Optimizing the Energy Delivery via V2G Systems based on Stochastic Inventory Theory

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***Abstract*—In this paper, we study the optimal energy delivery problem from viewpoints of both the vehicle owner and aggregator, in load shaving services of a vehicle-to-grid (V2G) system. We formulate the optimization problem based on a general plug-in hybrid electric vehicle (PHEV) model, taking into account the randomness in vehicle mobility, time-of-use electricity pricing, and realistic battery modeling. Stochastic inventory theory is applied to analyze the problem. We mathematically prove that a state-dependent (S, S') policy is optimal for the daily energy cost minimization of each vehicle, and develop an estimation algorithm to calculate the parameters of the optimal policy for practical applications. Furthermore, we investigate the multi-vehicle aggregator design problem by considering the power system constraints. A policy adjustment scheme is proposed to adjust the values of S and S' with respect to the optimal policy adopted by each PHEV, such that the aggregated recharging and discharging power constraints of the power system can be satisfied, while minimizing the incremental cost (or revenue loss) of PHEV owners. Based on characteristics of the state-dependent (S, S') policy and our proposed policy adjustment scheme, the optimal aggregator operation problem is transformed into a convex optimization one which can be readily solved by existing algorithms. The performance of our proposed schemes is evaluated via simulations based on real data collected from Canadian utilities, households, and commuters.**

***Index Terms*—Optimal energy delivery, plug-in hybrid electric vehicle, stochastic inventory theory, vehicle-to-grid.**

NOMENCLATURE

$C_{i,n,s_{i,n},x_{i,n}}(U_{i,n})$	Expected energy cost of PHEV i during periods $\{n, n+1, \cdots, N\}$ with respect to state $s_{i,n}$, energy level $x_{i,n}$, and decision $U_{i,n}$.
$C_{i,N}(x)$	End-of-day cost function of PHEV i with respect to energy level x.
$c^{rc}_{i,n}(z)$	Cost of increasing the energy level of the battery of PHEV i by z in period n.
$C^{s_{i,k}}_{i,k}(x_{i,k}, u_{i,k})$	Energy cost of PHEV i in period k with respect to state $s_{i,n}$, energy level $x_{i,n}$, and decision $u_{i,k}$.
$\mathcal{H}, \mathcal{W}, \mathcal{C}$	States (regions) of PHEV location: home, work, commute.
$\mathcal{I}_n$	Set of PHEVs connected to the aggregator in period n.
N	Index of the aggregate period.
$P_{i,n}(s_{i,n+1}\|s_{i,n})$	Transition probability of the state of PHEV i from $s_{i,n}$ in period n to $s_{i,n+1}$ in period $n+1$.
r_n	Time-of-use electricity price in period n.
r'_i	Average cost of using gasoline to satisfy a unit of energy demand of PHEV i.
$\tilde{r}_i$	Factor of battery value loss of PHEV i with respect to the recharged (or discharged) energy.
$s_{i,n}$	State of PHEV i in period n.
$S_{i,n}(s)$ $(S'_{i,n}(s))$	Recharging (discharging) threshold of PHEV i in period n with respect to state s.
$\tilde{S}_{i,n}(s)$ $(\tilde{S}'_{i,n}(s))$	Adjusted recharging (discharging) threshold of PHEV i in period n with respect to state s.
$\Delta S_{i,n}(s)$	Policy adjustment parameter of PHEV i in period n with respect to state s.
T	Duration of a period.
$u_{i,n}$	Energy level of the battery of PHEV i at the end of period n.
$u^*_{i,n}$	Optimal decision of PHEV i in period n.
$u^*_{i,n}(s,x)$	Optimal policy of PHEV i in period n with respect to state s and energy level x.
$u^{\max}_{agg,rc,n}$ $(u^{\max}_{agg,dc,n})$	Limit of the aggregated recharging (discharging) energy in period n.
$u^{\max}_{i,rc}$ $(u^{\max}_{i,dc})$	Maximum amount of energy that can be recharged into (discharged from) the battery of PHEV i in a period.
$\tilde{u}_{i,n}$	Adjusted decision of PHEV i in period n.
$\tilde{u}_{i,n}(s,x)$	Adjusted policy of PHEV i in period n with respect to state s and energy level x.
U_i	Decision variable of PHEV i.

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$U_{i,n}$	Decisions made by PHEV i from period n to period $N-1$.
U_i^*	Optimal decisions of PHEV i.
$U_{i,n}^*$	Optimal decisions of PHEV i from period n to period $N-1$.
$\tilde{U}_{i,n}$	Adjusted decisions of PHEV i from period n to period $N-1$.
$\mathscr{U}_i$	Class of all admissible decisions for PHEV i.
$\mathscr{U}_{i,n}$	Class of all admissible decisions for PHEV i from period n to period $N-1$.
$\tilde{\mathscr{U}}_{i,n}$	Class of all admissible adjusted decisions of PHEV i from period n to period $N-1$.
$V_{i,n}(s,x)$	Value function of PHEV i within periods $\{n, n+1, \cdots, N\}$ with respect to state s and energy level x.
$x_{i,n}$	Energy level of the battery of PHEV i at the beginning of period n.
$x_i^{\max}$	Capacity of the battery of PHEV i.
$x_i^{\min}$	Minimum energy level of the battery of PHEV i.
β_i	Self-discharging percentage of the battery of PHEV i.
η_i	Round-trip efficiency in recharging and discharging of PHEV i.
$\xi_{i,n}$	Household energy demand of PHEV i in period n.
$\bar{\xi}_{i,n}$	Expectation of the household energy demand of PHEV i in period n.
$\zeta_{i,n}$	Commute energy demand of PHEV i in period n.

I. Introduction

The next-generation electricity grid, commonly referred to as smart grid, incorporates information and communication technology into various aspects of a power system to deliver electricity in a more efficient, reliable, economic, and secure way [1]. As nearly 90% of power outages and disturbances are related to the distribution network, distributed energy storage has become one of the key technologies for the smart grid. With an increasing market penetration rate of electric vehicles, vehicle-to-grid (V2G) systems are expected to be a critical auxiliary energy storage infrastructure in the smart grid [2] [3] [4]. A key feature of a V2G system is a bidirectional energy delivery mechanism which enables an electric vehicle to either draw energy from or feed energy back to the grid. Aided by communication technologies, the energy delivery can be controlled in a smart way to reduce the transport cost while improving the grid stability. Two kinds of services can be provided by V2G systems [2] [3]. The ancillary services in terms of frequency regulation are used to mitigate the frequency fluctuations in the power system caused by the supply-demand imbalances. Since the imbalances are temporary and small-scale in nature, the ancillary services may not necessarily involve energy delivery but simply use the capacity of vehicle batteries. On the other hand, the load shaving services, which are the main focus of this paper, use the energy stored in vehicle batteries to compensate for the peak load of the power grid. From the vehicle owners' point of view, since electricity price is determined by demand, the energy cost can be relatively reduced by drawing "cheap" energy from the grid, and vice versa. In order to regulate the aggregated recharging/discharging power over a large number electric vehicles, aggregators are typically used in V2G systems [4]. However, unlike traditional energy storage systems such as the uninterrupted power supply (UPS) units [5] [6], vehicles are (randomly) mobile in nature and their mobility characteristics are highly non-stationary during different times of a day (e.g., rush and non-rush hours), which poses significant challenges in analyzing and solving the optimal energy delivery problem.

In this work, we use the stochastic inventory theory to solve the optimal energy delivery problem and investigate the basic structure of the optimal energy delivery policy. The study is based on an analogy between the state-of-charge (SOC) of a plug-in hybrid electric vehicle (PHEV) battery and the stock level of an inventory. This work extends our previous research [7] to solve the optimal aggregator operation problem. The PHEV mobility is modeled as a non-stationary Markov chain. A state-dependent (S, S') policy is proved to be optimal for the daily energy cost minimization problem of each vehicle, and an estimation algorithm is developed for policy parameter calculation in practical applications. We further propose a policy adjustment scheme for multi-vehicle aggregator design based on the optimality of the state-dependent (S, S') policy, where the S and S' values of each PHEV are adjusted to coordinate the recharging/discharging process of the aggregator. In this way, the power system constraints in terms of the aggregated recharging/discharging power can be satisfied. Then, the original optimal aggregator operation problem for minimizing the incremental cost (or revenue loss) of PHEV owners to satisfy the power system constraints is transformed into a convex optimization problem which can be readily solved by existing algorithms. Simulations are performed to evaluate the efficiency of our proposed schemes based on real data collected from Canadian utilities, households, and commuters. To the best of our knowledge, this is the first work in literature to study the optimal energy delivery problem in a V2G system based on a stochastic model (specifically, a non-stationary Markov chain model) of PHEV mobility, and utilize stochastic inventory theory to derive the optimal policy. The research outcomes should shed some light not only on shaving the peak load of the smart grid from the utility's point of view but also on reducing the energy cost of PHEV owners, which in turn stimulates the consumer adoption of PHEVs.

The remainder of this paper is organized as follows. Section II provides an overview of the related work. Section III and Section IV describe the system model and problem formulation, respectively. The solution technique based on stochastic inventory theory is discussed in Section V. Numerical results are presented in Section VI to demonstrate the performance of the proposed solution. Section VII concludes the research work.

II. Related Work

In literature, there is a large body of research on V2G ancillary services, including both unidirectional V2G [8] [9] and bidirectional V2G [4] [10]. The unidirectional V2G involves only recharging control of electric vehicles and is considered to be a logical first step of V2G implementation, since no significant update is required for the standard vehicle battery chargers [9]. By further introducing discharging control, significantly higher benefits can be achieved by bidirectional V2G. Although load shaving services also involve recharging and discharging control, the optimization techniques developed for V2G ancillary services cannot be directly applied. The main reason is that the ancillary services may not necessarily involve energy delivery, so that the revenue is typically evaluated based on the usage of the vehicle batteries in terms of available capacity and service provisioning duration [4] [8]. For load shaving services, on the other hand, the cost and revenue in energy transactions incurred by buying and selling electricity, respectively, should be investigated [3].

A major challenge in optimizing the energy delivery for load shaving services is to deal with the randomness in vehicle mobility. A widely used approach in the state-of-art research is to assume that each electric vehicle is stationary during a certain period of time, and the target SOC of the vehicle at the end of the stationary period can be estimated [3] [11] [12] [13]. The accuracy of the estimation is crucial for optimal energy delivery in load shaving services. For instance, the less economic gasoline engine should be used by a PHEV if insufficient energy is reserved for the commute demand, while simply reserving more energy may result in a less revenue in energy transactions. However, the estimation is not a straightforward task, as it needs to take account of not only the randomness in vehicle mobility and the associated commute energy demand, but also the potential electricity price variations after the stationary period. For simplicity, constant approximation is typically used in the existing research based on average commute energy demand [3] [11] [12]. However, this kind of estimates can lead to suboptimal solutions for load shaving services. A practical example is given in [13] where a vehicle driver might be required to undertake an unexpected journey and the commute energy demand depends on the actual traffic condition. As a result, a significantly large amount of energy should be reserved to address the uncertainty [13]. Again, how to optimize the amount of energy reservation while taking into account the electricity price variation after the stationary period needs further research.

In order to fine-tune the load shaving services, dynamic programming can be used [5]. However, for a highly non-stationary vehicle mobility, dynamic programming over a relatively long time frame (e.g., one day) suffers from the curse of dimensionality. The complexity of searching for an optimal policy increases exponentially with the number of possible system states. To reduce the computational complexity, the structure of an optimal energy delivery policy should be investigated. Further, to optimize the operation of an aggregator which manages a large number of energy storage devices (i.e., the electric vehicles in this research), the computational complexity of dynamic programming becomes prohibitive [14]. One promising approach to this problem is to distribute the computational load among the energy storage devices [15]. However, since the calculation by each device is electricity price oriented, the aggregated recharging/discharging power of energy storage devices may violate the power system constraints. As a result, a central controller should be developed to make realtime adjustments to regulate the recharging/discharging power of each energy storage device [15]. Yet, how to exploit the optimal energy delivery policy calculated by each vehicle such that the aggregator can make efficient realtime adjustments for V2G load shaving services needs further research.

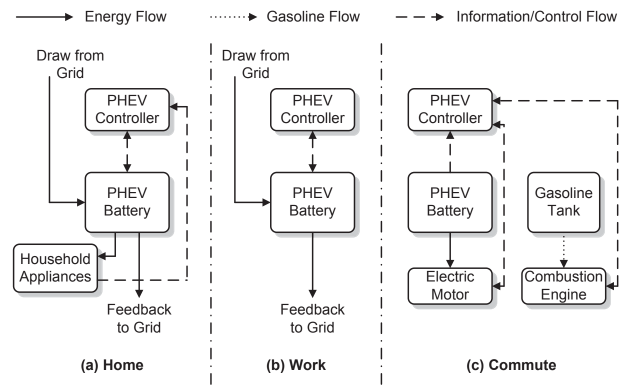

Fig. 1: An illustration of the individual PHEV model.

Different from the existing works, a stochastic model is used in this research to characterize the randomness in PHEV mobility. We use stochastic inventory theory to develop the optimal energy delivery policy for each PHEV. The properties of the optimal policy is further investigated such that the optimal aggregator operation problem can be solved efficiently based on existing algorithms.

III. System Model

Both individual PHEV and aggregator models are considered for a V2G system. An illustration of the individual PHEV model is given in Fig. 1. The PHEV controller manages battery recharging/discharging when the PHEV is connected to the power grid, and operates the electric motor and combustion engine when the PHEV is commuting. The battery status and commute energy demand can be directly monitored by the PHEV controller. Based on the communication functionality of the V2G system, the information of electricity price, PHEV mobility status, and household energy demand is acquired by the PHEV controller via wireline/wireless links. On the other hand, when a large number of vehicles in a small area (e.g., a parking lot) are involved in V2G services, an aggregator is used to regulate the vehicle recharging/discharging behaviors. An illustration of the model for an aggregator is shown

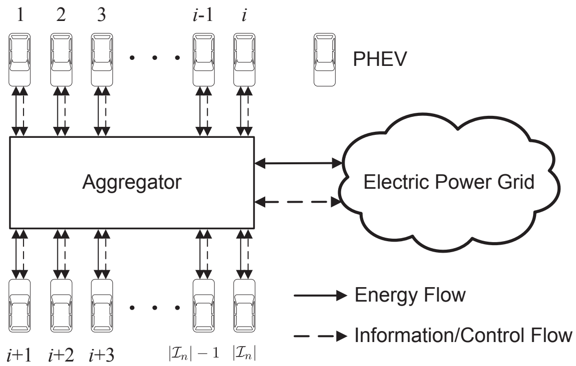

Fig. 2: An illustration of the aggregator model.

in Fig. 2. In this work, we focus on a single aggregator in the smart grid with certain recharging/discharging power constraints, while the coordination among multiple aggregators is left for future work.

Consider a time slotted system. Time is partitioned into periods with equal duration T (e.g., 10 minutes). The PHEV mobility is represented by the dynamics of PHEV locations and is modeled by a Markov chain. The state transition probabilities of the Markov chain can be obtained based on a typical vehicular communication network [16]. Time-of-use (TOU) electricity pricing is considered for the V2G system with a net metering arrangement for the pricing of energy feedback to the grid [3] [17]. Within each period, the location region of the PHEV stays unchange and the price of electricity remains constant. Moreover, a realistic battery model is considered for each PHEV, which includes energy and value loss in recharging/discharging, limited battery capacity, limited recharging/discharging rate, and self-discharge effect [3] [5] [6] [18]. More details of the system model are given in the following subsections.

A. PHEV Mobility and Energy Demand

Without loss of generality, we consider three states (regions) of the PHEV location, i.e., home ($\mathcal{H}$), work ($\mathcal{W}$), and commute ($\mathcal{C}$). An extension is straightforward to include more locations by increasing the state space. Consider a specific PHEV i. Denote the state of PHEV i in period n as $s_{i,n}$. When $s_{i,n} = \mathcal{H}$ or $\mathcal{W}$, energy can be either drawn from or fed back to the grid based on the bi-directional plugs of a V2G system, as shown in Fig. 1. The household appliances can use the energy in the battery only when $s_{i,n} = \mathcal{H}$. Consider one PHEV for each household [3] and let random variable $\xi_{i,n}$ denote the household energy demand in period n based on the smart meter readings. When $s_{i,n} = \mathcal{C}$, the energy in the battery is used to drive the electric motor. Let random variable $\zeta_{i,n}$ denote the commute energy demand in period n given $s_{i,n} = \mathcal{C}$. If a battery is depleted when $s_{i,n} = \mathcal{C}$, gasoline is used to drive the PHEV combustion engine with an additional cost. The variations of PHEV location are modeled as a Markov chain [16]. Taking account of the non-stationary PHEV mobility, the state transition probabilities of the Markov chain are time-dependent. Given period n ($n = 0, 1, 2, \cdots$) and the current state $s_{i,n}$ of PHEV i, the transition probability to state $s_{i,n+1}$ in period $n+1$ is denoted by $P_{i,n}(s_{i,n+1}|s_{i,n})$ ($s_{i,n}, s_{i,n+1} \in \{\mathcal{H}, \mathcal{W}, \mathcal{C}\}$), where period 0 corresponds to the first period of a day.

Note that, if the vehicle mobility is highly predictable, the Markov chain model can be simplified. Specifically, the state transition probability associated with two predictable vehicle locations in two consecutive periods can be set to one. Consider an example where the owner of PHEV i is scheduled to participate in a multi-hour meeting at work. Then, for any two consecutive periods (e.g., periods k and $k+1$) during the meeting, we can set $s_{i,k} = \mathcal{W}$, $s_{i,k+1} = \mathcal{W}$, and $P_{i,k}(s_{i,k+1}|s_{i,k}) = 1$. In this way, the number of potential mobility trajectories of PHEV i is reduced, so is the complexity associated with the dynamic programming in Section V. In addition, if the commute energy demand is predictable based on traffic condition forecast, a constant can be used to replace the random variable $\zeta_{i,n}$, which further reduces the complexity in optimizing the energy delivery via a V2G system.

B. Electricity Pricing

TOU electricity pricing is considered for the V2G system [3]. We do not consider the spatial difference in electricity price for a single aggregator, under an assumption that most daily commute is over a relatively short distance. For instance, 75% of Americans commute 65 km or less round-trip [19]. However, an extension is straightforward by considering the specific locations of home and work. In period n, the cost of drawing z units of energy from the grid is given by $r_n z$, where r_n is the TOU price. The net metering arrangement is considered for the pricing of energy feedback to the grid. Rather than paying cash, credit is given to the PHEV owner as an amount of excess energy is fed into the grid [17]. As a result, the same TOU price can be achieved for buying and selling energy[1], while selling energy can be considered the same as reducing the energy drawn from the grid by the neighborhood (when $s_{i,n} = \mathcal{H}$) or workplace appliances (when $s_{i,n} = \mathcal{W}$) [3]. If battery is depleted when $s_{i,n} = \mathcal{C}$, gasoline is used to drive the PHEV combustion engine[2]. For PHEV i, the average cost of using gasoline to satisfy z units of energy demand in period n is $r'_{i,n} z$. Since the gasoline price fluctuates at a much slower rate than the electricity price, constant approximation is typically used, i.e., $r'_{i,n} = r'_i$ [19].

C. Battery Model

A realistic battery model is considered in this work. For each recharge and discharge of the battery, a certain amount of energy is lost due to the battery conversion loss. Therefore, we use a virtual capacity of the battery such that all stored energy can be used. Specifically, when z units of energy is used to recharge the battery of PHEV i, the energy that can be fed back to the grid or used by the household appliances is $\eta_i z$ ($0 < \eta_i < 1$), where η_i is the round-trip efficiency

[1]Specifically, in September 2009, the Delaware governor has signed a bill for V2G system which requires electric utilities to compensate for the energy feedback to the grid at the same price as it is drawn from the grid [20].

[2]For instance, either a series hybrid or a series-parallel hybrid operation mode can be used by Chevrolet Volt based on the vehicle speed [19].

which merges the energy loss in both recharging and discharging [6]. Moreover, the lifetime of a battery is shortened for each recharging/discharging cycle since the capacity of the battery slowly deteriorates following the depth-of-discharge (DoD). As the deterioration is almost imperceptible on a daily basis [18], the capacity of the battery (denoted by $x_i^{\max}$) is approximately unchanged for the time frame that we consider. However, the loss of the battery value is modeled as a cost which is proportional to the recharged (or discharged) energy with a factor $\tilde{r}_i$ [3]. As a result, the cost to increase the energy level of the battery by z $(z > 0)$ in period n $(n = 0, 1, 2, \cdots)$ is given by

$$c_{i,n}^{rc}(z) = r_n \left(\frac{z}{\eta_i}\right) + \tilde{r}_i z = \left(\frac{r_n}{\eta_i} + \tilde{r}_i\right) z. \tag{1}$$

In order to prolong the battery lifetime, the energy level of the battery should not drop below $x_i^{\min}$ according to certain SOC [4]. With the virtual capacity, we consider $x_i^{\min} = 0$ for simplicity. We assume $r_i' \geq \left(\frac{r_n}{\eta_i} + \tilde{r}_i\right)$ for the PHEVs. That is, the cost of using combustion engine is higher than using the electric motor, taking account of all cost in the recharging/discharging process, which is one of the key features of PHEVs [19]. Within each period, the battery can be either recharged or discharged, but not both [5]. Because of a limited recharging/discharging rate, the maximum amount of energy that can be recharged into and discharged from the battery of PHEV i in a period is given by $u_{i,rc}^{\max}$ and $u_{i,dc}^{\max}$, respectively. Because of the self-discharge effect, the energy stored in the battery decreases by a percentage, β_i $(0 < \beta_i < 1)$, for each period. Specifically, suppose the remaining energy in the battery of PHEV i at the end of a period is x, the maximum amount of energy that can be used in the next period is $\beta_i x$.

D. Aggregator Model

Consider a specific aggregator in the V2G system. In period n, there is a set, $\mathcal{I}_n$, of PHEVs connected to the aggregator, as shown in Fig. 2. The aggregator provides an interface to the main electric power grid (potentially via power electronic converters and/or transformers) such that energy can be exchanged between the grid and PHEVs. Different from the frequency regulation services, the load shaving services considered in this work involve a significant amount of energy delivery [2] [3]. Therefore, the aggregated power output (either aggregated recharging power or aggregated discharging power) of the aggregator is subject to a set of constraints. Firstly, the aggregated recharging/discharging power is limited by the maximum current that can be transmitted through the aggregator [12]. Secondly, the aggregated recharging power is subject to an additional load limit which is mandated by the utility to prevent excessive loading of the system [8] [12]. In addition, if the aggregator is connected in a distribution system, the aggregated recharging/discharging power is further capped since excessive loading may cause undervoltage in the distribution system, and vice versa [21]. Taking into account all the constraints, we use $u_{agg,rc,n}^{\max}$ and $u_{agg,dc,n}^{\max}$ to denote the limits of the aggregated recharging and discharging energy in period n, respectively, for the set $\mathcal{I}_n$ of connected PHEVs. Note that if multiple constraints coexist for the aggregated recharging (or discharging) energy in a period, the most stringent constraint is used to define $u_{agg,rc,n}^{\max}$ (or $u_{agg,dc,n}^{\max}$). Detailed calculations of $u_{agg,rc,n}^{\max}$ and $u_{agg,dc,n}^{\max}$ are out the scope of this research. In the following, we assume that the aggregator obtains the values of $u_{agg,rc,n}^{\max}$ and $u_{agg,dc,n}^{\max}$ via wireline/wireless communications with the utility at the beginning of period n, and coordinates the recharging/discharging processes of the connected PHEVs in period n accordingly.

IV. Problem Formulation

The problem formulation consists of two parts. Each individual PHEV owner strives to minimize his/her own daily energy cost based on the fluctuations of electricity price. On the other hand, because of the power system constraints $u_{agg,rc,n}^{\max}$ and $u_{agg,dc,n}^{\max}$, the optimal energy delivery policy adopted by each PHEV may not be feasible from the aggregator's point of view and need to be adjusted. As an incremental cost is inevitable for modifying the optimal policies adopted by the PHEVs, the objective of optimal aggregator operation is to minimize the incremental cost of the connected PHEVs for a certain period under consideration. To investigate the daily energy cost minimization problem, we partition each day into $N+1$ periods, i.e., $n \in \{0, 1, \cdots, N\}$. The duration of period $n \in \{0, 1, \cdots, N-1\}$ is T. Since the PHEV mobility is negligible during the midnight off-peak period, we define period N as an aggregate period to reduce the computational complexity. For instance, if $\{0, 1, \cdots, N-1\}$ corresponds to the periods between 6:00am and 10:00pm, then period N is the remaining time between 10:00pm and next 6:00am. For simplicity, we neglect the self-discharge effect in period N for the off-peak hours since the battery can be recharged at a low cost. The analytical method of stochastic inventory theory [22] [23] [24] is used. For the V2G system under consideration, we extend the theory by incorporating the bi-directional energy flow, PHEV mobility pattern, and realistic battery model.

A. Energy Cost Minimization of Each Vehicle

Denote the energy level of the battery of PHEV i at the beginning of period n $(n \in \{0, 1, \cdots, N\})$ as $x_{i,n}$, where $x_{i,0}$ is the initial energy level. The decision variable is given by

$$U_i = (u_{i,0}, u_{i,1}, \cdots, u_{i,N-1}) \tag{2}$$

where $u_{i,n}$ denotes the energy level of the battery of PHEV i at the end of period n. The decision of $u_{i,n}$ is made at the beginning of period n given that PHEV i is connected to the grid, i.e., $s_{i,n} = \mathcal{H}$ or $\mathcal{W}$. Since recharge and discharge cannot be performed simultaneously, we have $u_{i,n} > x_{i,n}$ and $u_{i,n} < x_{i,n}$ if the battery is recharged and discharged, respectively, while $u_{i,n} = x_{i,n}$ if the battery is not used (or idle) in period n. Taking account of the self-discharge effect, the battery energy that can be used in period $n+1$ is $\beta_i u_{i,n}$. For a limited battery capacity and recharging/discharging rate, we have

$$u_{i,n} \in \left[\max\{0, x_{i,n} - u_{i,dc}^{\max}\}, \min\{x_i^{\max}, x_{i,n} + u_{i,rc}^{\max}\}\right]. \tag{3}$$

Denote the energy cost with respect to PHEV i in period k ($k \in \{0, 1, \cdots, N-1\}$) as $C_{i,k}^{s_{i,k}}(x_{i,k}, u_{i,k})$ which depends on the system states ($s_{i,k}$ and $x_{i,k}$) and the decision variable $u_{i,k}$. When $s_{i,k} = \mathcal{H}$, we have

$$\begin{aligned} C_k^{\mathcal{H}}(x_{i,k}, u_{i,k}) &= \left(r_k \xi_{i,k} + c_{i,k}^{rc}(u_{i,k} - x_{i,k})\right) I_{u_{i,k} > x_{i,k}} \\ &\quad + r_k \xi_{i,k} I_{u_{i,k} = x_{i,k}} \\ &\quad - r_k(x_{i,k} - u_{i,k} - \xi_{i,k}) I_{u_{i,k} < x_{i,k}} \\ &= r_k \xi_{i,k} + H_{i,k}(x_{i,k}, u_{i,k}) \end{aligned} \tag{4}$$

where I_A is an indication function which equals 1 if A is true and 0 otherwise, while $H_{i,k}(x, u)$ is defined as

$$H_{i,k}(x, u) = \left(\frac{r_k}{\eta_i} + \tilde{r}_i\right)(u - x) I_{u > x} + r_k(u - x) I_{u \le x}. \tag{5}$$

Note that for a recharging or idle period with $u_{i,k} \ge x_{i,k}$, all household demand resorts to drawing energy from the grid at a cost $r_k \xi_{i,k}$ since the battery cannot be discharged at the same time. For a discharging period with $u_{i,k} < x_{i,k}$, if $x_{i,k} - u_{i,k} < \xi_{i,k}$, the unsatisfied demand resorts to drawing energy from the grid, while if $x_{i,k} - u_{i,k} > \xi_{i,k}$, the energy unused by the demand is fed back to the grid with a negative cost representing the revenue. Since the same price is used for buying and selling energy, the benefit of selling energy back to the grid is the same as compensating for the household demand. Therefore, the cost of household demand, $r_k \xi_{i,k}$, can be considered as irrelevant to the decision $u_{i,k}$, as shown in (4). When $s_{i,k} = \mathcal{W}$, we have

$$\begin{aligned} C_{i,k}^{\mathcal{W}}(x_{i,k}, u_{i,k}) &= c_{i,k}^{rc}(u_{i,k} - x_{i,k}) I_{u_{i,k} > x_{i,k}} \\ &\quad - r_k(x_{i,k} - u_{i,k}) I_{u_{i,k} \le x_{i,k}} \\ &= H_{i,k}(x_{i,k}, u_{i,k}) \end{aligned} \tag{6}$$

where the only difference from (4) is that the household demand is not considered since the PHEV is away from the home. When $s_{i,k} = \mathcal{C}$, we have

$$C_{i,k}^{\mathcal{C}}(x_{i,k}, u_{i,k}) = r_i'(\zeta_{i,k} - x_{i,k})^+ \tag{7}$$

where $(x)^+$ equals x if $x > 0$ and 0 otherwise, $(\zeta_{i,k} - x_{i,k})^+$ represents the energy deficit in commute which needs to be compensated for by using gasoline. In (7), there is no decision on buying or selling of energy since the PHEV is not connected to the grid.

For period N, we consider that the PHEV battery should be fully recharged during the off-peak hours overnight [3], based on the assumption that all the energy can be used or sold during the daytime. Otherwise, a battery with a smaller capacity and lower cost should be equipped by the PHEV. Therefore, the end-of-day cost function ($C_{i,N}(x)$) is proportional to the energy to be recharged according to the off-peak price r_N, and is given by

$$C_{i,N}(x) = \left(\frac{r_N}{\eta_i} + \tilde{r}_i\right)(x_i^{\max} - x). \tag{8}$$

The daily energy cost minimization problem is defined as

$$(\mathbf{P1}) \quad \min_{U_i \in \mathscr{U}_i} C_{i,0,s_{i,0},x_{i,0}}(U_i) \tag{9}$$

where $\mathscr{U}_i$ represents the class of all admissible decisions for PHEV i satisfying (3). Denote $U_{i,n} = (u_{i,n}, u_{i,n+1}, \cdots, u_{i,N-1})$ as the decisions made by PHEV i from period n to period $N-1$ with the corresponding class of all admissible decisions given by $\mathscr{U}_{i,n}$. Denote $C_{i,n,s_{i,n},x_{i,n}}(U_{i,n})$ as the expected energy cost during periods $\{n, n+1, \cdots, N\}$ which is given by

$$C_{i,n,s_{i,n},x_{i,n}}(U_{i,n}) = E\left[\sum_{k=n}^{N-1} C_{i,k}^{s_{i,k}}(x_{i,k}, u_{i,k})\right] + C_{i,N}(x_{i,N}) \tag{10}$$

where the expectation is taken with respect to the household energy demand $\xi_{i,k}$ and commute energy demand $\zeta_{i,k}$ in (4) and (7), respectively. Given $U_{i,0} = U_i$ for the first period under consideration, the value of $C_{i,0,s_{i,0},x_{i,0}}(U_i)$ in (9) can be calculated by letting $n = 0$ in (10). Taking account of the self-discharge effect, the energy level of period k ($k \in \{0, 1, \cdots, N-1\}$) evolves as

$$x_{i,k+1} = \begin{cases} \beta_i u_{i,k}, & \text{if } s_{i,k} = \mathcal{H} \text{ or } \mathcal{W} \\ \beta_i (x_{i,k} - \zeta_{i,k})^+, & \text{otherwise} \end{cases} \tag{11}$$

where $(x_{i,k} - \zeta_{i,k})^+$ denotes the remaining energy in the battery after the commute in period k.

B. Optimal Aggregator Operation

Suppose the optimal decisions of PHEV i with respect to problem P1 is given by $U_i^* = (u_{i,0}^*, u_{i,1}^*, \cdots, u_{i,N-1}^*)$. Accordingly, denote $U_{i,n}^* = (u_{i,n}^*, u_{i,n+1}^*, \cdots, u_{i,N-1}^*)$ as the optimal decisions of PHEV i from period n to period $N-1$. Then, the output of the aggregator in period n can be calculated as $\sum_{i \in \mathcal{I}_n} [x_{i,n} - u_{i,n}^*]$, which is positive if the aggregator is discharging, and vice versa. Since the aggregated output may violate the power system constraints, the decisions from period n should be adjusted such that the power system constraints are satisfied and the increment cost of PHEVs is minimized. Therefore, the optimal aggregator design problem in period n is formulated as

$$(\mathbf{P2}) \quad \min_{\tilde{U}_{i,n} \in \tilde{\mathscr{U}}_{i,n}} \sum_{i \in \mathcal{I}_n} [C_{i,n,s_{i,n},x_{i,n}}(\tilde{U}_{i,n}) - C_{i,n,s_{i,n},x_{i,n}}(U_{i,n}^*)] \tag{12}$$

$$\text{subject to} \quad \sum_{i \in \mathcal{I}_n} [x_{i,n} - \tilde{u}_{i,n}] \ge -u_{agg,rc,n}^{\max} \tag{13}$$

$$\sum_{i \in \mathcal{I}_n} [x_{i,n} - \tilde{u}_{i,n}] \le u_{agg,dc,n}^{\max} \tag{14}$$

where $\tilde{U}_{i,n} = (\tilde{u}_{i,n}, \tilde{u}_{i,n+1}, \cdots, \tilde{u}_{i,N-1})$ is the adjusted decisions from period n to period $N-1$, while $\tilde{\mathscr{U}}_{i,n}$ is the corresponding class of all admissible decisions of PHEV i satisfying (3). In (12), the objective is to minimize the increment costs of all PHEVs connected to the aggregator in period n, in comparison with the optimal decisions ($U_{i,n}^*$). Constraints (13) and (14) correspond to the power system constraints with respect to the recharging and discharging energy of the aggregator in period n, respectively.

V. Energy Delivery Optimization

In this section, we first transform the original problem formulation P1 into a dynamic programming formulation and show the existence of an optimal Markov policy. In order to reduce the computational complexity, we further investigate the problem and prove the optimality of a state-dependent (S, S') policy for the energy cost minimization of each PHEV. Due to space limitation, we only present the key results with respect to the existence and optimality in Subsection V-A and Subsection V-B, respectively. Details of the proofs are given in [7]. Based on the properties of the (S, S') policy, we further investigate the optimal aggregator operation problem P2. A policy adjustment scheme is proposed for the aggregator to adjust the values of S and S' for each PHEV. Then, we prove that the original problem P2 can be transformed into a convex optimization problem based on our proposed policy adjustment scheme and can be readily solved efficiently by existing algorithms.

A. Existence of Optimal Markov Policy

For notation simplicity, letting $s = s_{i,n}$ and $x = x_{i,n}$, we define the value function of PHEV i within periods $\{n, n+1, \cdots, N\}$ as $V_{i,n}(s,x) = \min_{U_{i,n}\in\mathscr{U}_{i,n}} C_{i,n,s,x}(U_{i,n})$. For $n \in \{0, 1, \cdots, N-1\}$, the dynamic programming equation of the value function is given by

$$V_{i,n}(s,x) = \begin{cases} r_n\bar{\xi}_{i,n} + \min_u\{H_{i,n}(x,u) \\ \quad +E[V_{i,n+1}(s_{i,n+1}, \beta_i u)|s_n = \mathcal{H}]\}, \text{ if } s = \mathcal{H} \\ \min_u\{H_{i,n}(x,u) \\ \quad +E[V_{i,n+1}(s_{i,n+1}, \beta_i u)|s_n = \mathcal{W}]\}, \text{ if } s = \mathcal{W} \\ E[V_{i,n+1}(s_{i,n+1}, \beta_i(x-\zeta_{i,n})^+)|s_n = \mathcal{C}] \\ \quad +r'_i E[(\zeta_{i,n} - x)^+], \text{ if } s = \mathcal{C} \end{cases} \tag{15}$$

where the values of u are taken from a set as defined in (3), while $\bar{\xi}_{i,k}$ represents the expectation of $\xi_{i,k}$. For $n = N$, we have

$$V_{i,N}(s,x) = C_{i,N}(x). \tag{16}$$

A policy is a Markov policy if it depends only on the current state $(n, s_{i,n}, x_{i,n})$ for decision making and not on the past states $(k, s_{i,k}, x_{i,k})$ $(k \in \{0, 1, \cdots, n-1\})$. The following theorem shows the existence of an optimal Markov policy for problem P1.

Theorem 1. *There exists a function $u^*_{i,n}(s,x) : \mathbb{I}\times\mathbb{I}\times\mathbb{R} \to \mathbb{R}$, which provides the minimum of $V_{i,n}(s,x)$ in (15) for any x and $s = \mathcal{H}$ or $\mathcal{W}$. Moreover, the decision $U^*_i = (u^*_{i,0}, u^*_{i,1}, \cdots, u^*_{i,N-1})$ is optimal for the problem P1, where $u^*_{i,n} = u^*_{i,n}(s_{i,n}, x^*_{i,n})$ and x^*_{n+1} evolves according to (11) with respect to $u^*_{i,n}$ and $x^*_{i,n}$.*

B. Optimality of the State-Dependent (S, S') Policy

Based on Theorem 1, the optimal Markov policy exists. However, the computational complexity for the optimal policy is prohibitive even for a small N since function $u^*_{i,n}(s,x)$ should be optimized for each combination of s and x [25]. Therefore, we further investigate the properties of the value functions, and show that a state-dependent (S, S') policy is optimal. An (S, S') policy is defined as follows:

Definition 1. *Consider PHEV i. Given constants S_i and S'_i, $S_i \le S'_i$, and the current energy level x, an (S, S') policy is defined as*

$$u^*_i(x) = \begin{cases} x + u^{\max}_{i,rc}, & \text{if } x < S_i - u^{\max}_{i,rc} \\ S_i, & \text{if } S_i - u^{\max}_{i,rc} \le x < S_i \\ x, & \text{if } S_i \le x \le S'_i \\ S'_i, & \text{if } S'_i < x < S'_i + u^{\max}_{i,dc} \\ x - u^{\max}_{i,dc}, & \text{otherwise.} \end{cases} \tag{17}$$

Note that the (S, S') policy is essentially a double-threshold policy by incorporating the limited recharging/discharging rates of each PHEV. When the energy level is below S (above S'), the battery is recharged (discharged) as much as possible up to S (down to S'). When the energy level is between S and S', the battery is kept in an idle state. Moreover, the policy is user specific which is reflected by the PHEV index i.

For presentation clarity, we define a convex function $q_i(u)$, $u \in [0, x^{\max}_i]$, with minimum value $q^*_i = \min_{u\in[0,x^{\max}_i]}\{q_i(u)\}$. Since $\arg\min_{u\in[0,x^{\max}_i]}\{q_i(u)\}$ is a convex set [26], we can define its boundary points as

$$S_i = \min\{u \in [0, x^{\max}_i] | q_i(u) = q^*_i\} \tag{18}$$
$$S'_i = \max\{u \in [0, x^{\max}_i] | q_i(u) = q^*_i\}. \tag{19}$$

In the following, we first denote the state-dependent S and S' as functions of x by $S_{i,n}(s,x)$ and $S'_{i,n}(s,x)$, respectively, with respect to state s in period n. Then, we show that $S_{i,n}(s,x)$ and $S'_{i,n}(s,x)$ are indeed independent of x based on the properties of the value function. The main result for the optimality of a state-dependant (S, S') policy is given by the following Theorem.

Theorem 2. *Given state $s \in \{\mathcal{H}, \mathcal{W}\}$ in period n, a state-dependant (S, S') policy is optimal. The optimal policy $u^*_{i,n}(s,x)$ is given by (17) with $S_{i,n}(s,x)$ and $S'_{i,n}(s,x)$ given by (18) and (19), respectively, based on the following convex function with respect to u:*

$$q_{i,n}(s,x,u) = H_{i,n}(x,u) + E\left[V_{i,n+1}(s_{i,n+1}, \beta_i u)|s_{i,n} = s\right]. \tag{20}$$

Based on (20), $q_{i,n}(s,x,u)$ can be rewritten as

$$q_{i,n}(s,x,u) = \left[H^r_{i,n}(s,u) - \left(\frac{r_n}{\eta_i} + \tilde{r}_i\right)x\right] I_{u>x} + \left[H^d_{i,n}(s,u) - r_n x\right] I_{u\le x} \tag{21}$$

where

$$H^r_{i,n}(s,u) = \left(\frac{r_n}{\eta_i} + \tilde{r}_i\right)u + E\left[V_{i,n+1}(s_{i,n+1}, \beta_i u)|s_{i,n} = s\right] \tag{22}$$

$$H^d_{i,n}(s,u) = r_n u + E\left[V_{i,n+1}(s_{i,n+1}, \beta_i u)|s_{i,n} = s\right]. \tag{23}$$

Obviously, $H^r_{i,n}(s,u)$ and $H^d_{i,n}(s,u)$ are convex with respect to u because of the convexity of the value function ($V_{i,n+1}(s,x)$) and the fact that the convexity is preserved by the linear combination in terms of conditional expectation [26]. Denote the minimum of $H^r_{i,n}(s,u)$ with respect to u as

$$H^{r*}_{i,n}(s) = \min_{u\in[0,x_i^{\max}]} \left\{H^r_{i,n}(s,u)\right\}. \quad (24)$$

Since $\arg\min_{u\in[0,x_i^{\max}]}\{H^r_{i,n}(s,u)\}$ is a convex set, we define the minimum and maximum values of u to achieve $H^{r*}_{i,n}(s)$ as

$$u^{r1}_{i,n}(s) = \min\{u \in [0, x_i^{\max}] | H^r_{i,n}(s,u) = H^{r*}_{i,n}(s)\} \quad (25)$$
$$u^{r2}_{i,n}(s) = \max\{u \in [0, x_i^{\max}] | H^r_{i,n}(s,u) = H^{r*}_{i,n}(s)\}. \quad (26)$$

Similarly, we can define $u^{d1}_{i,n}(s)$ and $u^{d2}_{i,n}(s)$ for $H^d_{i,n}(s,u)$.

Letting the two state-dependent thresholds be

$$S_{i,n}(s) = u^{r1}_{i,n}(s) \quad (27)$$
$$S'_{i,n}(s) = u^{d2}_{i,n}(s) \quad (28)$$

we can transform the optimal policy $u^*_{i,n}(s,x)$ in Theorem 2 to that in the following

$$u^*_{i,n}(s,x) = \begin{cases} x + u^{\max}_{i,rc}, & \text{if } x < S_{i,n}(s) - u^{\max}_{i,rc} \\ S_{i,n}(s), & \text{if } S_{i,n}(s) - u^{\max}_{i,rc} \le x < S_{i,n}(s) \\ x, & \text{if } S_{i,n}(s) \le x \le S'_{i,n}(s) \\ S'_{i,n}(s), & \text{if } S'_{i,n}(s) < x < S'_{i,n}(s) + u^{\max}_{i,dc} \\ x - u^{\max}_{i,dc}, & \text{otherwise.} \end{cases} \quad (29)$$

As a result, the optimal policy can be simply denoted as a state-dependent (S,S') policy.

Based on the optimality of state-dependent (S,S') policy, we need to calculate only two parameters to obtain the optimal policy with respect to each state and each period (instead of a function $u^*_{i,n}(s,x)$ with respect to x), which significantly reduces the computational complexity for practical applications. The calculation of the optimal policy requires the daily statistics of the PHEV mobility ($P_{i,n}(j|s)$), average household energy demand ($\bar{\xi}_{i,n}$), and probability density function (PDF) of commute energy demand ($f_{\zeta_{i,n}}(y), y \ge 0$), for $n \in \{0,1,\cdots,N-1\}$, and $s,j \in \{\mathcal{H},\mathcal{W},\mathcal{C}\}$. With the statistics unknown a priori, we can use the historic information to estimate the statistics. The estimation is based on the fact that both commute pattern and electricity demand are periodic in nature on a daily basis. In [7], we propose an estimation algorithm to calculate the values of the state-dependent thresholds ($S_{i,n}(s)$ and $S'_{i,n}(s)$), which is based on a modified backward iteration algorithm for threshold calculation and an exponentially weighted moving average (EWMA) algorithm [27] for statistical estimation.

C. Optimal Policy Adjustment by Aggregator

In order to adjust the recharging/discharging energy of each PHEV in a period such that the power system constraints can be satisfied, the policy adopted by each PHEV should be adjusted by the aggregator. In the following, we first introduce a policy adjustment scheme with one adjustable parameter for each PHEV and prove that all admissible decisions can be achieved based on the policy adjustment. Then, we transform problem P2 into a convex optimization problem based on our proposed policy adjustment scheme.

According to Theorem 1, the state-dependant (S,S') policy belongs to the class of Markov policies. Therefore, the policy adjustment by the aggregator in period n with respect to PHEV i does not affect the optimality of the policy in periods $\{n+1,n+2,\cdots,N-1\}$. Therefore, we can rewrite $\tilde{U}_{i,n}$ in Problem P2 as

$$\tilde{U}_{i,n} = (\tilde{u}_{i,n}, u^*_{i,n+1},\cdots,u^*_{i,N-1}) \quad (30)$$

where the value of each decision depends on the energy level of the battery of PHEV i and the state-dependent (S,S') policy. The policy adjustment is performed by adjusting the two thresholds. Consider an adjusted policy $\tilde{u}_{i,n}(s,x)$ for PHEV i in period n, which is given by (29) with the two state-dependent thresholds ($S_{i,n}(s)$ and $S'_{i,n}(s)$) being replaced with $\tilde{S}_{i,n}(s)$ and $\tilde{S}'_{i,n}(s)$, respectively, given by

$$\tilde{S}_{i,n}(s) = \left(\min\{u^{r1}_{i,n}(s) + \Delta S_{i,n}(s), x_i^{\max}\}\right)^+ \quad (31)$$
$$\tilde{S}'_{i,n}(s) = \left(\min\{u^{d2}_{i,n}(s) + \Delta S_{i,n}(s), x_i^{\max}\}\right)^+. \quad (32)$$

Note that the policy adjustment in (31) and (32) is essentially based on the adjustment parameter $\Delta S_{i,n}(s)$ and then bound the two new thresholds by the battery capacity of the PHEV (in $[0,x_i^{\max}]$). In order to achieve all possible values of the energy level of PHEV i according to (3), $\Delta S_{i,n}(s)$ should take a value from the following set:

$$\Delta S_{i,n}(s) \in [x_{i,n} - u^{\max}_{i,dc} - u^{d2}_{i,n}(s), x_{i,n} + u^{\max}_{i,rc} - u^{r1}_{i,n}(s)]. \quad (33)$$

Note that the policy adjustment scheme may not be unique. However, our proposed policy adjustment scheme relies on only one parameter and can be easily implemented for the transformation of the optimal aggregator operation problem. For the properties of the policy adjustment scheme, we first have the following Lemma.

Lemma 1. *Given an energy level $x_{i,n}$ ($x_{i,n} \in [0,x_i^{\max}]$), the decision made by PHEV i in period n based on the state-dependant (S,S') policy with adjusted thresholds (31) and (32) can achieve any point in the set defined in (3) by adjusting parameter $\Delta S_{i,n}(s)$.*

Proof: Three cases should be considered for the admissible decision ($u_{i,n}$). Case 1: $u_{i,n} = x_{i,n}$; Case 2: $u_{i,n} \in [\max\{0, x_{i,n} - u^{\max}_{i,dc}\}, x_{i,n})$; Case 3: $u_{i,n} \in (x_{i,n}, \min\{x_i^{\max}, x_{i,n} + u^{\max}_{i,rc}\}]$. For Case 1, let $\Delta S_{i,n}(s) = x_{i,n} - u^{r1}_{i,n}(s)$, we have

$$\tilde{S}_{i,n}(s) = \left(\min\{u^{r1}_{i,n}(s) + \Delta S_{i,n}(s), x_i^{\max}\}\right)^+ = x_{i,n}. \quad (34)$$

Based on (29), we have $\tilde{u}_{i,n}(s_{i,n},x_{i,n}) = x_{i,n}$. For Case 2, let $\Delta S_{i,n}(s) = u_{i,n} - u^{d2}_{i,n}(s)$, we have

$$\tilde{S}'_{i,n}(s) = \left(\min\{u^{d2}_{i,n}(s) + \Delta S_{i,n}(s), x_i^{\max}\}\right)^+ = u_{i,n}. \quad (35)$$

Since $u_{i,n} < x_{i,n}$ in Case 2, we have $x_{i,n} > \tilde{S}'_{i,n}(s)$. Moreover, for $u_{i,n} \ge \max\{0, x_{i,n} - u^{\max}_{i,dc}\}$ in Case 2, we have $x_{i,n} \leq \tilde{S}'_{i,n}(s) + u^{\max}_{i,dc}$. Taking account of (29), we have

$$\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) = \tilde{S}'_{i,n}(s) = u_{i,n}. \tag{36}$$

For Case 3, let $\Delta S_{i,n}(s) = u_{i,n} - u^{r1}_{i,n}(s)$, we have

$$\tilde{S}_{i,n}(s) = \left(\min\{u^{r1}_{i,n}(s) + \Delta S_{i,n}(s), x^{\max}_i\}\right)^+ = u_{i,n}. \tag{37}$$

Since $u_{i,n} > x_{i,n}$ in Case 3, we have $x_{i,n} < \tilde{S}_{i,n}(s)$. Moreover, for $u_{i,n} \leq \min\{x^{\max}_i, x_{i,n} + u^{\max}_{i,rc}\}$ in Case 3, we have $x_{i,n} \geq \tilde{S}_{i,n}(s) - u^{\max}_{i,rc}$. Taking account of (29), we have $\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) = \tilde{S}_{i,n}(s) = u_{i,n}$. ■

According to Lemma 1, all admissible decisions can be achieved based on our proposed policy adjustment scheme for any energy level of the PHEV battery. On the other hand, it is straightforward to show that the decision made by PHEV i based on the adjusted state-dependent (S, S') policy always lies in the set defined by (3), provided that $\Delta S_{i,n}(s)$ takes value from (33). In other words, the optimal aggregator operation can be achieved by adjusting $\Delta S_{i,n}(s)$ with respect to each PHEV in $\mathcal{I}_n$. Therefore, problem P2 can be transformed as follows:

$$(\textbf{P3}) \min_{\substack{\Delta S_{i,n}(s) \\ \in[0,x^{\max}_i]}} \sum_{i\in\mathcal{I}_n} [C_{i,n,s_{i,n},x_{i,n}}(\tilde{U}_{i,n}) - C_{i,n,s_{i,n},x_{i,n}}(U^*_{i,n})] \tag{38}$$

$$\text{subject to} \quad \sum_{i\in\mathcal{I}_n} [\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) - x_{i,n}] \leq u^{\max}_{agg,rc,n}, \quad \text{if} \sum_{i\in\mathcal{I}_n} [\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) - x_{i,n}] \geq 0; \tag{39}$$

$$\sum_{i\in\mathcal{I}_n} [x_{i,n} - \tilde{u}_{i,n}(s_{i,n}, x_{i,n})] \leq u^{\max}_{agg,dc,n}, \quad \text{otherwise}. \tag{40}$$

For problem P3, we have the following Theorem.

Theorem 3. *Given $s_{i,n}$ and $x_{i,n}$, the expected energy cost $C_{i,n,s_{i,n},x_{i,n}}(\tilde{U}_{i,n})$ is convex with respect to $\Delta S_{i,n}(s)$. The inequality constraint functions $\sum_{i\in\mathcal{I}_n}[\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) - x_{i,n}]$ and $\sum_{i\in\mathcal{I}_n}[x_{i,n} - \tilde{u}_{i,n}(s_{i,n}, x_{i,n})]$ in (39) and (40), respectively, are convex with respect to $\Delta S_{i,n}(s)$.*

Proof: Given the domain of $\Delta S_{i,n}(s)$ in (33), we consider three cases for possible values of $\Delta S_{i,n}(s)$. Case 1: $\Delta S_{i,n}(s) \in [x_{i,n} - u^{\max}_{i,dc} - u^{d2}_{i,n}(s), x_{i,n} - u^{d2}_{i,n}(s))$; Case 2: $\Delta S_{i,n}(s) \in [x_{i,n} - u^{d2}_{i,n}(s), x_{i,n} - u^{r1}_{i,n}(s)]$; Case 3: $\Delta S_{i,n}(s) \in (x_{i,n} - u^{r1}_{i,n}(s), x_{i,n} + u^{\max}_{i,rc} - u^{r1}_{i,n}(s)]$. For Case 1, we have $x_{i,n} - u^{\max}_{i,dc} \leq \Delta S_{i,n}(s) + u^{d2}_{i,n}(s) < x_{i,n}$. Based on (32), we have $\tilde{S}'_{i,n}(s) < x_{i,n} \leq \tilde{S}'_{i,n}(s) + u^{\max}_{i,dc}$ since $0 \leq x_{i,n} \leq x^{\max}_i$. According to (29), we have

$$\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) = \tilde{S}'_{i,n}(s) = \Delta S_{i,n}(s) + u^{d2}_{i,n}(s) \tag{41}$$

Similarly, $\tilde{u}_{i,n}(s_{i,n}, x_{i,n})$ for Case 3 is given by

$$\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) = \tilde{S}_{i,n}(s) = \Delta S_{i,n}(s) + u^{r1}_{i,n}(s) \tag{42}$$

For Case 2, we have $\tilde{S}_{i,n}(s) \leq x_{i,n} \leq \tilde{S}'_{i,n}(s)$. Therefore, the value of $\tilde{u}_{i,n}(s_{i,n}, x_{i,n})$ is given by

$$\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) = x_{i,n}. \tag{43}$$

We first investigate the inequality constraint function in (39), while the proof with respect to the inequality constraint function in (40) follows the same steps. Obviously, for both Case 2 and Case 3, we have $\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) \geq x_{i,n}$. For the boundary point between Case 2 and Case 3 (i.e., $\Delta S_{i,n}(s) = x_{i,n} - u^{r1}_{i,n}(s)$), we have

$$\begin{aligned} &\tilde{u}_{i,n}(s_{i,n}, x_{i,n})|_{\Delta S_{i,n}(s)=(x_{i,n}-u^{r1}_{i,n}(s))^-} = x_{i,n} \\ &= (x_{i,n} - u^{r1}_{i,n}(s)) + u^{r1}_{i,n}(s) \\ &= \tilde{u}_{i,n}(s_{i,n}, x_{i,n})|_{\Delta S_{i,n}(s)=(x_{i,n}-u^{r1}_{i,n}(s))^+}. \end{aligned} \tag{44}$$

That is, $\tilde{u}_{i,n}(s_{i,n}, x_{i,n})$ (thus, $[\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) - x_{i,n}]$) is continuous with respect to $\Delta S_{i,n}(s)$ at the boundary point between Case 2 and Case 3. Also, $[\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) - x_{i,n}]$ is convex with respect to Case 2 (or Case 3) since it is a linear function. Moreover, at the boundary point between Case 2 and Case 3, we have

$$\left.\frac{d\tilde{u}_{i,n}(s_{i,n}, x_{i,n})}{d\Delta S_{i,n}(s)}\right|_{\Delta S_{i,n}(s)=[x_{i,n}-u^{r1}_{i,n}(s)]^-} = \frac{dx_{i,n}}{d\Delta S_{i,n}(s)} = 0 \tag{45}$$

$$\left.\frac{d\tilde{u}_{i,n}(s_{i,n}, x_{i,n})}{d\Delta S_{i,n}(s)}\right|_{\Delta S_{i,n}(s)=[x_{i,n}-u^{r1}_{i,n}(s)]^+} = \frac{d[\Delta S_{i,n}(s) + u^{r1}_{i,n}(s)]}{d\Delta S_{i,n}(s)} = 1. \tag{46}$$

Since the derivative of $\frac{d\tilde{u}_{i,n}(s_{i,n},x_{i,n})}{d\Delta S_{i,n}(s)}$ on the left side of the boundary is less than that on the right side of the boundary, $\tilde{u}_{i,n}(s_{i,n}, x_{i,n})$ (thus, $[\tilde{u}_{i,n}(s_{i,n}, x_{i,n}) - x_{i,n}]$) is convex with respect to $\Delta S_{i,n}(s)$ for Case 2 and Case 3. Similar proof can be done for Case 1 and Case 2. Further, the summation (over $i \in \mathcal{I}_n$) of convex functions is also convex [26], which completes the proof with respect to the convexity of inequality constraint functions.

For the objective function, without loss of generality, consider $s_{i,n} = \mathcal{W}$. The proof with respect to $s_{i,n} = \mathcal{H}$ is identical, taking account of an additional constant term $r_n\bar{\xi}_{i,n}$ according to (15). Since the decisions in periods $\{n+1, n+2, \cdots, N-1\}$ follow the optimal policy according to (30), we can rewrite $C_{i,n,\mathcal{W},x_{i,n}}(\tilde{U}_{i,n})$ as

$$\begin{aligned} C_{i,n,\mathcal{W},x_{i,n}}(\tilde{U}_{i,n}) &= H_{i,n}(x_{i,n}, \tilde{u}_{i,n}(\mathcal{W}, x_{i,n})) \\ &+ E[V_{i,n+1}(s_{i,n+1}, \beta_i\tilde{u}_{i,n}(\mathcal{W}, x_{i,n}))|s_n = \mathcal{W}]. \end{aligned} \tag{47}$$

Note that $H_{i,n}(x, u)$ and $V_{i,n+1}(s, x)$ are convex functions with respect to u and x, respectively, based on our preliminary study [7]. Therefore, the convexity of $H_{i,n}(x_{i,n}, \tilde{u}_{i,n}(\mathcal{W}, x_{i,n}))$ and $V_{i,n+1}(s_{i,n+1}, \beta_i\tilde{u}_{i,n}(\mathcal{W}, x_{i,n}))$ can be verified by checking the continuity and convexity with respect to the boundary points of Case 1, Case 2, and Case 3, similar to the proof of the inequality constraint functions. The convexity is preserved with respect to the linear combination in terms of the conditional expectation and summation over $i \in \mathcal{I}_n$. The detailed proof is omitted here for conciseness. ■

Based on Theorem 3, problem P3 belongs to the class of convex optimization problems and can be readily solved efficiently by existing algorithms [26]. In order to solve problem P3, the aggregator needs to acquire the statistics of PHEV mobility and energy demand (via the information/control flow as shown in Fig. 2) in terms of $P_{i,n}(j|s)$, $\bar{\xi}_{i,n}$, and $f_{\zeta_{i,n}}(y), y \geq 0$, for $i \in \mathcal{I}_n$, $n \in \{0, 1, \cdots, N-1\}$, and $s, j \in \{\mathcal{H}, \mathcal{W}, \mathcal{C}\}$. The state-dependent thresholds ($S_{i,n}(s)$ and $S'_{i,n}(s)$) calculated by each individual PHEV should be passed to the aggregator for optimal policy adjustment. It is worth mentioning that, in order to protect the privacy of PHEV owner, decentralized algorithms can potentially be used based on the convexity of problem P3 [28], which needs further research.

VI. Numerical Results

The performance of our proposed schemes are evaluated by simulation. The simulation is based on the ONE simulator [29] version 1.4.1 with an additional implementation of the V2G components. The simulator uses sample parameters and data from a real-life scenario. Hourly household energy demand data during the month of June 2011, obtained from volunteers of two different households (John and Terry) subscribed to the Waterloo North Hydro, is used [7] [30]. The TOU pricing of Waterloo North Hydro is in accordance with the Ontario electricity time-of-use price of the summer schedule [31]. The on-peak, mid-peak, and off-peak electricity prices are 10.7 cent/kWh, 8.9 cent/kWh, and 5.9 cent/kWh, respectively. The on-peak and off-peak hours are from 11:00am to 5:00pm and from 7:00pm to next 7:00am, respectively, while the remaining hours are mid-peak hours. Traces of vehicle mobility patterns of these individuals are generated using the ONE simulator based on the survey information which includes the locations of their homes, work places, points of interests, and commute patterns. For example, John usually leaves from home to work between 9:55am and 10:05am, works until 5:00pm. About 3 times a week, he goes to a grocery or a mall after work, and spends 30 to 60 minutes. Finally, he returns home and stays at home until the next morning. The trace is generated considering a map based mobility model where the vehicle follows main roads of the Waterloo Region as shown in Fig. 3. The energy cost during weekdays is considered since the TOU price is constant during weekends.

The default battery parameters are given in Table I, which is based on a lithium-ion battery [32]. The round-trip efficiency of lithium-ion battery is typically between 80% and 90% [33]. The mean value 85% is used as the default parameter in our simulation. High efficiency (97%) of lithium-ion battery is also reported in literature [34]. We only use the value for comparison purpose since experimental data for PHEV applications is not available. The equivalent gasoline cost is based on the average gasoline price in Waterloo Region in June 2011 (130.0 cent/L) and the eletricity/gasoline efficiency reported by Chevrolet Volt (0.125 kWh/km in all-electric mode and 6.4 L/100 km in gasoline-only mode) [19]. The battery value loss in recharging is calculated based on the statistics of Chevrolet Volt with a 8000-dollar battery pack (rated at 16 kWh and about 10 kWh available for use) and a 10% capacity loss after 10 years of normal daily use (i.e., two full recharging/discharging cycles for daily commute). Since long-term statistics of battery replacement are not available,

TABLE I: Default battery parameters.

Parameter	Value
Round-trip efficiency (η_i)	85%
Self-discharging percentage (β_i)	99.993%
Battery value loss in recharging ($\tilde{r}_i$)	1.1 c / kWh
Equivalent gasoline cost (r'_i)	67 c / kWh
Battery capacity ($x_i^{\max}$)	8 kWh

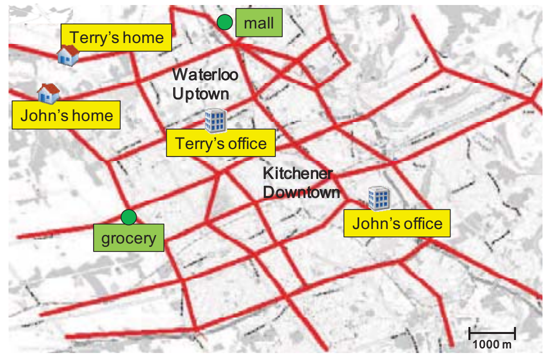

Fig. 3: The topology of the main roads in Waterlo Region and the locations of homes, work places, and points of interests of John and Terry.

we assume that the PHEV battery should be fully replaces after 15 years of use, with an average cost of 1.46 dollar per day. The recharging/discharging rates of individual PHEV are in accordance with level 2 (3.3 kW) infrastructures [35]. For performance optimization, we consider 16 hours from 6:00am to 10:00pm for the periods in $\{0, 1, \cdots N-1\}$ with a period duration $T = 10$ minutes. The commute energy demand $\zeta_{i,n}$ is quantized by a 0.5 kWh stepsize.

A. Recharging/Discharging Pattern

In order to demonstrate how a PHEV battery is managed according to the optimal policy, we consider a specific day of Terry and John, and assume that the PHEV mobility and energy demand information is known a priori. Suppose John leaves home and work at 10:00am and 5:00pm, respectively, with a single-trip commute time 20 minutes and energy consumption 2 kWh. Terry leaves home and work at 8:00am and 5:00pm, respectively, with a single-trip commute time 10 minutes and energy consumption 1 kWh. For comparison purpose, we assume that the PHEV of Terry has a lower recharging/discharging rate (1.2 kW in accordance with level 1 infrastructures [35]) and a higher round-trip energy efficiency (97%).

The results are presented in Fig. 4 and Fig. 5 for John and Terry, respectively, where the lower bound of discharging region and the upper bound of idle region correspond to S', while the lower bound of idle region and the upper bound of recharging region correspond to S. The midnight off-peak periods (which only include a recharging region) are not shown. We can see that, the round-trip energy efficiency has a critical impact on the optimal policy. For Terry, since the round-trip energy efficiency is high (97% as compared with

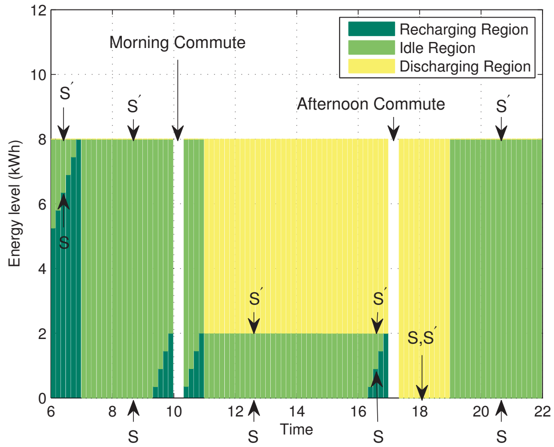

Fig. 4: The values of S and S' at different times of a day (John).

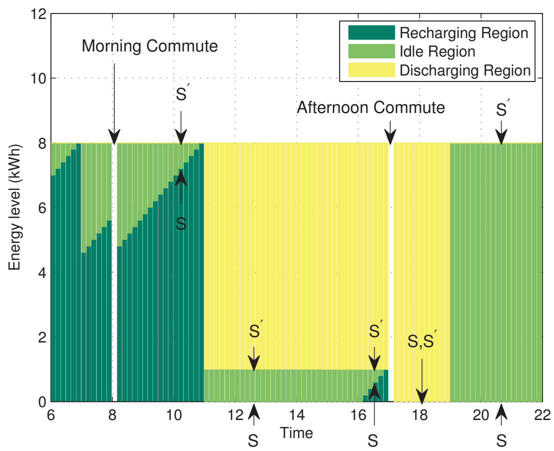

Fig. 5: The values of S and S' at different times of a day (Terry).

85% of John's PHEV), energy is bought as much as possible during the mid-peak periods, and sold during the on-peak periods. On the contrary, John buys energy during the mid-peak periods only for commute use since no benefit can be gained by selling the "mid-peak energy" during the on-peak periods, taking account of the energy loss in recharging and discharging. For instance, Terry buys energy at 9:30am (which is a mid-peak hour) as much as possible by setting a large S, while John buys energy at the same time only for commute use with a small S. Moreover, due to the self-discharge effect, energy for selling or commute is bought as late as possible with the maximum recharging rate.

B. Performance Evaluation of Individual PHEV

We evaluate the performance of the proposed scheme in comparison with three other schemes: i) W/O V2G – Without a V2G system, the energy drawn from the grid can only be used for daily commute, and the battery should be fully recharged during the off-peak periods to minimize the cost [20]; ii) SD – The energy store-and-deliver scheme utilizes commute statistics [3]. The PHEV battery is fully recharged during off-peak periods and a certain amount of energy is reserved for average daily commute demand. The remaining energy is used to compensate for the household demand during the on-peak and mid-peak periods when the PHEV is at home. The benefit of the SD scheme is the same as feeding energy back to the grid, but the amount of energy is bounded by the household demand; iii) OPT – As our proposed scheme uses estimated statistics, we also show the performance of a scheme with a priori knowledge of the PHEV mobility and energy demand information, which provides the best performance but cannot be realized in practical applications.

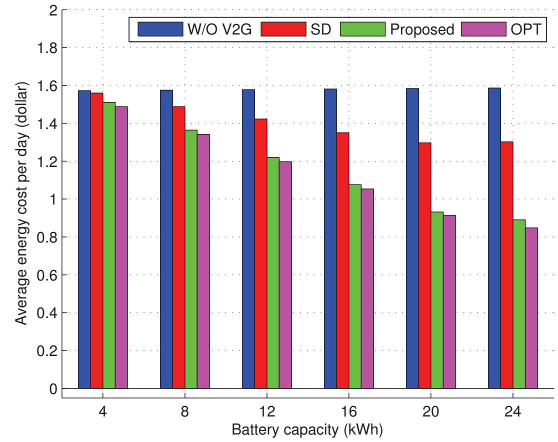

Fig. 6: Average energy cost per day versus battery capacity.

The average energy cost per day versus battery capacity is shown in Fig. 6. The energy cost is averaged over 20 days. The trace of John is used as an example. With a constant cost of household demand every day, we only consider the cost of operating the PHEV, the value loss of the PHEV battery, and the benefit of feeding energy back to the grid. We can see that, the cost without V2G is the highest since the energy in the PHEV battery cannot be used by the household demand or fed back to the grid. By compensating for the household demand during on-peak and mid-peak periods, the SD scheme can reduce the energy cost. But the reduction is not as significant as in with our proposed scheme, since most on-peak periods are not considered for energy feedback when the PHEV is at work place. Our proposed scheme using estimated statistics (with estimation errors) achieves slightly higher cost than the scheme with a priori knowledge. In terms of battery capacity, the cost without a V2G scheme increases as the battery capacity increases since the self-discharge effect gradually decreases the level of the unused energy in battery. The cost achieved by the SD scheme decreases as the battery capacity increases since more battery energy can be used by household appliances. However, the decrement is saturated from 20 kWh because of the limited household demand. For all battery capacities, the cost achieved by our proposed scheme is close to that of the scheme with a priori knowledge. As compared with the SD scheme, the cost reduction of our proposed scheme is more evident for a larger battery capacity.

C. Performance Evaluation of Aggregator

For performance evaluation of aggregator operation, we consider an aggregator with 30 PHEVs, which corresponds to the scale of a suburban residential area [21]. The average commute distance is 8.7 km with a distribution given by the data from the 2006 census of Waterloo Region [36]. The starting time of morning commute for each PHEV owner is randomly selected from 8:00am to 10:00am. Each PHEV owner spends 9 hours at work (including breaks) and commutes back home. The PHEV battery capacity ($x_{i,max}$) and energy loss (η_i) are randomly selected from $[6, 12]$ kWh and $[0.8, 0.95]$, respectively. Consider two tagged periods begin at 11:00am and 16:00pm, respectively. Without loss of generality, we set the constraints on recharging and discharging energy of the aggregator to be the same, i.e., $u^{\max}_{agg,rc,n} = u^{\max}_{agg,dc,n}$. Both level 1 and level 2 infrastructures are considered. All other parameters of the PHEV batteries follow the default settings in Table I. For comparison, we consider a recharging/discharging energy adjustment scheme (denoted by Uniform) such that the output of the PHEVs is adjusted by the same amount to compensate for the mismatch in recharging/discharging energy in a period. For instance, if the aggregated discharging energy in period n exceeds $u^{\max}_{agg,dc,n}$ by 2 kWh and there are 5 PHEVs discharging their batteries, the discharging energy of each PHEV is reduced by 0.4 kWh to meet the power system constraint. The scheme is similar to the generation and load curtailment scheme [15] when considering the PHEVs as domestic electric devices with the same priorities in curtailment.

The incremental cost in a period begins at 16:00pm is shown in Fig. 7. The cost is scaled by multiplying the number of periods of a day for the consistency of results. The initial battery level of each PHEV is randomly selected from $[1, 6]$ kWh. In this period, the recharging demand is dominating since the PHEVs will commute back home shortly and the PHEVs without enough energy for commute need to be recharged immediately. We can see that, the incremental cost decreases as the recharging/discharging energy limit increases as more electric power can be used to recharged the PHEV batteries. Thus, the commute back home relies less on gasoline which is a more expensive energy source than electricity. Our proposed policy adjustment scheme achieves lower incremental cost than the uniform policy adjustment scheme by taking into account the optimal energy delivery policy adopted by each individual PHEV. The incremental cost of level 2 infrastructure is low since the average commute distance in Waterloo Region is relatively short (8.7 km). Even the recharging energy of the aggregator is limited for one period begins at 16:00pm, the batteries of PHEVs can be sufficiently recharged in other periods before commuting back home by level 2 infrastructure which has a relatively high recharging power (3.3 kW). The incremental cost in the period begins at 11:00am is shown in Fig. 8. The initial battery level of each PHEV is randomly selected from $[4, 6]$ kWh. In this period, the discharging demand is dominating since the electricity is at the on-peak price. The incremental cost of level 2 infrastructure is higher than that of the level 1 infrastructure. As the average commute distance in Waterloo Region is relatively short, each PHEV has more energy to feed back to the grid for energy cost reduction. Without a constraint on aggregator discharging energy, the energy cost reduction is more obvious for level 2 infrastructure because of a higher discharging power of each PHEV. On the other hand, when the discharging energy of the aggregator is limited as shown in Fig. 8, the negative impact on level 2 infrastructure (corresponding to the incremental cost) is larger since the high discharging power of each PHEV cannot be exploited under the constraint of aggregated output. Still, our proposed policy adjustment scheme achieves lower incremental cost than that of the uniform policy adjustment scheme.

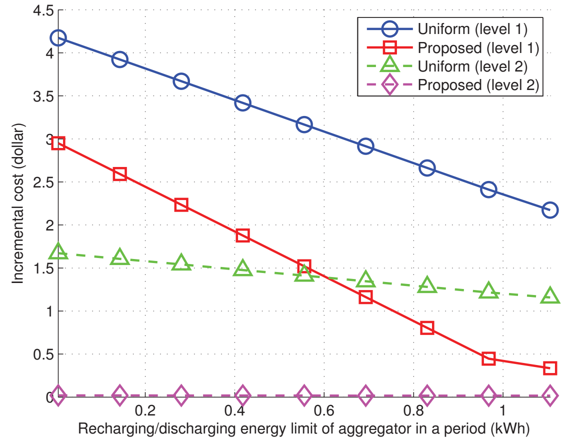

Fig. 7: Incremental cost in a period begins at 16:00pm.

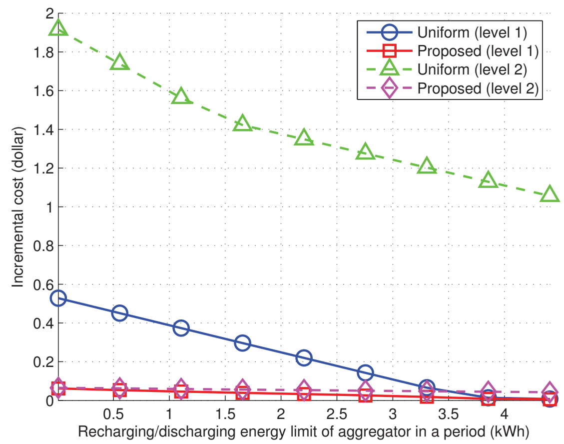

Fig. 8: Incremental cost in a period begins at 11:00am.

VII. Conclusions

In this paper, we have studied a V2G system with load shaving services by taking into account the randomness in vehicle mobility. The energy cost minimization problem of individual PHEV has been investigated under TOU electricity pricing and a realistic battery model. A state-dependent (S, S') policy has been proved to be optimal. For practical applications, we have also proposed an estimation algorithm to calculate the values of S and S' based on the estimations of the statistics of PHEV mobility and energy demand. Furthermore, we have investigated the optimal operation problem of a multi-vehicle aggregator and transformed the original problem into a convex optimization problem based on our proposed policy adjustment scheme which is evaluated based on real data collected from Canadian utilities, households, and commuters, and the results are compared with other existing schemes.

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