Throughput Analysis of Cooperative Communication in Wireless Ad Hoc Networks with Frequency Reuse

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***Abstract*— In this paper, we investigate the network throughput achieved by both spatial diversity and spatial frequency reuse in a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays. Compared to conventional direct transmissions, cooperative communication can enhance single-link transmission reliability, but reduce network-wide spatial frequency reuse due to relay transmissions. To study the tradeoff between these two competing effects, we construct a geographically constrained region for relay selection based on channel state information. The network throughput, defined as the product of the success probability of each link and the expected number of concurrent transmissions, is derived as a function of the total number of links, relay density, size of relay selection region, and distance between the source and destination. The performance analysis is carried out for both selection combining and maximum ratio combining at the destination. Such analytical results can evaluate the effectiveness of cooperative communication and provide useful insights on the design of large-scale networks. Finally, extensive simulations are conducted to validate the performance analysis.**

***Index Terms*—Cooperative communication, spatial diversity, spatial frequency reuse, constrained relay selection region, Poisson point process, selection combining, maximum ratio combining.**

I. Introduction

Wireless ad hoc networks have received extensive attentions from both academia and industry because of their low costs and wide applications [2]. To meet the ever increasing traffic demand and quality-of-service (QoS) requirement, the main causes of performance degradation (e.g., channel fading and transmission interference) should be addressed. As a result, cooperative communication as an effective technique for realizing spatial diversity has been proposed [3]. Via coordinating multiple nearby nodes to work together and form a virtual antenna array, cooperative communication can improve channel capacity and enhance transmission reliability [4]–[7].

Due to the scarcity of the radio spectrum, it is almost impossible to allocate an exclusive channel for each source-destination pair, especially in a wireless ad hoc network. By separating the concurrent transmissions in space using the same radio channel, spatial frequency reuse is an efficient method to enhance spectrum utilization [8]. Hence, two types of gains can be achieved by utilizing spatial resources, namely spatial diversity gain and spatial reuse gain. Existing works mainly focus on how to exploit either maximal spatial diversity gain or maximal spatial reuse gain [9], [10]. Maximizing one type of gain, however, does not necessarily maximize the other. We take a network scenario with two source-destination pairs, as shown in Fig. 1, as an example. Under the protocol interference model, while relay R can enhance the transmission reliability of the concerned link $S-D$ by achieving a spatial diversity gain, it can also enlarge the interference region (IR) of the concerned link to block the transmission of its neighboring link $B-A$. Generally, cooperation occupies more spatial resources to achieve the spatial diversity gain, at a potential cost of reducing the spatial reuse gain due to relay transmissions. There exists a tradeoff between single-link cooperation gain and network-wide reduced spatial frequency reuse. Hence, the effectiveness of cooperation in a wireless ad hoc network should be evaluated from a perspective of overall network performance [1], rather than the performance of a single source-destination pair.

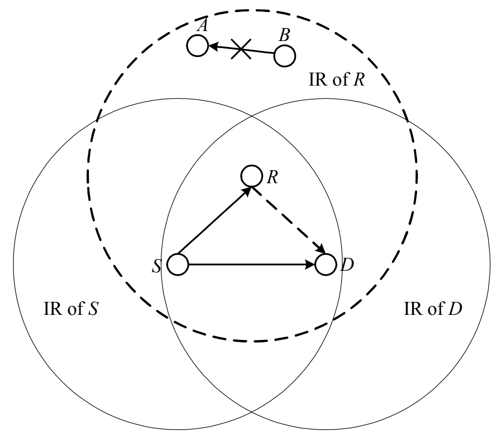

Fig. 1: IR enlargement due to relay transmission under the protocol interference model.

In a wireless ad hoc network, node locations are dynamic due to node mobility. Such characteristics pose challenges on analyzing the overall network performance achieved by both spatial diversity and spatial frequency reuse. Firstly, the single-link cooperation gain depends on relay selection, which should take account of both the spatial distribution of relays and time-varying channel fading. Secondly, the location randomness of both source-destination pairs and relays makes

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it difficult to analyze the reduction in spatial frequency reuse of the whole network. Without network-wide information, a locally beneficial cooperation decision is not guaranteed to be network-wide beneficial. Although there have been significant efforts in exploiting the benefits and demonstrating the effectiveness of cooperation for a single source-destination pair, the performance analysis for cooperation in a wireless ad hoc network is very limited. Such analytical results can provide insights for the network design. Hence, it is desirable to fully understand the benefits and limitations of cooperation in a wireless ad hoc network, which motivates this work.

In this paper, we analyze the network throughput of cooperative transmissions in a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays, to study the tradeoff between the performance achieved by spatial diversity and spatial frequency reuse. Under the protocol interference model, we construct a diamond-shaped relay selection region to restrict the detrimental effect of the enlarged IR due to relay transmissions. Based on stochastic geometry, we model the relay locations by a homogeneous Poisson point process (PPP). Within a relay selection region, the relay with the best channel to the destination that received a packet from the source is selected as the best relay. Over such a network model, we derive the outage probabilities for both selection combining (SC) and maximum ratio combining (MRC) at the destination. The single-link cooperation gain is characterized by comparing the success probabilities of the cooperative and direct transmissions. On the other hand, we model the source locations by a binomial point process (BPP) and approximate the link interference region by an elliptical region. Based on a randomized scheduling scheme, we derive the expected number of concurrent transmissions that can be accommodated within the network coverage area. The network-wide reduced spatial frequency reuse is characterized by comparing the expected numbers of concurrent cooperative and direct transmissions. As a result, we can evaluate the effectiveness of cooperation from a perspective of overall network performance, by checking whether or not the achieved single-link cooperation gain can compensate for the reduction in network-wide spatial frequency reuse.

The main contributions of this paper are three-fold:

$i)$ We develop a theoretical performance analysis framework for cooperation in a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays. It can be extended to analyze more complicated relay selection schemes. The effectiveness of cooperation is evaluated from a perspective of overall network performance by considering both single-link cooperation gain and network-wide reduced spatial frequency reuse;

$ii)$ We construct a diamond-shaped relay selection region to balance the tradeoff between spatial diversity and spatial frequency reuse. The amount of spatial resources allocated to achieve each type of gain is controlled by the size of relay selection region. Such a relay selection region covers the best relay locations by introducing a small enlarged IR;

$iii)$ The network throughput is derived for both SC and MRC at the destination in terms of important network and protocol parameters. Extensive simulations are conducted to validate the performance analysis. The analytical results can be used to evaluate the network performance and provide guidance on the network design.

The rest of this paper is organized as follows. The related work is reviewed in Section II. In Section III, the system model under consideration is described. We present the proposed cooperation scheme and relay selection strategy in Section IV. In Section V, we characterize the single-link cooperation gain. The network-wide reduced spatial frequency reuse is analyzed in Section VI. Numerical results are given in Section VII. Finally, Section VIII concludes this work. The important symbols used in this paper are summarized in Table I, and the proofs of three propositions are given in Appendices.

II. Related Work

Two categories of performance analysis for cooperative communication can be distinguished in the literature. In the first category, the relay locations are known and fixed; while the second category includes the scenario with random relay locations.

The scenario with fixed relay locations is studied in [11]–[15]. Without power control, employing more relays leads to a higher spatial diversity gain, but incurs a larger IR for each cooperative link. In contrast, a single-relay cooperative scheme is easier to implement and achieves full-order spatial diversity by selecting the best relay [11]. Because of its simplicity and efficiency, the single-relay cooperative scheme is used in many existing studies. Opportunistic relaying [11] and selection cooperation [12] are two representative single relay selection approaches. In opportunistic relaying, the relay that maximizes the minimum signal-to-noise ratio (SNR) of the source-relay and relay-destination links is selected. The outage probabilities of opportunistic amplify-and-forward (AF) and decode-and-forward (DF) relaying under different fading channels are derived in [13] and [14], respectively. On the other hand, in selection cooperation, the relay with the best channel to the destination is selected. The outage probability of selection cooperation is derived in closed form in [15]. Although these studies demonstrate the potential benefits of the single relay selection approach, they do not take account of the spatial distribution of relays and cannot be directly extended to a wireless ad hoc network.

The scenario with randomly positioned relays is studied in [16]–[19]. The stochastic geometry [20], [21] as an effective mathematical tool is used to deal with random network topologies by treating node locations in a probabilistic manner. Via modeling the relay locations by a homogeneous PPP, the outage probabilities of both opportunistic relaying and selection cooperation are analyzed for Rayleigh fading channels in [16]. The performance analysis is extended for general fading channels in [17]. An uncoordinated cooperation scheme is proposed in [18], where each relay contends to cooperate with a specific probability calculated based on both local channel state information (CSI) and spatial distribution of relays. Moreover, the authors in [19] introduce a QoS region within which any relay can be selected to satisfy a specified QoS constraint. These studies focus on the performance analysis

TABLE I. Summary of Important Symbols

Symbol	Definition
$\mathcal{A}_{DR_0(\omega)}$	Area of $DR_0(\omega)$
d_{xy}	Euclidean distance between nodes x and y
$DR_0(\omega)$	Diamond-shaped relay selection region with angle ω for link L_0
G	Network throughput gain in using cooperative transmissions
H_{xy}	Random distance-independent fading coefficient between nodes x and y
K_0	Number of potential relays for link L_0
N	Total number of links within the network coverage area
$N_{(D/C)T}$	Expected number of concurrent direct/cooperative transmissions
$P(k,n)$	Probability that k links can be scheduled after checking the first n links
$q_{CT}^{(S/MR)C}$	Outage probability of the cooperative transmission with SC/MRC
q_{DT}	Outage probability of the direct transmission
$Q_{(D/C)T}$	Interference-free probability between any two direct/cooperative transmissions
R_b^0	Best relay of link L_0 for the cooperative transmission with SC
$R_{b'}^0$	Best relay of link L_0 for the cooperative transmission with MRC
$\mathcal{R}_{(D/C)T}^I$	Interference range of the direct/cooperative transmission
$T_{(D/C)T}$	Network throughput of the direct/cooperative transmission
$\mathcal{Z}_i$	Location of relay R_i
α	Path loss exponent
β_r	Required reception threshold for the packets transmitted at rate r
γ_{xy}	SNR of the link between nodes x and y
λ_R	Relay density
Φ_0	Set of qualified relays for link L_0
Φ_R	PPP formed by relay nodes

for a single source-destination pair, which although providing useful insights on the potential benefits of cooperation, cannot characterize the performance of cooperation in a wireless ad hoc network.

A study on cooperation in a wireless ad hoc network is presented in [22], which employs a unit disk graph model to analyze the penalty of the enlarged IR. This method suffers from three limitations: First, the link density is not considered, which is an influential factor for the reduction in network-wide spatial frequency reuse; Second, the unit disk graph model cannot accurately characterize the IRs for both direct and cooperative links; Third, the relay selection is not considered, which determines the single-link cooperation gain.

In this paper, we characterize both single-link cooperation gain and network-wide reduced spatial frequency reuse in a wireless ad hoc network, while taking into account the spatial distributions of sources and relays, constrained relay selection region, and time-varying channel fading. We evaluate the effectiveness of cooperation from a perspective of overall network performance, in terms of the total number of links, relay density, size of relay selection region, and distance between the source and destination.

III. System Model

A. Network Topology

Consider a wireless ad hoc network with nodes randomly located in the network coverage area. The locations of sources at any time instant can be specified by a BPP. Specifically, N sources are independently and uniformly distributed within the network coverage area. To explicitly illustrate the impact of the distance between the source and destination on the effectiveness of cooperative communication, we assume that each source (S_i) has an associated destination (D_i) located at a fixed distance d_{SD} away with a random direction [22]–[24]. Assuming that the relays do not have their own packets to transmit and they are always willing to forward packets from the sources [11]–[19]. At any time instant, they form a homogeneous PPP Φ_R with density λ_R (the number of nodes per unit area). Let $\mathcal{Z}_i$ denote the location of relay R_i.

B. Propagation Channel

The channel between any pair of nodes is characterized by both Rayleigh fading and path loss. The channel impact between nodes x and y on the received signal power is represented by $H_{xy}d_{xy}^{-\alpha}$, where H_{xy} denotes the random distance-independent fading coefficient, d_{xy} denotes the Euclidean distance between the nodes, and $\alpha \geq 2$ denotes the path loss exponent. All the fading coefficients are independent and identically distributed (i.i.d.) exponential random variables with unit mean. All nodes transmit with the same power. For a packet transmission from source S_0 to destination D_0, the received SNR is given by

$$\gamma_{S_0D_0} = \frac{P_t H_{S_0D_0} d_{SD}^{-\alpha}}{W} \quad (1)$$

where P_t and W denote the transmission power and noise power, respectively.

C. Interference Model

We study the impact of interference on spatial reuse under the protocol interference model. The interference range defines a region within which the transmission from an interferer interrupts a packet reception [25]. The impact of interference is binary with respect to the interference range. The interference from an interferer within the interference range of a receiver is intolerable, while the interference from an interferer outside the interference range is negligible. Thus, a packet transmission from S_0 to D_0 is successful only if the following conditions are satisfied: $\gamma_{S_0D_0} \geq \beta_r$ and $d_{S_kD_0} > \mathcal{R}_{DT}^I$, for every S_k, $k > 0$, transmitting concurrently, where β_r denotes the required reception threshold for the packets transmitted at rate r (in bit/s), and $\mathcal{R}_{DT}^I$ denotes the interference range for the direct transmission. Define the required reception threshold $\beta_r \equiv 2^{r/B} - 1$ so that $r = B \cdot \log_2(1 + \beta_r)$, based on Shannon's formula, where B denotes the channel bandwidth in Hz. In two-hop cooperative transmission, rate $2r$ is utilized in both hops to achieve the target rate r between the source and destination. The required reception threshold and interference range for the cooperative transmission are denoted as β_{2r} and $\mathcal{R}_{CT}^I$, respectively.

D. Packet Transmission

We consider a time-slotted packet transmission scheme, in which the time-slot duration is a constant and all sources are synchronized in time. The fading coefficients and node locations remain invariant during one time-slot. There is only one communication channel and the concurrent transmissions are enabled across different locations. Before a packet transmission, the coordination signaling among a source, its intended destination, and neighboring relays is required. Each source always has a packet for transmission [26] and it transmits to its intended destination via either direct or cooperative transmission. For both direct and cooperative transmissions, all packets have equal length and each packet is transmitted in exactly one time-slot. Each node has a single omni-directional antenna and operates in half-duplex mode. As the main focus of this paper is to study the tradeoff between spatial diversity and spatial frequency reuse, the protocol overhead incurred by coordination signaling and relay selection is not considered.

IV. Cooperative Transmission and Relay Selection

A single relay is considered for the cooperative transmission; hence, each source and its best relay share one time-slot in transmitting the same packet. The DF scheme is adopted by the best relay. A time-slot is partitioned equally to two sub-time-slots [27]. Each source transmits a packet at rate $2r$ in the first sub-time-slot. Due to the broadcast nature of wireless communications, the intended destination and neighboring relays can successfully receive the packet, depending on both instantaneous channel fading and path loss. A CSI-based relay selection strategy is employed to select the best relay, which forwards the packet to the intended destination at rate $2r$ in the second sub-time-slot. Finally, the destination decodes the packet using either SC or MRC. In SC, the destination selects only one link from the direct and forward link for packet decoding. On the other hand, in MRC, the destination combines the signals from both the direct and forward link for packet decoding. Hence, two cooperative transmission schemes are considered, that is CSI-based relay selection with SC and MRC, respectively. Studying both SC and MRC can provide insights for the tradeoff between performance and complexity.

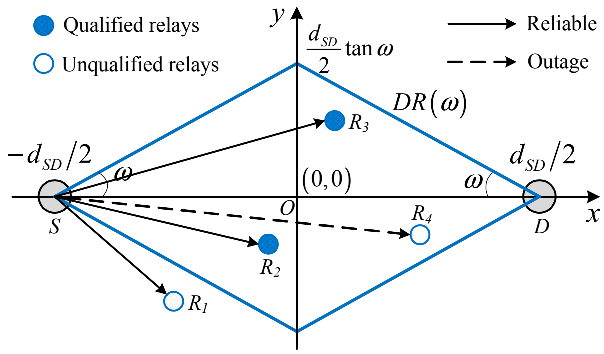


Fig. 2: A diamond-shaped constrained relay selection region under the rectangular coordinate system. Only the relays within the constrained relay selection region (e.g., R_2, R_3, and R_4) and successfully receive the packet from the source (e.g., R_2 and R_3) are qualified relays (e.g., R_2 and R_3).

At the link layer, as each transmission requires handshaking, each node of one link becomes both a transmitter and receiver during the transmission of one packet. The IR of a direct (or cooperative) link is the combination of the IRs of the source and destination (and relays). As more nodes take part in the transmission of one packet, a cooperative link occupies more spatial resources, which reduces the spatial frequency reuse. As shown in Fig. 1, the size of the IR occupied by a cooperative link is determined by the relay location. Because all relays are randomly distributed across the network, the size of the IR of each cooperative link is random and it may be large enough to significantly reduce the spatial frequency reuse. To restrict the size of the IR occupied by each cooperative link, a simple way is to construct a constrained geographical region for relay selection. The relays within the relay selection region are called potential relays and only the potential relays can contend to be the best relay. In order to select the best relay and in turn achieve full-order spatial diversity, the packet collision at each potential relay should be avoided and the signaling among the potential relays for relay selection should be collision-free. As a result, all potential relays should be protected from interference. The size of the IR of a cooperative link is determined by the relative location of the furthermost potential relay, which can be controlled by adjusting the size of relay selection region. Such a relay selection region establishes a connection between single-link cooperation gain and network-wide reduced spatial frequency reuse. A larger relay selection region leads to a higher cooperation gain by incorporating more potential relays, at the cost of reducing the network-wide spatial frequency reuse.

We consider a diamond-shaped relay selection region for each cooperative link, as it covers the best relay locations by introducing a small enlarged IR and only has one parameter. As shown in Fig. 2, a source and its intended destination locate at two endpoints of one diagonal. Such a diamond region (DR) is characterized by an angle ω, which determines the size of the constrained relay selection region. The potential relays that successfully receive the packet from source S_0 in the first sub-time-slot are referred to as qualified relays, which form a decoding set, Φ_0. Mathematically, the decoding set Φ_0 can be expressed as

$$\Phi_0 = \{\mathcal{Z}_i \in \Phi_R \cap DR_0(\omega), \gamma_{S_0 R_i} \geq \beta_{2r}\} \tag{2}$$

where $DR_0(\omega)$ denotes the constrained relay selection region with angle ω for link L_0.

We assume that each qualified relay knows its qualification status and has instantaneous SNR information of the channel between itself and its intended destination. A back-off scheme can be used to select the best relay in a distributed manner, which requires only local CSI [28]. When the decoding set Φ_0 is not empty, a qualified relay with the best channel to the destination obtains the shortest back-off duration and contends to be the best relay first. Other qualified relays quit contention as soon as they receive the signaling from the best relay, R_b^0. The location of the best relay is given by

$$\mathcal{Z}_{R_b^0} = \arg \max_{\mathcal{Z}_i \in \Phi_0} \{H_{R_i D_0} d_{R_i D_0}^{-\alpha}\}. \tag{3}$$

In addition to spatial diversity, spatial frequency reuse is another way to enhance spectrum efficiency. As all links are randomly distributed across the network coverage area and interact with each other, the optimal scheduling problem is shown to be NP-hard in [29]. For simplicity, a randomized scheduling scheme proposed in [30] is employed to activate non-interfering links for concurrent transmissions. The main idea of the randomized scheduling scheme is to check all links in a random order and remove a new link if it interferes with existing ones. The remaining links can be activated concurrently without interrupting each other. Two links interfere with each other when any node of one link locates within the interference range of any node of the other link.

The network throughput, defined as the product of the success probability of each link and the expected number of concurrent transmissions, measures the expected number of concurrent successful transmissions within the network coverage area, given the total number of links and relay density. The network throughput for the direct and cooperative transmissions can be, respectively, expressed as

$$\begin{aligned} T_{DT} &= (1 - q_{DT}) \cdot N_{DT} \\ T_{CT} &= (1 - q_{CT}) \cdot N_{CT} \end{aligned} \tag{4}$$

where q_{DT} and q_{CT} represent the outage probabilities of the direct and cooperative transmissions respectively, N_{DT} and N_{CT} represent the expected numbers of concurrent direct and cooperative transmissions respectively. Due to the enlarged IR, N_{CT} is not larger than N_{DT}.

The expected number of concurrent direct transmissions is affected by the total number of links (i.e., N), while the expected number of concurrent cooperative transmissions is further affected by the size of relay selection region (i.e., ω) and relay density (i.e., λ_R). The values of parameters ω and λ_R can be set to balance the tradeoff between spatial diversity and spatial frequency reuse. Specifically, with an increase of ω or λ_R, the single-link cooperation gain is enhanced as more potential relays are available and the probability of selecting a better relay increases. On the other hand, with an increase of ω or λ_R, the IR of a cooperative link is enlarged and the network-wide spatial frequency reuse is reduced, which will be discussed in Section VI.

From a perspective of overall network performance, the network throughput gain in using the cooperative transmission over the direct transmission can be expressed as

$$G = \frac{(1 - q_{CT}) \cdot N_{CT}}{(1 - q_{DT}) \cdot N_{DT}} = \rho \cdot \eta \tag{5}$$

where $\rho = \frac{1-q_{CT}}{1-q_{DT}}$ represents the single-link cooperation gain and $\eta = \frac{N_{CT}}{N_{DT}}$ represents the network-wide reduced spatial frequency reuse.

According to (5), cooperation in a wireless ad hoc network is beneficial only when the achieved single-link cooperation gain can compensate for the reduction in network-wide spatial frequency reuse, that is, $\rho \cdot \eta > 1$. To evaluate the effectiveness of cooperation, we derive the single-link cooperation gain and network-wide reduced spatial frequency reuse in Sections V and VI, respectively.

V. Single-link Cooperation Gain ρ

In this section, we characterize the single-link cooperation gain by comparing the success probabilities of the cooperative and direct transmissions. Based on stochastic geometry, we derive the outage probability of the CSI-based relay selection strategy for both SC and MRC, while taking into account the spatial distribution of relays, constrained relay selection region, and time-varying channel fading.

A. CSI-based Relay Selection with SC

With SC at the destination, an outage occurs when both the direct and forward links cannot support the required transmission rate. Specifically, as a source transmits a packet at rate $2r$ in the first sub-time-slot, the direct link fails when the SNR at the destination is smaller than β_{2r}. On the other hand, the forward link fails when one of the following events occurs: 1) There are no potential relays within $DR_0(\omega)$; 2) Event $\mathcal{E}_1$: there are no qualified relays when there exists at least one potential relay; 3) Event $\mathcal{E}_2$: destination D_0 fails to decode the packet from the best relay in the second sub-time-slot when decoding set Φ_0 is not empty. Hence, the outage probability, denoted as q_{CT}^{SC}, is given by

$$\begin{aligned} q_{CT}^{SC} &= \mathbb{P}(\gamma_{S_0 D_0} < \beta_{2r}) \\ &\times \left[\mathbb{P}(K_0 = 0) + \sum_{k=1}^{\infty} \mathbb{P}(K_0 = k) \cdot \mathbb{P}(\mathcal{E}_1 \cup \mathcal{E}_2 | K_0 = k) \right] \end{aligned} \tag{6}$$

where K_0 represents the number of potential relays within $DR_0(\omega)$, and outage events $\mathcal{E}_1$ and $\mathcal{E}_2$ can be expressed as

$$\begin{aligned} \mathcal{E}_1 &= \{\Phi_0 = \emptyset, K_0 > 0\} \\ \mathcal{E}_2 &= \left\{ \Phi_0 \neq \emptyset, \gamma_{R_b^0 D_0} < \beta_{2r} \right\}. \end{aligned} \tag{7}$$

$$q_{CT}^{SC} = \left[1 - \exp\left(-M d_{SD}^{\alpha}\right)\right] \times \exp\left[-2\lambda_R \int_0^{\frac{d_{SD}}{2}\tan\omega} \int_{\frac{y}{\tan\omega} - \frac{d_{SD}}{2}}^{\frac{d_{SD}}{2} - \frac{y}{\tan\omega}} \exp\left(-M\left(d_{S_0R}^{\alpha} + d_{RD_0}^{\alpha}\right)\right) dxdy\right]. \tag{8}$$

$$\begin{aligned} q_{CT}^{SC} &= \left[1 - \exp\left(-M d_{SD}^{\alpha}\right)\right] \times \exp\left[-\frac{\sqrt{2\pi}}{\sqrt{M}}\lambda_R \int_0^{\frac{d_{SD}}{2}\tan\omega} \exp\left(-2My^2 - \frac{M d_{SD}^2}{2}\right) \operatorname{erf}\left(\sqrt{2M}\left(\frac{d_{SD}}{2} - y\right)\right) dy\right] \\ &= \left[1 - \exp\left(-M d_{SD}^{\alpha}\right)\right] \times \exp\left[-\sum_{k=0}^{\infty} \frac{\lambda_R 2^{2k+2} M^k}{(2k+1)!! \exp\left(M d_{SD}^2\right)} \mathcal{B}\left(4t^2 - 2d_{SD}t, \frac{d_{SD}}{2}(1 - \tan\omega), \frac{d_{SD}}{2}\right)\right]. \end{aligned} \tag{9}$$

The outage probability can be evaluated in terms of the relay density, size of relay selection region, and distance between the source and destination, as stated in the following proposition.

Proposition 1. *Given a diamond-shaped relay selection region $DR_0(\omega)$, as shown in Fig. 2, the outage probability of the CSI-based relay selection strategy with SC at the destination receiver, given by (6), can be written as (8), where $M = \frac{\beta_{2r} W}{P_t}$, $d_{S_0R} = \sqrt{(x + \frac{d_{SD}}{2})^2 + y^2}$, and $d_{RD_0} = \sqrt{(x - \frac{d_{SD}}{2})^2 + y^2}$.*

The outage probability in (8) decreases exponentially as the relay density increases, and it can be calculated numerically in MAPLE. For the special case of $\alpha = 2$, the integration can be replaced by the series summation in the following corollary, which can be calculated more efficiently [31].

Corollary 1. *For the special case of $\alpha = 2$, the outage probability given in (8) can be simplified as (9), where $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ is the error function and*

$$\mathcal{B}(g(t), m, n) = \int_m^n \exp\left[-Mg(t)\right] t^{2k+1} dt. \tag{10}$$

B. CSI-based Relay Selection with MRC

With MRC at the destination receiver, in the following analysis, the source is treated equivalently as a qualified relay and it transmits the packet in the second sub-time-slot only when it is selected as the best relay, i.e., all qualified relays keep silent. Let $R_{b'}^0$ denote the best relay in this cooperation scheme. The instantaneous SNR of the channel between $R_{b'}^0$ and D_0 is given by

$$\gamma_{R_{b'}^0 D_0} = \max\left\{\gamma_{R_b^0 D_0}, \gamma_{S_0 D_0}\right\}. \tag{11}$$

An outage occurs when the destination fails to decode the packet after combining the signals transmitted by the source and the best relay in the first and second sub-time-slots, respectively. Hence, the outage probability, denoted as q_{CT}^{MRC}, is given by

$$q_{CT}^{MRC} = \mathbb{P}\left(\gamma_{S_0 D_0} + \gamma_{R_{b'}^0 D_0} < \beta_{2r}\right). \tag{12}$$

Proposition 2. *Given a diamond-shaped relay selection region $DR_0(\omega)$, as shown in Fig. 2, the outage probability of the CSI-based relay selection strategy with MRC at the destination receiver, given by (12), can be written as (13).*

Similarly, the outage probability in (13) decreases exponentially as the relay density increases, and it can be numerically evaluated in MAPLE.

In the direct transmission, a source utilizes the whole time-slot to transmit a packet at rate r. An outage occurs when the SNR at the destination is smaller than β_r. Hence, the outage probability for the direct transmission is given by $q_{DT} = 1 - \exp\left(-\frac{\beta_r W}{P_t} d_{SD}^{\alpha}\right)$. Finally, by comparing the success probabilities of the cooperative and direct transmissions, the single-link cooperation gain ρ can be obtained.

VI. Network-wide Reduced Spatial Frequency Reuse η

In this section, we characterize the reduction in network-wide spatial frequency reuse by comparing the expected numbers of concurrent cooperative and direct transmissions that can be accommodated within the network coverage area. Taking into account the spatial distributions of both source-destination pairs and relays, we calculate the expected numbers of concurrent direct and cooperative transmissions based on a randomized scheduling scheme [30].

Let $P(k, n)$ denote the probability that k links can be scheduled for concurrent transmissions after checking the first n links. Denote Q as the interference-free probability between any two links. After checking the first n links, there are k links that can be scheduled concurrently if 1) $(k-1)$ links are scheduled after checking the first $(n-1)$ links, and link n does not interfere with the scheduled $(k-1)$ links; 2) k links are scheduled after checking the first $(n-1)$ links, and link n interferes with at least one of the scheduled k links. Hence, we have

$$P(k, n) = P(k-1, n-1) Q^{k-1} + P(k, n-1)(1 - Q^k). \tag{15}$$

With the initial values $P(1, 1) = 1$, $P(1, 2) = 1 - Q$, and $P(2, 2) = Q$, we can iteratively calculate $P(k, N)$ for all $k \leq N$. Hence, the expected number of concurrent transmissions can be calculated by $N_E = \sum_{k=1}^{N} kP(k, N)$. To calculate the expected number of concurrent transmissions, the interference-free probability between any two direct (cooperative) links should be derived. The interference-free probability between two direct links depends on the distance between the source and destination, while the interference-free probability

$$q_{CT}^{MRC} = \left[1-\exp\left(-\frac{M}{2}d_{SD}^{\alpha}\right)\right]\exp\left(-2\lambda_R\int_0^{\frac{d_{SD}}{2}\tan\omega}\int_{\frac{y}{\tan\omega}-\frac{d_{SD}}{2}}^{\frac{d_{SD}}{2}-\frac{y}{\tan\omega}}\mathcal{I}(x,y)dxdy\right) \tag{13}$$

where

$$\mathcal{I}(x,y) = \frac{-\exp\left[-M\left(d_{S_0R}^{\alpha}+d_{RD_0}^{\alpha}\right)\right]+\exp\left[-\frac{M}{2}\left(d_{SD}^{\alpha}+d_{RD_0}^{\alpha}\right)-Md_{S_0R}^{\alpha}\right]}{\left[1-\exp\left(-\frac{M}{2}d_{SD}^{\alpha}\right)\right]\left[\left(d_{RD_0}/d_{SD}\right)^{\alpha}-1\right]}. \tag{14}$$

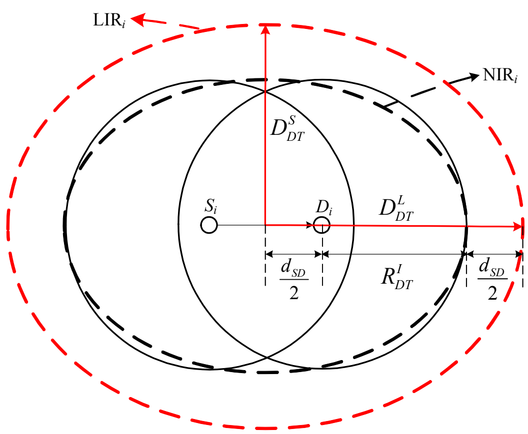


Fig. 3: Approximated IR of a direct link.

between two cooperative links is further affected by the size of relay selection region and relay density.

As the IR of a direct (cooperative) link is the combination of the IRs of the source and destination (and relays), it can be approximated as an elliptical region [1]. We calculate the interference-free probability between any two direct (cooperative) links in the following two subsections.

A. Direct Link

As shown in Fig. 3, we approximate the IR of a direct link, L_i, by an elliptical region and refer to it as node interference region (NIR$_i$). Any node of other active links (e.g., $L_j, j \neq i$) should locate outside NIR$_i$ to avoid interrupting link L_i. However, to guarantee both S_j and D_j locate outside NIR$_i$ is not trivial, as the locations of S_j and D_j are not independent and they are placed d_{SD} apart. For simplicity, we introduce a link interference region (LIR), which is also an elliptical region but larger than the NIR, to capture the interference relationship among different links. Specifically, LIR$_i$ is an elliptical region centered at the center of link L_i, and the lengths of the semi-major axis and semi-minor axis are given by

$$D_{DT}^L = \mathcal{R}_{DT}^I + d_{SD}$$
$$D_{DT}^S = \mathcal{R}_{DT}^I + \frac{d_{SD}}{2}. \tag{16}$$

As illustrated in Fig. 3, the LIR radii are increased by $d_{SD}/2$ from those of the NIR. Thus, for any two direct links (e.g., L_i and L_j), if the center of link L_j is outside LIR$_i$, both S_j and D_j are guaranteed to locate outside NIR$_i$, which implies that two links are interference-free. Note that the reverse condition, the center of link L_i locating outside LIR$_j$, is not required, as

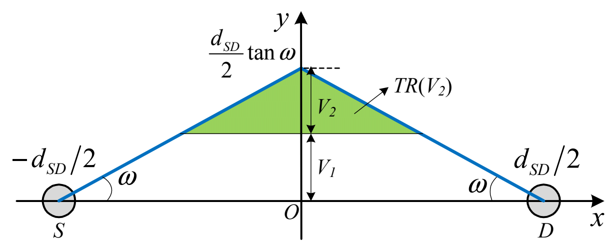


Fig. 4: The upper half of the relay selection region.

the interference relationship between the nodes is reciprocal.

The elliptical LIR area of a direct link is given by $\mathcal{A}_{DT} = \pi D_{DT}^L D_{DT}^S$. Because of the uniform distribution of nodes, the interference-free probability between any two direct links can be calculated by $Q_{DT} = 1 - \frac{\mathcal{A}_{DT}}{\mathcal{A}_N}$, where $\mathcal{A}_N$ is the network coverage area.

B. Cooperative Link

As discussed in Section IV, all potential relays should be protected from interference and the IR of a cooperative link is determined by the relative location of the furthermost potential relay in each side of the link. Due to the symmetry, we take the upper half of the relay selection region, as shown in Fig. 4, as an example. The furthermost potential relay refers to the potential relay that has the largest Y-coordinate. Denote V_1 as the Y-coordinate of the furthermost potential relay. As the potential relays are randomly distributed within $DR_0(\omega)$, V_1 is a random variable, which takes value in $\left[0, \frac{d_{SD}}{2}\tan\omega\right]$. To calculate the average size of the IR occupied by a cooperative link, we calculate the expected value of V_1 as follows.

Proposition 3. *The expected value of the Y-coordinate of the furthermost potential relay in the upper half of the relay selection region, $E[V_1]$, is given by*

$$E\left[V_1\right] = \frac{d_{SD}}{2}\tan\omega - \sqrt{\frac{\pi\tan\omega}{4\lambda_R}}\,\mathrm{erf}\left(\frac{d_{SD}}{2}\sqrt{\lambda_R\tan\omega}\right). \tag{17}$$

From (17), $E[V_1]$ increases with λ_R and ω. This is because, with an increase of λ_R and ω, the probability of having a faraway potential relay increases. Let R_{F1} and R_{F2} denote the furthermost potential relays at both sides, at $(0, E(V_1))$ and $(0, -E(V_1))$ on average, respectively. The IR of a cooperative link is calculated according to the average locations of the furthermost potential relays.

The IR of a cooperative link, L_i, is also approximated as an elliptical region, referred to as cooperative node interference

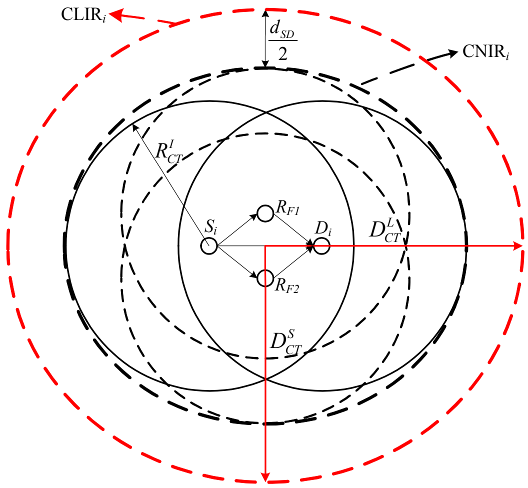


Fig. 5: Approximated IR of a cooperative link.

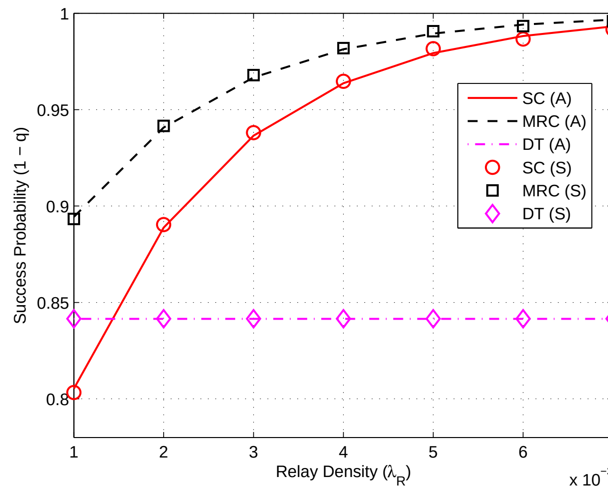


Fig. 6: Success probability versus relay density (in nodes/m^2) when $\omega = \pi/6$ and $d_{SD} = 50$ m.

region (CNIR$_i$). Similarly, as shown in Fig. 5, we define a cooperative link interference region (CLIR$_i$), which is centered at the center of link L_i and the lengths of the semi-major axis and semi-minor axis are given by

$$\begin{aligned} D^L_{CT} &= \mathcal{R}^I_{CT} + d_{SD} \\ D^S_{CT} &= \mathcal{R}^I_{CT} + \frac{d_{SD}}{2} + E(V_1). \end{aligned} \tag{18}$$

For any two cooperative links (e.g., L_i and L_j), if the center of CLIR$_j$ locates outside CLIR$_i$, both L_i and L_j can transmit concurrently without interrupting each other. The elliptical CLIR area can be calculated by $\mathcal{A}_{CT} = \pi D^L_{CT} D^S_{CT}$. Accordingly, the interference-free probability between any two cooperative links is given by $Q_{CT} = 1 - \frac{\mathcal{A}_{CT}}{\mathcal{A}_N}$.

The calculation of the interference-free probability between any two links is slightly conservative, as some links may be blocked although they do not actually cause collisions. According to the simulation results in Section VII, the results calculated by this method is rather accurate and the computation complexity is quite low. The expected numbers of concurrent direct and cooperative transmissions, N_{DT} and N_{CT}, can be calculated by substituting Q_{DT} and Q_{CT} into (15). By comparing these two numbers, the network-wide reduced spatial frequency reuse η can be obtained. Finally, we can derive the network throughput and network throughput gain by substituting the corresponding values into (4) and (5), respectively.

VII. Numerical Results

This section presents analytical (A) and simulation (S) results for both direct and cooperative transmissions in a wireless ad hoc network. In the simulation, a circular network coverage area with radius 2000 m is considered. Based on [32] and [33], the interference ranges of direct and cooperative transmissions, $\mathcal{R}^I_{DT}$ and $\mathcal{R}^I_{CT}$, are set to be 60 m and 70 m, respectively. The transmission rate (r) and channel bandwidth (B) are normalized to be 1 Mbit/s and 1 MHz, respectively. The reception threshold of direct transmissions (β_r) is set to be $\sqrt{2}-1$, which is calculated based on Shannon's formula and the normalized reception threshold of cooperative transmissions (i.e., $\beta_{2r} = 1$). In addition, we set $P_t = 0.06$ mW, $W = -50$ dBm, and $\alpha = 2$. The simulation results are obtained by averaging 10^5 realizations of the random network topology.

A. Transmission Success Probability and Single-Link Cooperation Gain

In this subsection, we study the impact of relay density λ_R, angle of relay selection region ω, and link distance d_{SD} on the transmission success probability and single-link cooperation gain.

Fig. 6 shows the success probabilities of direct transmission (DT) and cooperative transmissions (SC and MRC) versus the relay density with parameters $\omega = \pi/6$ and $d_{SD} = 50$ m, where the analytical results are obtained based on (8) and (13). When the relay density is low, the cooperative transmission with an SC receiver performs worse than the direct transmission, because the probability of having a reliable relay is low and the cooperative transmission requires a higher reception threshold for the SNR. It is observed that the success probabilities of both cooperative transmission schemes increase with the relay density, while that of the direct transmission does not change. This is due to the fact that, with an increase of λ_R, more potential relays are available for relay selection, which results in a higher probability of selecting a reliable relay. By combining the signal from the direct link, the cooperative transmission with an MRC receiver outperforms that with an SC receiver. However, the performance gap between the cooperative transmission schemes reduces with an increase of λ_R, because the channel quality of the forward link becomes better, which reduces the importance of the direct link for packet decoding.

Fig. 7 illustrates the single-link cooperation gain achieved by cooperative transmission versus the angle of relay selection region for $d_{SD} = 50$ m and 40 m when $\lambda_R = 0.003$ nodes/m^2. The cooperation gain increases with the size of

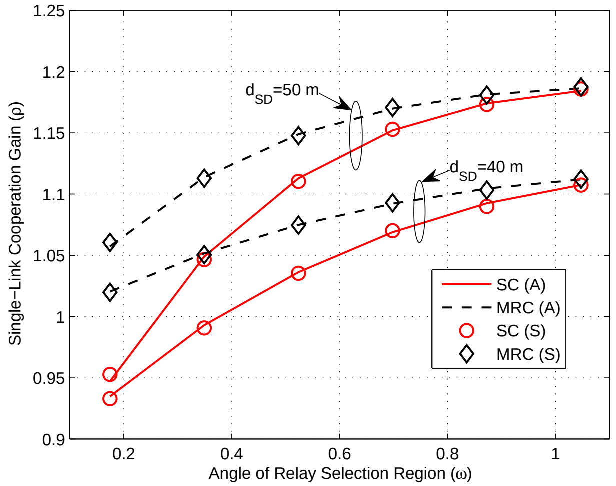


Fig. 7: Single-link cooperation gain versus angle of relay selection region for $d_{SD} = 50$ m and 40 m when $\lambda_R = 0.003$ nodes/m^2.

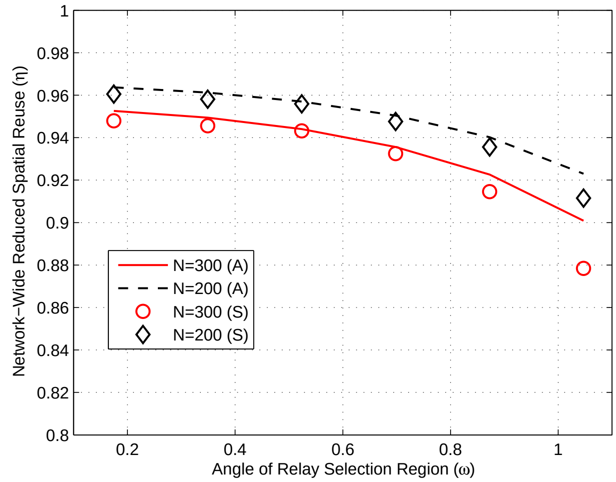


Fig. 9: Network-wide reduced spatial frequency reuse versus angle of relay selection region for $N = 200$ and 300 when $\lambda_R = 0.003$ nodes/m^2 and $d_{SD} = 50$ m.

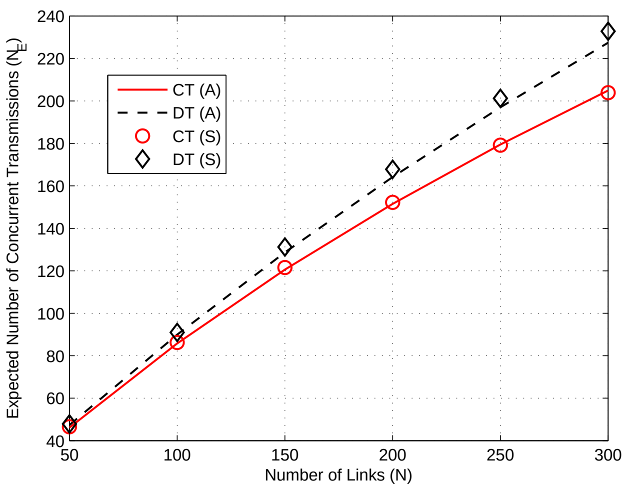


Fig. 8: Expected numbers of concurrent direct and cooperative transmissions versus total number of links when $\omega = \pi/3$, $\lambda_R = 0.003$ nodes/m^2, and $d_{SD} = 50$ m.

relay selection region, as more spatial resources are allocated to each cooperative link and more potential relays are available for spatial diversity. Similarly, the performance gap between the cooperative transmission schemes shrinks with an increase of ω. The cooperation gain at $d_{SD} = 40$ m is smaller than that at $d_{SD} = 50$ m. This observation shows that the cooperative transmission is preferable when the link distance is large (i.e., the channel quality of the direct link is poor).

B. Expected Number of Concurrent Transmissions and Network-Wide Reduced Spatial Reuse

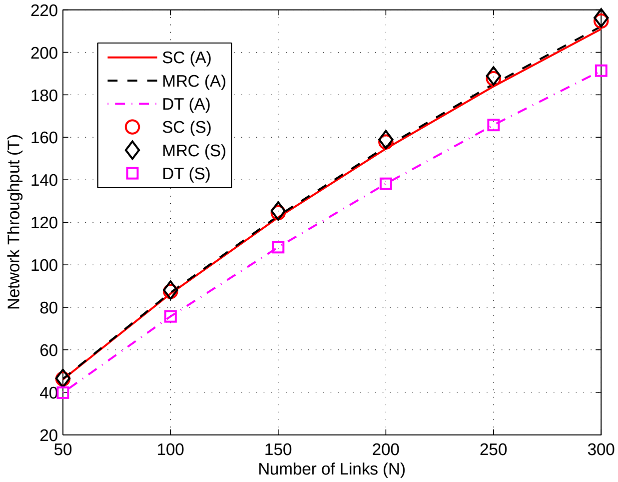
Fig. 8 plots the expected numbers of concurrent direct transmissions and cooperative transmissions (CT) versus the total number of links with parameters $\omega = \pi/3$, $\lambda_R = 0.003$ nodes/m^2, and $d_{SD} = 50$ m. The expected numbers of concurrent transmissions increase with the total number of links. As expected, the expected number of concurrent cooperative transmissions is always smaller than that of concurrent direct transmissions, as the enlarged IR due to relay transmissions reduces the network-wide spatial frequency reuse. The gap between the expected numbers of concurrent direct and cooperative transmissions expands with an increase of N. This is because the adverse effect of the enlarged IR becomes more significant as N increases.

Fig. 9 shows the network-wide reduced spatial frequency reuse versus the angle of relay selection region for $N = 200$ and 300 with parameters $\lambda_R = 0.003$ nodes/m^2 and $d_{SD} = 50$ m. The spatial frequency reuse reduces as the angle of relay selection region increases. As ω increases, each cooperative link occupies more spatial resources on average for spatial diversity, which reduces the spatial resources available for spatial reuse. The reduction in network-wide spatial frequency reuse at $N = 300$ is larger than that at $N = 200$. This observation shows that, when the link density is high, the cooperative transmission is more likely to cause link blockage (i.e., the limitation of the cooperative transmission becomes more significant). Due to the conservative calculation of the interference-free probability between any two links, there exists a small deviation between the analytical and simulation results.

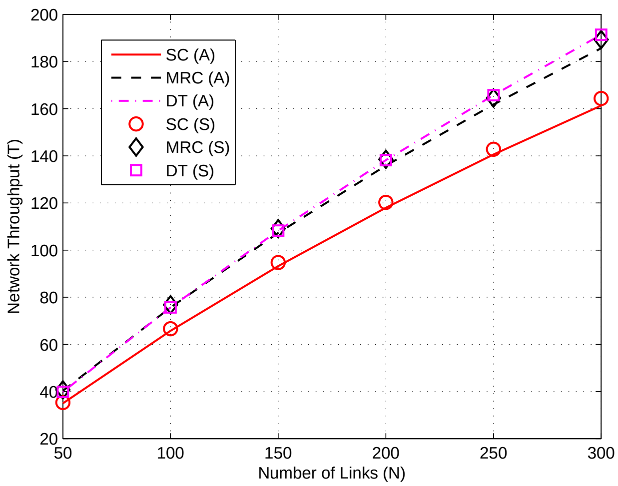
C. Network Throughput

In this subsection, we account for both the transmission success probability and expected number of concurrent transmissions, and study the impact of the total number of links, size of relay selection region, relay density, and link distance on the network throughput.

Fig. 10 illustrates the network throughput of the direct and cooperative transmissions versus the total number of links when $d_{SD} = 50$ m. The network throughput of all schemes increase with N. With a small relay selection region and a high relay density, as shown in Fig. 10(a), both cooperative transmission schemes outperform the direct transmission, as



(a) $\omega = \pi/6$ and $\lambda_R = 0.006$ nodes/m^2



(b) $\omega = \pi/3$ and $\lambda_R = 0.0002$ nodes/m^2

Fig. 10: Network throughput versus total number of links when $d_{SD} = 50$ m.

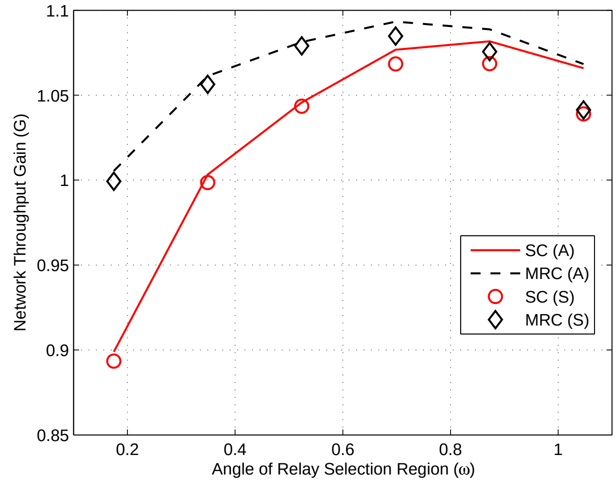


Fig. 11: Network throughput gain versus angle of relay selection region when $\lambda_R = 0.003$ nodes/m^2, $d_{SD} = 50$ m, and $N = 300$.

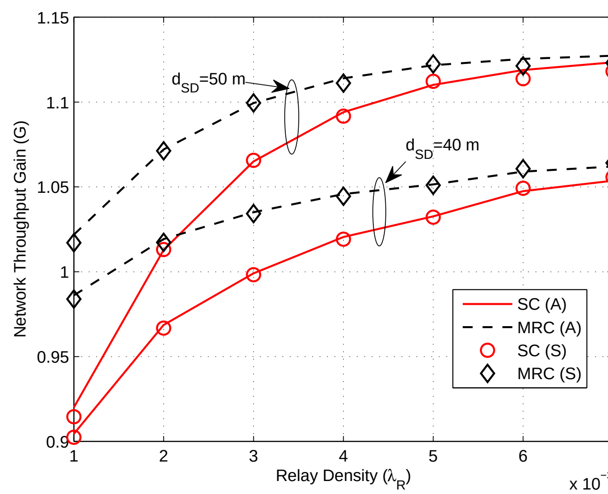


Fig. 12: Network throughput gain versus relay density (in nodes/m^2) for $d_{SD} = 40$ m and 50 m when $\omega = \pi/6$ and $N = 200$.

the single-link cooperation gain outweighs the reduction in network-wide spatial frequency reuse. On the other hand, with a large relay selection region and a low relay density, as shown in Fig. 10(b), the direct transmission achieves higher network throughput than both cooperative transmission schemes. This implies that the cooperative transmission is not always beneficial and its effectiveness depends on the size of relay selection region and relay density.

Fig. 11 plots the network throughput gain versus the angle of relay selection region with parameters $\lambda_R = 0.003$ nodes/m^2, $d_{SD} = 50$ m, and $N = 300$. With the variation of the angle of relay selection region, there exists a peak point of the network throughput gain. Take the cooperative transmission with MRC as an example. The network throughput gain increases with ω when $\omega < 2\pi/9$, and decreases with ω when $\omega > 2\pi/9$. Note that when $\omega > 2\pi/9$ the increasing rate of the single-link cooperation gain is smaller than the decreasing rate of the network-wide reduced spatial frequency reuse. This implies that the size of relay selection region can be set to balance the tradeoff between spatial diversity and spatial reuse. In addition, we observe that the direct transmission outperforms the cooperative transmission with SC when $\omega < \pi/9$, as the number of potential relays is not large enough to enhance the overall network performance. Comparing the results in Figs. 7 and 11, due to the reduced spatial frequency reuse, the network throughput gain is always smaller than the single-link cooperation gain, and the cooperative transmission that is beneficial for a single source-destination pair may not be beneficial for the whole network. Hence, the effectiveness of cooperation should be evaluated from a perspective of overall network performance.

Fig. 12 shows the network throughput gain of both cooperative transmission schemes versus the relay density for $d_{SD} = 40$ m and 50 m with parameters $\omega = \pi/6$ and $N = 200$. The network throughput gain increases with the relay density, as more potential relays are available for each cooperative link. When $d_{SD} = 40$ m, the cooperative transmission with SC is beneficial only if $\lambda_R > 3 \times 10^{-3}$ nodes/m^2, while the cooperative transmission with MRC is beneficial only if $\lambda_R > 1.4 \times 10^{-3}$ nodes/m^2. This implies that a smaller number of potential relays is required for the cooperative transmission with MRC, at the cost of requiring higher implementation complexity. On the other hand, the network throughput gain at $d_{SD} = 40$ m is smaller than that at $d_{SD} = 50$ m. This confirms that the effectiveness of cooperation depends on the link distance and the cooperative transmission is effective when the link distance is large.

VIII. Conclusions

In this paper, we investigate the network throughput of cooperative communication in a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays. The objective is to evaluate the effectiveness of cooperation from a perspective of overall network performance. We construct a diamond-shaped relay selection region to study the tradeoff between spatial diversity gain of a single link and reduced spatial frequency reuse of the whole network. We derive the network throughput of the proposed cooperation scheme in terms of the total number of links, relay density, size of relay selection region, and link distance. The cooperative transmission is not always beneficial and its effectiveness depends on all these influential factors. Due to the reduced spatial frequency reuse, the network throughput gain is always smaller than the single-link cooperation gain. Extensive simulations are conducted to validate the performance analysis. The analytical results can be used to evaluate the network performance and provide guidance on the design of large-scale networks. For further work, we aim to take account of the protocol overhead incurred by coordination signaling and relay selection, in order to fully understand the benefits and limitations of cooperative communication.

Appendix A. Proof of Proposition 1

The probability that the direct link fails in the first sub-time-slot is given by

$$\mathbb{P}\left(\gamma_{S_0D_0} < \beta_{2r}\right) = \mathbb{P}\left(H_{S_0D_0} < \frac{\beta_{2r}W}{P_t} d_{SD}^{\alpha}\right) \stackrel{(a)}{=} 1 - \exp(-M d_{SD}^{\alpha}) \tag{19}$$

where (a) follows from the exponential distribution of $H_{S_0D_0}$.

Similarly, with a relay located at $\mathcal{Z}_i$ instead of at a distance d_{SD} away, the success probabilities of the source-relay and relay-destination links can be expressed as

$$\begin{aligned} \mathbb{P}\left(\gamma_{S_0R_i} \ge \beta_{2r}\right) &= \exp\left(-M d_{S_0R_i}^{\alpha}\right) \\ \mathbb{P}\left(\gamma_{R_iD_0} \ge \beta_{2r}\right) &= \exp\left(-M d_{R_iD_0}^{\alpha}\right). \end{aligned} \tag{20}$$

As the potential relays form a homogeneous PPP, the probability of existing k relays within $DR_0(\omega)$ is given by

$$\mathbb{P}\left(K_0 = k\right) = \frac{\left(\lambda_R \mathcal{A}_{DR_0(\omega)}\right)^k}{k!} \exp\left(-\lambda_R \mathcal{A}_{DR_0(\omega)}\right) \tag{21}$$

where $\mathcal{A}_{DR_0(\omega)} = \frac{d_{SD}^2}{2} \tan\omega$ represents the area of $DR_0(\omega)$.

Outage event $\mathcal{E}_1$ means that no potential relays have a reliable link to the source. Outage event $\mathcal{E}_2$ means that no qualified relays have a reliable link to the destination when the decoding set is not empty. Hence, outage event $(\mathcal{E}_1 \cup \mathcal{E}_2)$ is equivalent to the event that no potential relays have a reliable link to both the source and destination. Given that k potential relays locate within $DR_0(\omega)$, we have

$$\begin{aligned} &\mathbb{P}\left(\mathcal{E}_1 \cup \mathcal{E}_2 \,|K_0 = k\right) \\ &= \mathbb{E}\left[\prod_{i=1}^{k}\left[1 - \mathbb{P}\left(\gamma_{S_0R_i} \ge \beta_{2r}\right) \cdot \mathbb{P}\left(\gamma_{R_iD_0} \ge \beta_{2r}\right)\right]\right]. \end{aligned} \tag{22}$$

As k potential relays are uniformly distributed within $DR_0(\omega)$, we have

$$\begin{aligned} &\mathbb{P}\left(\mathcal{E}_1 \cup \mathcal{E}_2 \,|K_0 = k\right) \\ &\stackrel{(a)}{=} \left(\frac{\int_{DR_0(\omega)} \left[1 - \mathbb{P}\left(\gamma_{S_0R_i} \ge \beta_{2r}\right) \mathbb{P}\left(\gamma_{R_iD_0} \ge \beta_{2r}\right)\right] ds}{\mathcal{A}_{DR_0(\omega)}}\right)^k \\ &\equiv \mathcal{J}^k \end{aligned} \tag{23}$$

where (a) follows from the probability generating functional (PGFL) of the BPP [21].

Combining (21) and (23), we can obtain (24), shown at the top of the next page.

Consider a rectangular coordinate system with origin at link center O, as shown in Fig. 2. Due to the constrained relay selection region, the coordinate of any qualified relay should satisfy one of the following two constraints

$$\begin{cases} x \in \left[\frac{y}{\tan\omega} - \frac{d_{SD}}{2}, -\frac{y}{\tan\omega} + \frac{d_{SD}}{2}\right], & \text{if } y \in \left[0, \frac{d_{SD}}{2}\tan\omega\right] \\ x \in \left[-\frac{y}{\tan\omega} - \frac{d_{SD}}{2}, \frac{y}{\tan\omega} + \frac{d_{SD}}{2}\right], & \text{if } y \in \left[-\frac{d_{SD}}{2}\tan\omega, 0\right] \end{cases}. \tag{25}$$

Due to the symmetry, we focus on the upper half of the constrained relay selection region, where $y \in \left[0, \frac{d_{SD}}{2}\tan\omega\right]$. By substituting (19), (20), and (24) into (6), we get the result in (8).

Appendix B. Proof of Proposition 2

As the qualified relay with the best channel to the destination is selected, the outage probability can be expressed as

$$\begin{aligned} q_{CT}^{MRC} &= \mathbb{P}\left(\gamma_{S_0D_0} + \max\left\{\gamma_{R_b^0D_0}, \gamma_{S_0D_0}\right\} < \beta_{2r}\right) \\ &= \mathbb{P}\left(\gamma_{S_0D_0} + \max_{\mathcal{Z}_i \in \Phi_0}\left\{\gamma_{R_iD_0}\right\} < \beta_{2r}, 2\gamma_{S_0D_0} < \beta_{2r}\right) \\ &= \mathbb{P}\left(\gamma_{S_0D_0} + \max_{\mathcal{Z}_i \in \Phi_0}\left\{\gamma_{R_iD_0}\right\} < \beta_{2r} \,\middle|\, \gamma_{S_0D_0} < \frac{\beta_{2r}}{2}\right) \\ &\times \mathbb{P}\left(\gamma_{S_0D_0} < \frac{\beta_{2r}}{2}\right). \end{aligned} \tag{26}$$

With $H_{S_0D_0}$ following an exponential distribution, we have $\mathbb{P}(\gamma_{S_0D_0} < \beta_{2r}/2) = 1 - \exp(-M d_{SD}^{\alpha}/2)$.

By selecting the best relay, the outage event is equivalent to that all qualified relays are in outage. The conditional probability of transmission failure with MRC can be obtained

$$\begin{aligned}&\mathbb{P}\left(K_0=0\right)+\sum_{k=1}^{\infty}\mathbb{P}\left(K_0=k\right)\cdot\mathbb{P}\left(\mathcal{E}_1\cup\mathcal{E}_2\left|K_0=k\right.\right)\\&=\exp\left(-\lambda_R\mathcal{A}_{DR_0(\omega)}\right)+\sum_{k=1}^{\infty}\frac{\left(\lambda_R\mathcal{A}_{DR_0(\omega)}\right)^k}{k!}\exp\left(-\lambda_R\mathcal{A}_{DR_0(\omega)}\right)\cdot\mathcal{J}^k\\&=\exp\left[-\lambda_R\mathcal{A}_{DR_0(\omega)}\left(1-\mathcal{J}\right)\right]\\&=\exp\left[-\lambda_R\int_{DR_0(\omega)}\mathbb{P}\left(\gamma_{S_0R_i}\geq\beta_{2r}\right)\cdot\mathbb{P}\left(\gamma_{R_iD_0}\geq\beta_{2r}\right)ds\right].\end{aligned}\tag{24}$$

$$\begin{aligned}&\mathbb{P}\left(\gamma_{S_0D_0}+\max_{\mathcal{Z}_i\in\Phi_0}\left\{\gamma_{R_iD_0}\right\}<\beta_{2r}\left|\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right.\right)\\&=\exp\left[-\int_{DR_0(\omega)}\left(1-\mathbb{P}\left(\gamma_{S_0D_0}+\gamma_{RD_0}<\beta_{2r}\left|\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right.\right)\right)\mu\left(ds\right)\right].\end{aligned}\tag{28}$$

$$\begin{aligned}&\mathbb{P}\left(\gamma_{S_0D_0}+\gamma_{RD_0}<\beta_{2r},\gamma_{S_0D_0}<\tfrac{\beta_{2r}}{2}\right)\\&=\mathbb{P}\left(d_{SD}^{-\alpha}H_{S_0D_0}+d_{RD_0}^{-\alpha}H_{RD_0}<M,H_{S_0D_0}<\tfrac{M}{2}d_{SD}^{\alpha}\right)\\&=\int_0^{\frac{M}{2}d_{SD}^{\alpha}}\int_0^{Md_{SD}^{\alpha}-\left(\frac{d_{SD}}{d_{RD_0}}\right)^{\alpha}x}\exp\left(-x\right)\exp\left(-y\right)dydx\\&=\begin{cases}1-\exp\left(-\frac{M}{2}d_{SD}^{\alpha}\right)+\frac{\exp\left(-Md_{RD_0}^{\alpha}\right)-\exp\left(-\frac{M}{2}\left(d_{SD}^{\alpha}+d_{RD_0}^{\alpha}\right)\right)}{\left(d_{RD_0}/d_{SD}\right)^{\alpha}-1}, & \text{if } d_{SD}\neq d_{RD_0}\\1-\exp\left(-\frac{M}{2}d_{SD}^{\alpha}\right)\left[1+\frac{M}{2}d_{SD}^{\alpha}\exp\left(-\frac{M}{2}d_{SD}^{\alpha}\right)\right], & \text{if } d_{SD}=d_{RD_0}.\end{cases}\end{aligned}\tag{31}$$

as

$$\begin{aligned}&\mathbb{P}\left(\gamma_{S_0D_0}+\max_{\mathcal{Z}_i\in\Phi_0}\left\{\gamma_{R_iD_0}\right\}<\beta_{2r}\left|\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right.\right)\\&=\mathbb{E}\left[\prod_{\mathcal{Z}_i\in\Phi_0}\mathbb{P}\left(\gamma_{S_0D_0}+\gamma_{R_iD_0}<\beta_{2r}\left|\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right.\right)\right].\end{aligned}\tag{27}$$

As different source-relay links experience independent channel fading, the set of qualified relays Φ_0 is an independent thinning of Φ_R. Statistically, a relay closer to the source has a higher probability to receive the packet successfully in the first sub-time-slot. Hence, the probability of successful packet reception at each relay is location-dependent, which results in an inhomogeneous PPP Φ_0. According to the PGFL of the PPP [21], we obtain (28), where $\mu\left(ds\right)$ represents the intensity measure.

The intensity measure of Φ_0 is equal to the average number of qualified relays in $DR_0(\omega)$. It is given by

$$\begin{aligned}\mu\left(DR_0\left(\omega\right)\right)&\overset{(a)}{=}\mathbb{E}\left[\sum_{\mathcal{Z}_i\in\Phi_R}\mathbf{1}\left(\mathcal{Z}_i\in\Phi_0\cap DR_0(\omega)\right)\right]\\&\overset{(b)}{=}\int_{DR_0(\omega)}\lambda_R\exp\left(-Md_{S_0R_i}^{\alpha}\right)ds\end{aligned}\tag{29}$$

where $\mathbf{1}(\cdot)$ is the indicator function, (a) follows from the definition of intensity measure, and (b) follows from the Campbell's theorem [21].

According to the definition of conditional probability, we have

$$\begin{aligned}&\mathbb{P}\left(\gamma_{S_0D_0}+\gamma_{RD_0}<\beta_{2r}\left|\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right.\right)\\&=\frac{\mathbb{P}\left(\gamma_{S_0D_0}+\gamma_{RD_0}<\beta_{2r},\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right)}{\mathbb{P}\left(\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right)}.\end{aligned}\tag{30}$$

In (30), we should first calculate $\mathbb{P}\left(\gamma_{S_0D_0}+\gamma_{RD_0}<\beta_{2r},\gamma_{S_0D_0}<\frac{\beta_{2r}}{2}\right)$. As both $H_{S_0D_0}$ and H_{RD_0} follow an exponential distribution, we obtain (31).

Due to the constrained relay selection region, d_{SD} is not equal to d_{RD_0}. By substituting (29), (30), and (31) into (28), we obtain (13).

Appendix C. Proof of Proposition 3

Let random variable $V_2=\frac{d_{SD}}{2}\tan\omega-V_1$, as shown in Fig. 4. The expected value of V_2 can be expressed as

$$\begin{aligned}E\left[V_2\right]&=\int_0^{\frac{d_{SD}}{2}\tan\omega}v\cdot f_{V_2}\left(v\right)dv\\&=\int_0^{\frac{d_{SD}}{2}\tan\omega}\overline{F}_{V_2}\left(v\right)dv\end{aligned}\tag{32}$$

where $f_{V_2}(v)$, $\overline{F}_{V_2}(v)$ represent the probability density function and complementary cumulative density function of random variable V_2. Note that $\overline{F}_{V_2}(v)$ is the same as the probability that there are no potential relays within the shaded triangular region. Hence, we have

$$\begin{aligned}\overline{F}_{V_2}(v) &= \mathbb{P}(V_2 > v) \\ &= \mathbb{P}(\text{No potential relays within } TR(v)) \\ &\overset{(a)}{=} \exp\left(-\frac{\lambda_R}{\tan\omega}v^2\right)\end{aligned} \tag{33}$$

where (a) follows from the definition of the PPP, and $TR(v)$ represents the shaded triangular region and its area is $\mathcal{A}_{TR(v)} = v^2/\tan\omega$.

By substituting (33) into (32), we have

$$E[V_2] = \sqrt{\frac{\pi\tan\omega}{4\lambda_R}}\,\mathrm{erf}\left(\frac{d_{SD}}{2}\sqrt{\lambda_R\tan\omega}\right). \tag{34}$$

As $E[V_1] = \frac{d_{SD}}{2}\tan\omega - E[V_2]$, we get the result in (17).

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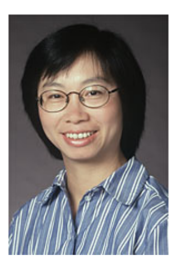

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