{\lambda}} \int_x^\infty \frac{1}{K_3 y^r} y^{K_1} e^{-(m_y/2)}\, dy \geq K_4 x^{K_1 - r - 1 + \beta} e^{-(m_x/2)} \quad \forall x \geq x_1. \tag{55}$$

and

$$\frac{\overline{\lambda_1}}{\overline{\lambda}} \int_x^\infty y^{K_2}\, e^{-(m_y/2)}\, dy \le K_5 x^{K_2-1+\beta}\, e^{-(m_x/2)} \qquad \forall x \ge x_1. \quad (56)$$

From (52) and (54)–(56),

$$K_4 x^{K_1-r-1+\beta} e^{-(m_x/2)} \le \overline{\lambda_1} P_L^1(x) \le K_5 x^{K_2-1+\beta}\, e^{-(m_x/2)} \qquad \forall x \ge x_1.$$

Take logarithm and rearrange to get

$$\ln\left(\frac{K_4}{\overline{\overline{\lambda_1}}}\right) + (K_1 - r - 1 + \beta)\ln x \le \ln P_L^1(x) + \frac{m_x}{2} \le \ln\left(\frac{K_5}{\overline{\overline{\lambda_1}}}\right) + (K_2 - 1 + \beta)\ln x \qquad \forall x \ge x_1.$$

Then divide by $\ln x$ and take $x \to \infty$ to get

$$-\infty < \liminf_{x\to\infty} \frac{1}{\ln x}\left(\ln P_L^1(x) + \frac{m_x}{2}\right) \le \limsup_{x\to\infty} \frac{1}{\ln x}\left(\ln P_L^1(x) + \frac{m_x}{2}\right) < \infty. \quad (57)$$

The expression in (57) can be rewritten as

$$\ln P_L^1(x) + \frac{m_x}{2} = O(\ln x). \quad (58)$$

It is also given in [15] that

$$\ln \mathbb{P}\{Q_I^m > x\} + \frac{m_x}{2} = O(\ln x). \quad (59)$$

Therefore,

$$\ln \mathbb{P}\{Q_I^m > x\} - \ln P_L^1(x) = O(\ln x). \quad (60)$$

In the infinite partitioned buffer, when newly arrived class 1 traffic is lost, the buffer content should be above x, and vice versa. Therefore, $P_L^1 \approx \mathbb{P}\{Q_I > x\}$ in the fluid queue, and

$$\ln \mathbb{P}\{Q_I^m > x\} - \ln \mathbb{P}\{Q_I > x\} = O(\ln x). \quad (61)$$

Now, let us go back to the original J-region-partitioned buffer. As $\mathbb{P}\{Q_I^m > x\}$ and $\mathbb{P}\{Q_I > x\}$ in the earlier analysis are the generalized representations of $P_V^{m,j}(B_j|B_{j-1})$ and $P_V(B_j|B_{j-1})$, respectively, we have

$$\ln P_V^{m,j}(B_j|B_{j-1}) - \ln P_V(B_j|B_{j-1}) = O\left(\ln(B_j - B_{j-1})\right), \qquad 1 \le j \le J-1. \quad (62)$$

Comparing the level j promoted system with the original partitioned buffer, it is easily seen that $P_V^{m,j}(B_k) \approx P_V(B_k)$ for $0 \le k \le j-1$ when $B_j - B_{j-1} \to \infty$. By adding the item $\ln P_V^{m,j}(B_{j-1}) - \ln P_V(B_{j-1})$, which is approximately equal to 0, to the left-hand side of (62), we obtain (31). ■

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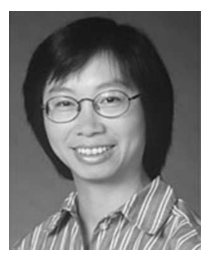

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