Packet Assignment under Resource Constraints with D2D Communications

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Abstract

Device-to-device (D2D) communications offer various benefits such as coverage extension and high energy efficiency. For example, a cluster of in-coverage devices can be recruited to relay packets for an out-of-coverage device. As the helper devices can be subject to varying relay cost and resource constraint, a sequence of packets need to be properly assigned among the helpers. In this article, we study the packet assignment problem from a *mechanism design*'s perspective, taking into account the strategic interactions of self-interested helpers. Fundamental concepts in mechanism design are introduced for general readers to understand the background. Auctions are an important application of mechanism design and widely used in many fields. We develop a reverse auction mechanism, consisting of an allocation rule and a payment rule, to assign packets among the helpers and determine the payments based on their declared costs and resource constraints. As it is NP-hard to find an optimal packet assignment that minimizes the total cost, we use a low-complexity allocation rule, which performs closely to the optimal allocation on average. The payment rule implements the allocation rule in dominant strategies that the helpers disclose their true costs and resource constraints. The truthful revelation in turn justifies the allocation according to the helpers' declaration. Numerical analysis and simulations are conducted to evaluate the performance.

Index Terms

Mechanism design, incentive compatible, VCG mechanism, auction, generalized assignment problem, D2D communications.

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I. Introduction

The past decade has witnessed an explosive growth of tremendous data traffic and mobile devices. Device-to-device (D2D) communications offer a promising solution to accommodate the surging demands, via direct communications among mobile devices in a peer-to-peer fashion bypassing the base station (BS). Thus, D2D provides a virtual "cell tier" that underpins small cells to extend coverage, offload cellular traffic, and improve energy efficiency. In particular, there have been substantial research efforts toward D2D with underlay spectrum sharing, which allows D2D and cellular users to access the same spectrum simultaneously. Such an underlay to the cellular network improves spectral efficiency, but requires effective resource allocation schemes to address non-negligible co-channel interference [1,2].

In the literature, there have been various studies on spectrum sharing to limit the co-channel interference among D2D and cellular users. Interestingly, many solutions use a game-theoretical approach or auction-based design to align with the distributed and autonomous nature of D2D users. In [3], Zhou *et al.* formulate a noncooperative game for D2D underlay resource allocation. A distributed interference-aware algorithm is proposed to derive a Nash equilibrium while maximizing energy efficiency. In [4], Xiao *et al.* present a Bayesian overlapping coalition formation game to analyze the spectrum sharing problem. In [5], a reverse iterative combinatorial auction is proposed to obtain a resource allocation that improves system sum rate.

With effective spectrum sharing, D2D can enable promising application scenarios such as distributed content dissemination [6,7], collaborative crowdsensing, virtual coverage expansion, mobile cloud access, and load balancing [8]. For instance, a relay-by-smartphone prototype via multihop D2D is developed in [6] and successfully tested in field experiments simulating message delivery to isolated areas such as in a disaster situation. In [7], Antonopoulos *et al.* propose two game-theoretic strategies for content dissemination in D2D communication scenarios. Their strategies aim to reduce content dissemination completion time while increasing the energy efficiency of the network.

In this article, we study how to exploit D2D relay to extend BS coverage, as depicted in Fig. 1. Here, source device s is out of the service range of BS d. A cluster of helper devices with redundant spectrum and energy resources can be recruited to relay a message from the source device to the BS via D2D transmission. The cluster can also be viewed as a peer-based ad hoc mobile cloud, which pools together the helpers' resources to relay a sequence of packets that constitute the message. As a helper may have

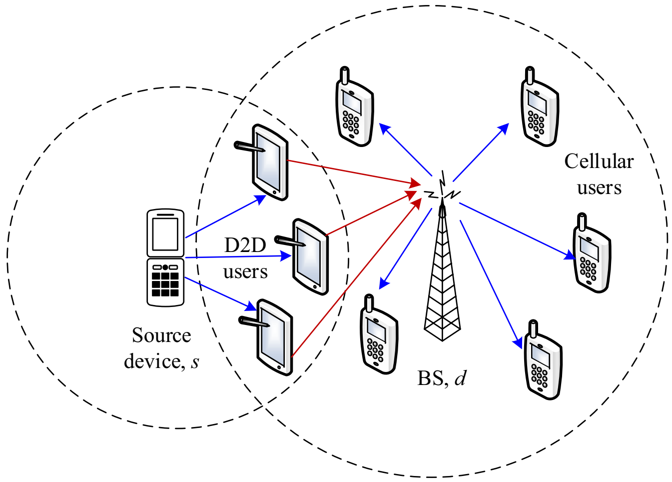


Fig. 1. Coverage expansion with D2D relay.

a capacity constraint due to its limited resources, it is essential to properly assign the packets among the helpers such that the message can be delivered with a minimum cost. We formulate this packet assignment problem as a special case of the NP-hard generalized assignment problem (GAP).

More importantly, the assignment should address any selfish behavior of the helper devices which are only concerned with their own payoffs. As a result, the packet assignment problem goes beyond a general optimization problem and becomes a *mechanism design* problem. Mechanism design is colloquially known as "inverse game theory" [9]. In general, mechanism design aims to design a game such that the game's equilibrium holds certain desired properties, whereas the analysis of a given game focuses on the strategic interaction among the agents to derive the equilibrium if exists. An important application of mechanism design is auctions. Thus, we develop a reverse auction mechanism for the packet assignment problem and analyze its performance in two essential aspects, *i.e.*, *efficiency* and *incentive compatibility (truthfulness)*. The proposed mechanism is proved to be truthful and shows reasonable efficiency in average-case performance.

The remainder of this article is organized as follows. In Section II, we describe the system model with D2D communications for coverage expansion and the packet assignment problem. Section III introduces some basics of mechanism design. In Section IV, we apply mechanism design to the packet assignment problem and analyze the reverse auction mechanism. Section V concludes this research.

II. The Packet Assignment Problem

A. System Model with D2D Relay

Consider the D2D relaying scenario in Fig. 1, where source device s is out of the coverage of BS d. To send a message to d, s can recruit a set of helper devices, denoted by $N = \{1, ..., n\}$, which are within the transmission range of both s and d. Source s segments the message into a sequence of packets, denoted by $M = \{1, ..., m\}$, and assigns them to helper devices in N for forwarding to d via D2D links. With limited radio spectrum, the D2D links share the downlink spectrum of regular cellular users. There exists interference between D2D user i and cellular user j who share the same downlink channel. Since the potential D2D helpers are often close to the edge, the downlink interference from the distant BS can be generally weaker than the uplink interference from nearby devices. Hence, such setting allows simple resource allocation for D2D communications, although it does not exclude the D2D links from sharing the uplink spectrum. On the other hand, the co-channel interference is negligible at the BS when the helpers use orthogonal uplink channels to forward packets to the BS.

For each packet $k \in M$ that source s wants to send to the BS, s can advertise its size z_k and quality of service (QoS) requirement, *e.g.*, a packet delay not greater than Δ_k. The source can also provide transmission-related information such as its transmit power and channel gain estimate. Then, helper i needs to check if the channel rate of the $s-i$ D2D link can meet the data rate requirement, *i.e.*, $B_{si} \cdot \log_2(1+\gamma_{si}) \geq \lambda_k = z_k/\Delta_k$, where B_{si} is the bandwidth of resource blocks of helper i for the D2D link, and γ_{si} is the signal to interference plus noise ratio (SINR) at helper i. Basically, SINR can be estimated from pilot bits plus user bits in some cases to increase accuracy with more available samples [10]. Here, we only require a reasonable accuracy since SINR is not used for sophisticated power allocation but for cost estimation. Further, helper i should estimate the minimum transmit power to provide the required data rate over the $i - d$ cellular link, *i.e.*, $B_{id} \cdot \log_2(1 + \gamma_{id}) \geq \lambda_k$, if helper i has bandwidth B_{id} for the cellular link and achieves a signal to noise ratio γ_{id} at d. In addition, helper i can evaluate its computing cost, *e.g.*, memory consumption at the device, to buffer the relaying packets.

B. The Packet Assignment Problem

Due to the helpers' resource constraints, the amount of resources that a helper can devote to relaying the source's packets is restricted. Let c_{ik} denote the resource cost for helper i to relay packet k, and w_i the overall resource budget of helper i. Define x_{ik} as a binary indicator, which takes value 1 if helper i is assigned packet k and value 0 otherwise. To assign all packets in M among the helpers in N while minimizing the total cost, the packet assignment problem is formulated as

$$\text{minimize} \quad \sum_{i=1}^{n}\sum_{k=1}^{m} c_{ik}x_{ik} \tag{1a}$$

$$\text{subject to} \quad \sum_{k=1}^{m} c_{ik}x_{ik} \leq w_i, \forall i \in N \tag{1b}$$

$$\sum_{i=1}^{n} x_{ik} = 1, \forall k \in M \tag{1c}$$

$$x_{ik} \in \{0,1\}, \forall i \in N, k \in M. \tag{1d}$$

Here, the objective function in (1a) aims to minimize the total resource cost for relaying all packets. Constraint (1b) accommodates the resource budget of each helper i, (1c) ensures that each packet is only assigned to one helper, and (1d) is the integer constraint for the decision variables x_{ik}. This problem is NP-hard, as its maximized counterpart generalizes the NP-hard multiple subset sum problem (MSSP) [11]. For MSSP, the weights c_{ik} in the objective function and those in the first constraint not only coincide but also are invariant with k. MSSP is a special case of the 0-1 multiple knapsack problem (MKP) where the above weights do not necessarily coincide. The generalized assignment problem (GAP) further generalizes MKP in that such weights vary with both i and k. As a result, MKP and GAP are also NP-hard [12].

The optimization problem for packet assignment is formulated from the perspective of the source. Even though there exist approximation algorithms [13,14] for this NP-hard problem, two important issues need to be addressed. First, it is implicitly assumed that the resource costs and constraints of the helpers are known to the source to solve the optimization problem. When the source attempts to collect such information from the helpers, the helpers may not disclose the true information to the source. If the optimization is based on incorrect information, the final performance is not optimal and not even predictable. Second, as an incentive, the helpers should be compensated for their costs. One simple way is to offer a reward proportional to the cost, *e.g.*, providing a payment $(1+\sigma)\sum_{k=1}^{m} c_{ik}x_{ik}$ to helper i, which is the cost plus a certain proportion ($\sigma > 0$) of the cost as profit. Such a reward method can result in further challenge to the first problem. A selfish helper who is interested in maximizing its own payoff tends to declare a low cost to have more assigned packets, since the payment includes a profit margin and thus induces certain manipulation space. On the other hand, a helper may intend to boost its cost to gain a higher payment as long as its packet assignment is not reduced. To address these two problems due to the strategic behaviors of helpers, we need to transform an algorithmic solution of the optimization problem into a mechanism.

III. Some Fundamentals of Mechanism Design

In game theory, complete-information games can be described in a normal form. In particular, the utility function of each player for a set of action profiles is *known* for a complete-information game. Accordingly, the game can be analyzed to derive a Nash equilibrium if exists. In mechanism design, the players' utility functions are *private*. A mechanism together with the players' secret utility functions forms a Bayesian game. In contrast to a complete-information game, a Bayesian game has incomplete information with uncertainties about the players, *e.g.*, in the action space and utilities.

A. Definition of General Mechanisms

There is a close relationship between game theory and mechanism design. In [9], a mechanism is defined within a Bayesian game setting, including

- N, a finite set of n agents;
- $\Theta = \Theta_1 \times \cdots \times \Theta_n$, a set of possible joint type vectors; the type of an agent is a simple way to encapsulate the private information held by the agent;
- p, a common-prior probability distribution on Θ;
- O, a set of outcomes; and
- $u = (u_1, ..., u_n)$, where $u_i : O \times \Theta \mapsto \mathbb{R}$ is the utility function for each agent i.

Then, a mechanism in the Bayesian game setting is defined by

- $A = A_1 \times \cdots \times A_n$, a set of action profiles, where A_i is the set of available actions to agent $i \in N$; the available actions of agent i depend on its private type in Θ_i; and
- $M : A \mapsto \Pi(O)$, which maps each action profile in A to an outcome or a distribution over outcomes.

Direct mechanisms are an important subset of mechanisms, where the only action available to each agent is to announce its type, *i.e.*, $A_i = \Theta_i$. Placing a mechanism in its Bayesian game setting forms a game. The key of mechanism design is to form a game with a desired equilibrium, which can implement a target outcome when the game is played in equilibrium. One desired equilibrium is *an equilibrium in dominant strategies*, in which the strategy of every agent weakly dominates any other strategy of that agent, no matter what strategies the other agents play. Furthermore, the equilibrium in dominant strategies is preferably a specific one that agents truthfully disclose their private information.

B. Direct Mechanisms in Quasilinear Setting

As a widely used assumption in mechanism design, *quasilinear preferences* define a restricted domain which splits the set of outcomes into two linearly related subsets, *i.e.*, $O = X \times \mathbb{R}^n$, where X represents a finite set of nonmonetary choices, *e.g.*, a good allocated in an auction, and $\mathbb{R}^n$ represents the monetary transfer each agent gives (if positive) or receives (if negative). Second, the general utility function is adapted to a *quasilinear utility function*. For an outcome $o = (x, p)$ in O, the utility of agent i is given by $u_i(o, \theta) = v_i(x, \theta) - f_i(p_i)$, where $v_i : X \times \Theta \mapsto \mathbb{R}$ and $f_i : \mathbb{R} \mapsto \mathbb{R}$ is a strictly monotonically increasing function. If the agents' utilities only depend on their own types, $v_i(x, \theta) = v_i(x, \theta_i)$ can be written as $v_i(x)$ for short and referred to as agent i's *valuation* for choice x. The function $f_i(p_i)$ expresses agent i's value for an amount of money p_i, *e.g.*, a linear function f_i for a *risk neutral* agent.

In a quasilinear setting, a direct mechanism includes an action space with $A_i = \Theta_i, \forall i \in N$, and a mapping function M that is split into two functions [9]:

- $\mathcal{X} : A \mapsto \Pi(X)$, is *a choice rule* that maps each action profile to a selected choice in X or a distribution over choices; and
- $\wp : A \mapsto \mathbb{R}^n$, is *a payment rule* that determines a payment for each agent according to an action profile in A.

C. The Vickrey-Clarke-Groves (VCG) Mechanism

The VCG mechanism is the most important direct quasilinear mechanism, which defines

$$\mathcal{X}(\hat{v}) = \arg\max_{x \in X} \sum_i \hat{v}_i(x) \tag{2}$$

$$\wp_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(\mathcal{X}(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(\mathcal{X}(\hat{v})). \tag{3}$$

Here, $\hat{v}_i(x)$ denotes the declared valuation of agent i for choice $x \in X$. The vector of all agents' declared valuations is denoted by $\hat{v}$, while the vector of the declarations from all agents excluding i is denoted by $\hat{v}_{-i}$. The payment by agent i to the mechanism can be interpreted as the "externality" caused by agent i, *i.e.*, the loss of total valuation inflicted on the other $(n-1)$ agents by i's presence.

The VCG mechanism simultaneously holds two desired properties.

- *Dominant-strategy incentive compatibility*: A direct mechanism is dominant-strategy incentive compatible (or truthful) if it induces a dominant-strategy equilibrium for the resulting Bayesian game, in which every agent's strategy is to announce its true valuation for each choice $x \in X$.
- *Efficiency in social welfare maximization*: A strictly Pareto efficient, or just efficient mechanism in equilibrium selects a choice $x = \arg\max_{x' \in X} \sum_i v_i(x')$, *i.e.*, the sum of agents' true valuations (also known as social welfare), disregarding the monetary payments in their utilities.

IV. Auction Mechanisms for The Packet Assignment Problem

To deal with the two concerns involved with the helpers' strategic interactions in the context of mechanism design, a dominant-strategy truthful mechanism provides a solution to the first concern. The mechanism can be designed so that, in equilibrium, the dominant strategy of each helper is to disclose its real information to the source. Based on such information, the source can assign its packets among the helpers to minimize total cost, as if such information were known a priori. Thus, the allocation rule should be implemented by a payment rule such that the mechanism is dominant-strategy truthful. This is the key to address the second concern.

A. Formulation of Reverse Auction

Here, we focus on direct mechanisms, specifically, sealed-bid auctions. The auction is single-sided with a set of n helpers as bidders, and the source device as the single buyer of the relaying service for m packets. Based on the packet and transmission information provided by the source, helper $i \in N$ estimates its transmission and computing costs, and converts the costs into monetary amounts, represented by an m-vector, $c_i = (c_{i1}, ..., c_{im})$. Also, helper i has a resource constraint, w_i, which limits the maximum cost it can afford. As seen, this is a multi-parameter environment, in which the private information of a bidder cannot be represented by a single parameter. The true type of helper i can be described by its cost vector and resource constraint, *i.e.*, $\theta_i = (c_i, w_i)$. All helpers simultaneously submit their bids to the source, denoted by $\hat{c} = (\hat{c}_1; ...; \hat{c}_n)$ and $\hat{w} = (\hat{w}_1, ..., \hat{w}_n)$. Here, we use $\hat{c}_i$ and $\hat{w}_i$ to represent the reported type of helper i to be distinguished from its true type (c_i and w_i) Then, the source applies a mechanism with

1) an allocation rule, $x = \{x_{ik} : i \in N, k \in M\}$, for packet assignment among bidding helpers; and
2) a payment rule, $p = \{p_{ik} : i \in N, k \in M\}$, for the amount of payment rewarded to each helper.

If allocation x satisfies resource constraint, *i.e.*, $\sum_{k=1}^{m} c_{ik}x_{ik} \leq w_i$, the utility of helper i of true type θ_i for an outcome $o = (x, p)$ is defined as $u_i(o, \theta_i) = \sum_{k=1}^{m} (p_{ik} - c_{ik}x_{ik})$, while the utility is $-\infty$ otherwise. Here, this problem is considered as a *reverse auction*, which is different from auctions for selling goods (known as forward auctions). The valuation of helper i for packet k is $-c_{ik}$. The payment that a bidder pays the mechanism is $-p_{ik}$, where a negative amount means the mechanism pays the bidder. As discussed in Section III-B, an efficient allocation maximizes the social welfare as the sum of agents' true valuations. In this reverse auction, it is translated to minimizing the total relay cost as formulated in (1).

It is worth noting that problem (1) may not have a feasible solution due to the resource constraint. To simplify feasibility analysis, we can reformulate (1) by including the source as the $(n+1)^{th}$ virtual bidder with $w_{n+1} = +\infty$. The source's cost vector, $c_{n+1} = (c_{n+1,1}, ..., c_{n+1,m})$, can be interpreted as the reserve price of the source, in the sense that the bidders' costs c_{ik} $(1 \leq i \leq n)$ should not be greater than $c_{n+1,k}$. Then, the set of bidders become $N' = N \cup \{n+1\}$. This implies that the packets unassigned to the helpers could roll back to the source and thus induce an additional cost if the source had to deliver them by itself. Here, we have $\hat{c}_{n+1,k} = c_{n+1,k}$ and $\hat{w}_{n+1} = w_{n+1}$ only for the source. For each helper i, the declared cost $\hat{c}_{ik}$ should replace the real cost c_{ik} in (1) to highlight that the truthfulness assumption remains to be validated. Then, the packet assignment problem in (1) can be reformulated as

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n+1}\sum_{k=1}^{m} \hat{c}_{ik}x_{ik} \\ \text{subject to} \quad & \sum_{k=1}^{m} \hat{c}_{ik}x_{ik} \leq \hat{w}_i, \forall i \in N' \\ & \sum_{i=1}^{n+1} x_{ik} = 1, \forall k \in M \\ & x_{ik} \in \{0,1\}, \forall i \in N', k \in M. \end{aligned} \tag{4}$$

B. Reverse Auction Mechanisms

Applying the VCG mechanism to the formulated reverse auction, we can obtain the allocation rule as an *optimal* solution to problem (4), and the payment to any helper i as the increase of total cost to the other agents if i were absent. When helper i is present or absent, the resulting optimal allocations will induce different total costs to the other agents excluding i. The increase of total cost when i is absent as opposed to that with its presence quantifies i's contribution to the relaying service and thus its payment. Though the VCG mechanism is efficient and truthful, it needs to solve $(n+1)$ NP-hard problem instances, one for the packet assignment and n for the payments to n helpers.

To reduce computational overhead, we design a low-complexity but non-optimal algorithm to compute the allocation and payment. Alg. 1 consists of two loops regarding all m packets and n helpers, respectively. For the first loop in Lines 2-10, each packet is first tentatively assigned to the bidder of the lowest valid bid, with a temporary payment of either the reserve price of the source if only one helper submits a valid bid, or the second-lowest valid bid otherwise. In Lines 11-20 of the second loop, the assignment of each helper is further checked to ensure that the assigned packets do not violate the resource constraints of the selected bidders. For each helper, the potential profits from the assigned packets are given by the tentative payments minus the declared costs. If all assigned packets cannot be handled within a helper's resource budget, a subset of the assigned packets are selected to maximize each helper's total profit while satisfying its resource constraint. Line 17 actually solves an instance of the knapsack problem. Though there is no polynomial time approximation scheme (PTAS) for the knapsack problem, it can be solved by a pseudo-polynomial time algorithm using dynamic programming. Therefore, Alg. 1 has a time complexity of $O(nm \cdot \max_{i,k}(p_{ik} - \hat{c}_{ik}))$, which is much lower than that of the VCG mechanism.

As seen from the input of Alg. 1, the source needs to collect bids from helpers in terms of per-packet costs $\hat{c}$ and total resource budgets $\hat{w}$. To facilitate a potential helper to derive such information, the source should advertise the packet and transmission-related information, as well as its reserved price, so that the helpers with higher costs will not bother to bid. Before the auction, the exchange of information between the source and helpers can be conducted over D2D links, which also assists the helpers in estimating their transmission costs. This exchange of information can also be supported by out-of-band short-range radio such as Wi-Fi. After the auction, the helpers are informed of the auction result, deliver the assigned

Algorithm 1 Computing the allocation and payment for the packet assignment problem.

Input: $\hat{c} = (\hat{c}_1; ...; \hat{c}_n)$, $\hat{w} = (\hat{w}_1, ..., \hat{w}_n)$

Output: x, p

$x_{ik} \leftarrow 0, p_{ik} \leftarrow 0, \forall i \in N, k \in M$

for $k \in M$ **do**

  $H_k = \{i : i \in N, c_{ik} < c_{n+1,k}\}$

  Sort H_k in non-increasing order such that $\hat{c}_{i_1 k} \leq \hat{c}_{i_2 k} \leq \cdots \leq \hat{c}_{i_\phi k}$

  if $|H_k| = 1$ **then**

    $x_{i_1 k} \leftarrow 1, p_{i_1 k} \leftarrow c_{n+1,k}$

  else

    $x_{i_1 k} \leftarrow 1, p_{i_1 k} \leftarrow \hat{c}_{i_2 k}$

  end if

end for

for $i \in N$ **do**

  $G_i = \{k : k \in M, x_{ik} = 1\}$

  Sort G_i in non-decreasing order such that $(p_{ik_1} - \hat{c}_{ik_1}) \geq (p_{ik_2} - \hat{c}_{ik_2}) \geq \cdots \geq (p_{ik_\psi} - \hat{c}_{ik_\psi})$

  if $|G_i| = 1$ and $\hat{c}_{ik_1} > \hat{w}_i$ **then**

    $x_{ik_1} \leftarrow 0, p_{ik_1} \leftarrow 0$

  else if $|G_i| \geq 2$ **then**

    Determine $S_i = \arg\max_{F \subseteq G_i} \sum_{k_l \in F} (p_{ik_l} - \hat{c}_{ik_l})$ such that $\sum_{k_l \in F} \hat{c}_{ik_l} \leq \hat{w}_i$

    $x_{ik'_l} \leftarrow 0, p_{ik'_l} \leftarrow 0$, for all $k'_l \notin S_i$

  end if

end for

return x, p

packets, and collect the payments (*e.g.*, cashable credits) that the source has deposited at the BS.

C. Truthfulness Analysis of The Proposed Mechanism

To prove Alg. 1 defines a truthful mechanism, we refer to the theorem in [9] that a direct, deterministic mechanism is dominant-strategy incentive compatible if and only if, for every $i \in N$ and every $\hat{v}_{-i} \in V_{-i}$,

1) the payment function $\wp_i(\hat{v})$ can be written as $\wp_i(\hat{v}_{-i}, \mathcal{X}(\hat{v}))$; and
2) for every $\hat{v}_i \in V_i$, $\mathcal{X}(\hat{v}_i, \hat{v}_{-i}) = \arg\max_{x \in \chi_i(\hat{v}_{-i})}(\hat{v}_i(x) - \wp_i(\hat{v}_{-i}, x))$.

The first condition requires that each agent's payment depend on the other agents' declaration and the selected choice, but not on its own declaration. In the second condition, $\chi_i(\hat{v}_{-i})$ denotes the set of choices that can be selected by choice rule $\mathcal{X}$, given the declaration $\hat{v}_{-i}$ by the agents other than i, *i.e.*, the range of $\mathcal{X}(\cdot, \hat{v}_{-i})$. For every agent, provided the other agents' declaration and the payment function that satisfies the first condition, the choice rule selects the choice that maximizes its *declared* valuation minus the payment. Note that an agent's utility is defined as its *true* valuation (not its declared valuation) minus the payment. Since the payment is independent of an agent's own declaration, an agent cannot manipulate its payment by its declaration. Therefore, the only way that an agent can maximize its utility is to align with the choice rule by declaring its true valuation.

Next, we analyze the allocation rule and payment rule in Alg. 1 based on the theorem. First, each bidder's tentative payments determined in the first for-loop are independent of its own declaration. Essentially, the payment to the selected helper for each packet is the minimum of the second-lowest bid and the source's reserve price. Thus, a bidder cannot increase its individual payment for each assigned packet by manipulating its declaration. Moreover, the second for-loop cancels certain assigned packets to ensure satisfaction of resource constraints. The procedure actually maximizes the bidders' utilities if they announce their true costs. Thus, the bidders do not have any incentive to misreport their costs.

Second, a bidder cannot increase its utility by misreporting its resource constraint. Since the first for-loop is independent of the bidders' resource constraints, we focus on the second for-loop. Consider bidder i with a set of tentatively assigned packets resulting from the first for-loop. If these packets induce a cost sum not greater than w_i, bidder i cannot change its best allocation by reporting a resource budget higher than w_i, while a reported budget lower than w_i will decrease its allocation. On the other hand, if bidder i cannot take all assigned packets without violating its resource constraint, declaring a budget higher than w_i will cause a negative utility, as it will cost more resources than bidder i can afford. Similarly, reporting a budget lower than w_i will only decrease i's utility. Thus, the bidders have no incentive to misreport their resource constraints.

D. Efficiency Analysis of The Proposed Mechanism

Though Alg. 1 gives a truthful mechanism, the allocation is not guaranteed optimal in minimizing the total cost. In [15], Nisan *et al.* study a more general problem, the cost minimization allocation problem (CMAP), which allows any restriction for the possible allocations. It is proved that an allocation algorithm of a non-optimal VCG-based mechanism for CMAP must be *degenerate*. For a degenerate algorithm, the resulting objective value can be arbitrarily far from optimal, both additively and multiplicatively. Hence, we next analyze the performance of Alg. 1 in the average case rather than in the worse case.

For simplicity, consider a special case where the helpers' costs (c_{ik}) follow a uniform distribution $U(0, 1)$, and the resource constraints (w_i) follow another uniform distribution $U(0, m)$. The following analysis can be extended to other probability distributions. First, we derive the average total cost of the allocations resulting from Alg. 1. Assuming truthful revelation, the bids submitted by the n helpers for packet k, *i.e.*, $c_{1k}, ..., c_{nk}$, are actually n independent and identically distributed (i.i.d.) uniform random variables (r.v.'s). Thus, the lowest bid selected for packet k is the first order statistic, which follows a beta distribution $Beta(1,n)$ with mean $\frac{1}{n+1}$. The probability that bidder i tentatively wins j $(0 \leq j \leq m)$ packets, denoted by $q_i(j)$, is the probability that j out of m lowest bids for the packets belong to bidder i. Thus, the number of packets won by each bidder follows a binomial distribution $B(m, \frac{1}{n})$. If all j packets tentatively assigned to bidder i fall within i's resource budget w_i, *i.e.*, $\sum_{l=1}^{j} c_{ik_l} \leq w_i$, they induce an average cost $\frac{j}{n+1}$. As $\sum_{l=1}^{j} c_{ik_l}$ is the sum of i.i.d. beta distributions, we can approximate the sum by a normal distribution $N(\mu, \sigma^2)$ with the same mean and variance, *i.e.*, $\mu = \frac{j}{n+1}$ and $\sigma^2 = \frac{jn}{(n+1)^2(n+2)}$. Then, we can numerically evaluate the probability of satisfying the resource constraint, denoted by $\eta_i(j)$.

To estimate an upper bound of the total cost, consider the packets are removed from the assignment if the resource budget is violated. These eliminated packets will be reallocated back to the source. Since the bidders' costs are expected to be not greater than that of the source, we can model $c_{n+1,k}$ by the n^{th} order statistic of n i.i.d. r.v.'s following $U(0,1)$. Thus, $c_{n+1,k}$ follows $Beta(n,1)$ with mean $\frac{n}{n+1}$. It means each packet is subject to average cost $\frac{n}{n+1}$ at the source. Therefore, when j packets assigned to i violate its resource budget with probability $1-\eta_i(j)$ and roll back to the source, these j packets will totally cost $\frac{jn}{n+1}$ on average. Treating all n bidders symmetrically, we can write the average total cost as

$$\overline{C}_{\text{ALG}} \leq \overline{C}'_{\text{ALG}} = \sum_{j=1}^{m} n \cdot q_i(j) \left[\eta_i(j)\frac{j}{n+1} + (1-\eta_i(j))\frac{jn}{n+1}\right] \tag{5}$$

where $\overline{C}'_{\text{ALG}}$ denotes an upper bound of the average total cost of Alg. 1, $\overline{C}_{\text{ALG}}$.

On the other hand, the average total cost of optimal solutions to (4), denoted by $\overline{C}_{\text{OPT}}$, is lower bounded by the sum of m lowest bids selected among all nm bids. Clearly, this lower bound relaxes the first and second constraints in (4). It consists of the first to m^{th} order statistics of nm i.i.d. r.v.'s following $U(0,1)$. It is known that the j^{th} order statistic follows $Beta(j, nm-j+1)$ with mean $\frac{j}{nm+1}$. The lower bound of average total cost is then obtained as

$$\overline{C}_{\text{LB}} = \sum_{j=1}^{m} \frac{j}{nm+1} = \frac{m(m+1)}{2(nm+1)}. \tag{6}$$

To evaluate the cost performance, we conduct some numerical and simulation experiments. Fig. 2 shows the ratios $\frac{\overline{C}'_{\text{ALG}}}{\overline{C}_{\text{LB}}}$, $\frac{\overline{C}_{\text{ALG}}}{\overline{C}_{\text{LB}}}$, $\frac{\overline{C}_{\text{OPT}}}{\overline{C}_{\text{LB}}}$, and $\frac{\overline{C}_{\text{ALG}}}{\overline{C}_{\text{OPT}}}$, with $m=40$ and n varying from 4 to 22. As seen, our analysis for the upper bound $\overline{C}'_{\text{ALG}}$ matches well the simulation results, and it provides a tight bound for the real

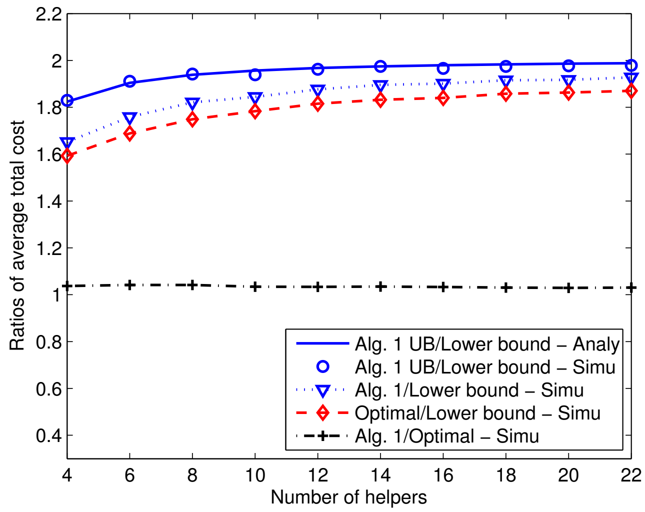


Fig. 2. Ratios of average total costs of Alg. 1 and optimal solutions.

cost of Alg. 1, $\overline{C}_{\text{ALG}}$. It is also observed that the average total cost of Alg. 1 is less than 1.1 times that of the optimal solutions, which shows Alg. 1 can approach the optimal performance on average.

E. Simulation Results in D2D Relay

To examine the performance of Alg. 1 in a D2D relay scenario in Fig. 1, we conduct simulations with parameters in Table I. There are a sequence of m packets whose rate requirements are randomly generated with a maximum rate of 100 kbit/s. The n helpers are deployed in a sector region within a symmetric angle interval of $(-\pi/4, \pi/4)$ with respect to the source-destination link. Regular cellular users are uniformly deployed in a circular area centered at the BS. The computing cost (*e.g.*, due to memory consumption) of helper i for packet k is simulated by a random unit cost. The overall cost of helper i for each packet is converted into a monetary term by using a weighted sum. The resource constraint of each helper is modeled by $w_i = \rho \sum_{k=1}^{m} c_{ik}$, where $\rho \sim U(0,1)$.

Fig. 3 compares the cost and payment of the assignments by Alg. 1 and the VCG mechanism. Note that Fig. 3 is based on a scenario randomly generated according to the above settings. The result can vary for a different set of data. For easy interpretation, the cost and payment are normalized by the maximum cost and payment of the VCG mechanism, respectively. As seen in Fig. 3(a), the packet assignment using Alg. 1 can lead to a total cost much higher than that of the VCG mechanism. This is because some packets remain unassigned among the helpers and thus generate a much higher cost if rolling back to the source. As shown in Fig. 3(b), because of these unassigned packets with Alg. 1, the helpers are paid less

TABLE I

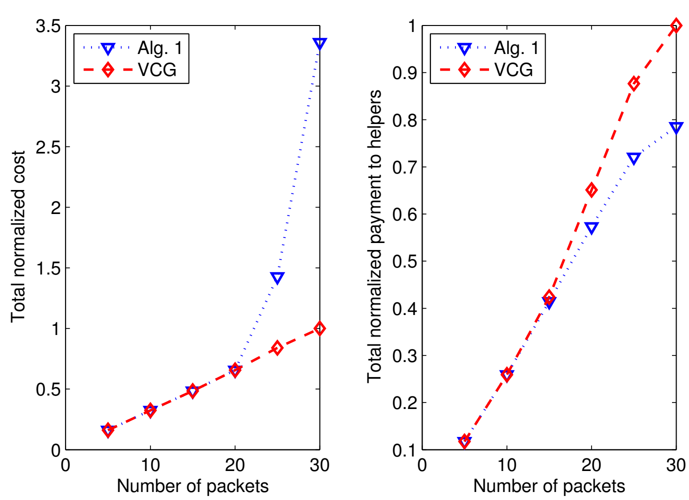
SYSTEM PARAMETERS.

Parameter	Value	Parameter	Value
Path loss exponent	3	Power spectral density of AWGN noise	-115.6 dBm/Hz
Coverage radius of BS	500 m	Coverage radius of source s	200 m
Transmit power of regular cellular users	26 dBm	Transmit power of source s	15.6 dBm
Number of D2D helpers	5	Number of packets	$6 \sim 30$
Number of regular cellular users	50	Bandwidth of source and helpers	180 kHz

in total. In general, Alg. 1 is not guaranteed to produce an optimal packet assignment. Nonetheless, when the cost bids of helpers are more diverse and have more consistent ordering with respect to the packets, Alg. 1 is more likely to approach an optimal packet assignment.

V. Concluding Remarks

In this article, we study how to design an auction mechanism which assigns packets from an out-of-range source to a set of resource-constrained helper devices for relaying via D2D communications. The packet assignment is formulated as a special case of GAP which aims to minimize the total cost. To address the strategic interactions of self-interested helpers, we develop a sealed-bid reverse auction and analyze its performance in two essential aspects, efficiency and dominant-strategy incentive compatibility. As the packet assignment problem is NP-hard, the efficiency requirement is relaxed to have a low-complexity allocation rule, which performs closely to the optimal allocation on average. The payment



(a) (b)

Fig. 3. Normalized cost and payment of the assignments by Alg. 1 and the VCG mechanism.

rule implements the allocation rule in dominant strategies that the helpers truthfully disclose their costs and resource constraints. Focusing on packet assignment, we assume that the resource allocation for the D2D links can be handled by existing spectrum sharing schemes. For future studies, it is interesting to jointly consider the two problems to improve resource utilization.

References

[1] Z. Zhou, M. Dong, K. Ota, J. Wu, and T. Sato, "Energy efficiency and spectral efficiency tradeoff in device-to-device (D2D) communications," *IEEE Wireless Commun. Lett.*, vol. 3, no. 5, pp. 485–488, 2014.

[2] Li, S. Guo, T. Miyazaki, and W. Zhuang, "Fine-grained resource allocation for cooperative device-to-device communication in cellular networks," *IEEE Wireless Commun. Mag.*, vol. 21, no. 5, pp. 35–40, 2014.

[3] Z. Zhou, M. Dong, K. Ota, R. Shi, Z. Liu, and T. Sato, "Game-theoretic approach to energy-efficient resource allocation in device-to-device underlay communications," *IET Communications*, vol. 9, no. 3, pp. 375–385, 2015.

[4] Y. Xiao, K.-C. Chen, C. Yuen, Z. Han, and L. A. DaSilva, "A Bayesian overlapping coalition formation game for device-to-device spectrum sharing in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 4034–4051, 2015.

[5] C. Xu, L. Song, Z. Han, Q. Zhao, X. Wang, X. Cheng, and B. Jiao, "Efficiency resource allocation for device-to-device underlay communication systems: A reverse iterative combinatorial auction based approach," *IEEE J. Select. Areas Commun.*, vol. 31, no. 9, pp. 348–358, 2013.

[6] H. Nishiyama, M. Ito, and N. Kato, "Relay-by-smartphone: Realizing multi-hop device-to-device communications," *IEEE Commun. Mag.*, vol. 52, no. 4, pp. 56–65, 2014.

[7] A. Antonopoulos, E. Kartsakli, and C. Verikoukis, "Game theoretic D2D content dissemination in 4G cellular networks," *IEEE Commun. Mag.*, vol. 52, no. 6, pp. 125–132, 2014.

[8] J. Liu, Y. Kawamoto, H. Nishiyama, N. Kato, and N. Kadowaki, "Device-to-device communications achieve efficient load balancing in LTE-advanced networks," *IEEE Wireless Commun. Mag.*, vol. 21, no. 2, pp. 57–65, 2014.

[9] Y. Shoham and K. Leyton-Brown, *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2009.

[10] D. R. Jeske and A. Sampath, "Signal-to-interference-plus-noise ratio estimation for wireless communication systems: Methods and analysis," *Naval Research Logistics*, vol. 51, no. 5, pp. 720–740, 2004.

[11] A. Caprara, H. Kellerer, and U. Pferschy, "A PTAS for the multiple subset sum problem with different knapsack capacities," *Information Processing Letters*, vol. 73, pp. 111–118, 2000.

[12] S. Martello and P. Toth, "An algorithm for the generalized assignment problem," *Operations Research*, pp. 589–603, 1981.

[13] R. Cohen, L. Katzir, and D. Raz, "An efficient approximation for the generalized assignment problem," *Information Processing Letters*, vol. 100, no. 4, pp. 162–166, 2006.

[14] C. Chekuri and S. Khanna, "A PTAS for the multiple knapsack problem," in *Proceedings of ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2000, pp. 213–222.

[15] N. Nisan and A. Ronen, "Computationally feasible VCG mechanisms," *Journal of Artificial Intelligence Research*, vol. 29, pp. 19–47, 2007.