Simple Channel Sensing Order in Cognitive Radio Networks

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***Abstract*— In cognitive radio networks (CRNs), effective and efficient channel exploitation is imperative for unlicensed secondary users to seize available network resources and improve resource utilization. In this paper, we propose a simple channel sensing order for secondary users in multi-channel CRNs without *a priori* knowledge of primary user activities. By sensing the channels according to the descending order of their achievable rates with optimal stopping, we show that the proposed channel exploitation approach is efficient yet effective in elevating throughput and resource utilization. Simulation results show that our proposed channel exploitation approach outperforms its counterparts by up to 18% in a single-secondary user pair scenario. In addition, we investigate the probability of packet transmission collision in a multi-secondary user pair scenario, and show that the probability of collision decreases as the number of channels increases and/or the number of secondary user pairs decreases. It is observed that the total throughput and resource utilization increase with the number of secondary user pairs due to increased transmission opportunities and multi-user diversity. Our results also demonstrate that resource utilization can be further improved via the proposed channel exploitation approach when the number of secondary user pairs approaches the number of channels.**

***Index Terms*—Channel exploitation, cognitive radio, multi-user diversity, optimal stopping, probability of collision, resource utilization.**

I. Introduction

In CONVENTIONAL wireless communication systems, radio spectrum resources are usually governed by license holders. This resource management allows licensed users to access the spectrum with no or minimal interference. However, recent studies show that many frequency bands in the radio spectrum are underutilized most of the time [1]. Due to the inefficiency of the current spectrum allocation, the notion of *cognitive radio* has emerged as an intelligent and promising solution, allowing dynamic spectrum access and hence alleviating the problem of spectrum congestion and low resource utilization. The success of cognitive radio is highly contingent upon the effectiveness of how (unlicensed) secondary users utilize the temporarily available spectrum bands that are licensed to primary users.

In cognitive radio networks (CRNs), primary users are guaranteed access to assigned radio resources, whereas secondary users can only access the spectrum when no primary users are active (i.e., no primary activity). As such, to guarantee the quality-of-service (QoS) of primary users, it is indispensable for secondary users to ensure that the spectrum is free of primary activities before transmitting their information. In the case of multi-channel networks, secondary users equipped with a simple transceiver have to sense the channels one at a time to determine which channel is available, if any. Therefore, an effective and efficient channel sensing order is crucial in both resource utilization melioration and QoS support. In fact, to realize efficient dynamic spectrum access in CRNs, spectrum exploration and spectrum exploitation are imperative [2]. Spectrum exploration requires a secondary user to determine whether or not a channel is free of primary activities as fast as possible, whereas spectrum exploitation refers to how efficiently a secondary user can access and utilize a channel or a set of channels. In this work, we focus on the latter and investigate a simple channel sensing order so as to efficiently yet effectively exploit the temporarily unoccupied spectrum.

In a typical multi-channel system (e.g., an orthogonal frequency division multiplexing (OFDM)-based network), due to distinct small-scale frequency-dependent multi-path propagation characteristics, a (secondary) user can experience different channel gains across different frequency channels [3]. In a multi-user environment, different users can experience different channel conditions over the same channel(s). This phenomenon gives rise to the notion of *multi-user diversity* [4]. The disparity in channel gain is more significant across a wide range of frequency bands, for instance, from the TV frequency bands (e.g., 30 to 300MHz) to the ISM frequency bands (e.g., 2.4 to 2.5GHz). In the context of CRNs, we can take advantage of the difference in the channel gains when devising a channel exploitation strategy for secondary users, thereby plausibly increasing resource utilization. In this paper, we focus on the situation where primary activities are unknown to secondary users, and study the problem of spectrum exploitation (i.e., channel sensing order) by considering heterogeneous channels. Closely related to our research work here are [5]–[8]. Compared to [5,6], we consider no *a priori* knowledge of primary (activity)-free probabilities; compared to [7], we take the intrinsic nature of multi-path channels into consideration; compared to [8], we do not consider any recall (i.e., a previously sensed channel can be accessed) or guessing (i.e., an un-sensed channel can be accessed) of channels when deriving a channel sensing order for secondary user pairs. For presentation clarity, all the proofs are given in Appendix.

The contributions and significance of this work are three-fold:

- First, we propose a simple channel sensing order for

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unlicensed secondary users. By sensing the channels according to the descending order of their achievable rates, we prove that a secondary user should stop at the first sensed free channel for maximal performance. In addition, our proposed channel exploitation approach does not require *a priori* knowledge of any primary activities;
- Second, in a multi-secondary user pair scenario, we analytically derive an expression for the probability of transmission collision, which is a function of primary-free probabilities, the number of channels, and the number of secondary user pairs. Our analysis and simulation results show that the probability of collision decreases as the number of channels increases, the number of secondary user pairs decreases, and/or the values of primary-free probabilities decrease. Our results also demonstrate that, as the number of secondary user pairs increases, both the total throughput and resource utilization rise because of increased transmission opportunities and multi-user diversity;
- Third, we compare our proposed channel exploitation approach with two other approaches. Simulation results show that the newly proposed sensing order achieves the best reward (i.e., throughput) performance, outperforming its counterparts. We also show that our simple sensing order can preserve its performance superiority over the other two approaches independent of the level of resource utilization in the system (i.e., good or poor resource utilization by primary users).

II. Related Work

In the literature, the topic of spectrum exploration (i.e. channel sensing) in CRNs has drawn a plethora of attention [9]–[11]. Without *a priori* knowledge of primary transmissions, energy detection-based sensing is shown to be optimal [10]. In this work, we consider that primary activities are unknown to secondary users, and each sensing attempt via energy detection-based sensing (e.g., [11]) is successful with a certain probability. As such, our focus is to devise a simple channel sensing order for efficient spectrum exploitation.

Concerning spectrum exploitation, many channel access strategies have been proposed [5]–[8,12]–[17], aiming to increase resource utilization and throughput performance. In [12], a channel selection problem is modeled as a multiarmed bandit problem, where an optimal strategy driven by Gittins indices is proposed to strike a desired balance between spectrum exploration and spectrum exploitation. The tradeoff between spectrum exploration and spectrum exploitation is also discussed in [13]. An optimal stopping rule is suggested in [14] to achieve high throughput performance gains in a conventional (non-cognitive) wireless network. However, directly applying the suggested stopping rule to CRNs can be ineffective or inefficient. In [15], a Markov-based model is proposed to characterize the channel availabilities of primary users. With knowledge of channel availabilities, secondary users can opportunistically sense and select the channels. In [16], it is shown that, with accurate knowledge of primary activities (i.e., channel availabilities), secondary users can better exploit the channels to avoid disrupting primary activities and to increase resource utilization. In [17], a multi-band joint detection approach for secondary users is proposed. Similar to [15], by sensing and exploiting multiple channels, the proposed multi-band joint detection approach is shown effective in improving spectrum utilization. For the sake of practical implementation, however, simple spectrum exploitation strategies are desired for low-cost off-the-shelf wireless devices, where channels can only be sensed one at a time. As we will discuss in Section IV-A2, exploiting multiple channels for data transmission generally requires *a priori* information of primary activities. In practice, however, the availability of each channel is hardly predictable [15].

Closely related to our work here are [5]–[8]. In [5], a channel sensing order with respect to the channel availabilities (or primary-free probabilities) is proposed for a single secondary user, referred to as *intuitive channel sensing*. As an extended work of [5], [6] studies the issue of channel sensing orders for two secondary users, where three medium access control (MAC)-layer contention-resolution protocols are proposed. Compared to [5,6], our work here considers neither *a priori* information nor the estimates of primary-free probabilities. In [7], channel exploitation is studied from an MAC-layer perspective, where a cognitive MAC protocol is proposed. Despite optimal stopping in place, homogeneous channels are considered, meaning that the intrinsic features of a multi-path channel are not taken advantage of, thereby leading to suboptimal system performance. By contrast, we consider heterogeneous channels in this paper. In [8], an optimal channel exploitation approach for a single secondary user pair is proposed with the knowledge of primary-free probabilities. Channel recall and channel guessing are considered in the proposed approach. In our work, however, we consider neither channel recall nor channel guessing. Besides, we consider no *a priori* information of primary-free probabilities. We also investigate the problem of channel exploitation for multiple secondary user pairs in a distributed manner.

III. System Model

In wireless networks targeting high-speed communications, OFDM has been demonstrated as a promising modulation technology to support high data-rate transmission with resistance to delay dispersion due to multi-path propagation. OFDM essentially converts a frequency-selective fading channel into a number of flat fading channels by employing multiple subcarriers [18], thereby reducing inter-symbol interference. With unique propagation characteristics across frequency spectrum, a secondary user pair can experience different channel gains on different frequency channels. In light of these channel gain variations, an effective channel exploitation strategy can be developed.

Consider a synchronized CRN with M non-mobile secondary user pairs and N available channels. Time is partitioned into slots of duration T. In each timeslot, each channel is either available (i.e., no primary activities) or busy (i.e., with primary activities). The availability of a channel is i.i.d. in both frequency channel and timeslot dimensions. In each timeslot, the source node of a secondary user pair senses the channels according to its sensing sequence. Denote $(s_1, s_2, ..., s_N)$ as a sensing sequence, which is a permutation of the set

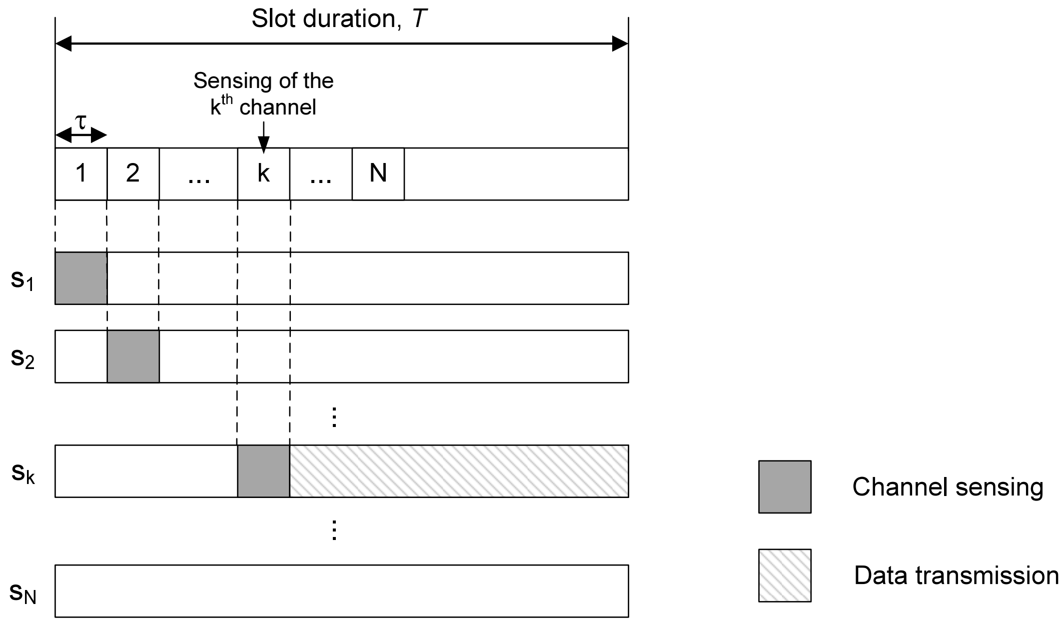

Fig. 1. An illustration of the channel sensing procedure for a secondary user.

$(1, 2, ..., N)$. Denote τ as the time needed for sensing a channel, where $N\tau < T$. Here, we consider that a sensing attempt (by means of energy detection [11]) is accurate[1] with probability p, and the success/failure of each sensing attempt is independent of other attempts. Each secondary user is equipped with a simple transceiver and, therefore, it can only sense one channel at a time. Fig. 1 depicts the slot structure and channel exploitation under consideration, with a negligible channel switching time, where the source node of a secondary user pair senses the first $(k-1)$ channels busy, and it stops at the k^{th} sensed channel as the optimal stopping criterion is satisfied (to be discussed in Section IV). As such, the secondary user transmits its information over the k^{th} sensed channel in the reminder of that timeslot. The period of time spent on channel sensing and that on data transmission are $k\tau$ and $T - k\tau$, respectively. Notice that, with a constant slot time, the actual data transmission time changes with k [14]. In the case of multiple secondary user pairs, each secondary user pair senses and, if feasible, accesses the wireless medium according to its own sensing sequence. A collision occurs should more than one user pair transmits over the same channel simultaneously (to be discussed in Section IV). Denote c_k as a scaling factor if a secondary user pair stops at the k^{th} sensed channel, which is given by

$$c_k = 1 - \frac{k\tau}{T}. \tag{1}$$

In other words, the larger the value of k, the smaller the value of c_k, and the lower the effective data transmission rate (due to sensing overhead). With non-mobile secondary user pairs, the channel gain of each transmission link can be estimated accurately via pilot symbols or training sequences [19]. In specific, once a channel is sensed free for the first time, the source of a secondary user pair transmits a packet with pilot symbols (or training sequences) to its destination node so as to estimate the channel gain of this transmission link. Since we consider non-mobile nodes, the channel gains remain more or less the same within a long *coherence time interval* [3], whereby only one estimation attempt per channel is sufficient for a long period of time. On the other hand, a secondary user pair can be active only when there is no primary activity. Together with the background noise power and its transmit power, the achievable transmission rate of a secondary user pair over each channel can be determined based on the physical layer model and parameters. Here, for simplicity, we use the well-known Shannon capacity formula [20]. Therefore, when the same channel is sensed free subsequently, the source of the secondary user pair of interest can transmit its information messages to its destination node at a desired transmission rate. A summary of important symbols is given in Table I for easy reference.

[1]If a sensing attempt is accurate, a secondary user can determine whether the sensed channel is available or busy in τ. In other words, there is no Type I error or false alarm (Type II error or missed detection) when the channel is sensed busy (idle) if a sensing attempt is accurate. This work focuses on the case when p is close to 1. Addressing Type I and Type II errors in channel sensing is beyond the scope of this work.

IV. Proposed Stopping Rule and Channel Exploitation

In CRNs, the gist of channel exploitation is to utilize a desired unoccupied channel as effective and efficient as possible. In fact, the problem of channel exploitation can be viewed as a general stopping rule problem, the objective of which is to stop at some channel that maximizes the (expected) reward for a secondary user pair. In our work, with a finite number of channels, we can formulate the channel exploitation problem as a finite-horizon stopping problem [21]. In the

TABLE I
SUMMARY OF IMPORTANT SYMBOLS.

Symbol	Definition
p	probability of accurate sensing
s_k	the k^{th} channel to be sensed in a sensing sequence
θ_{s_k}	primary-free probability of the k^{th} sensed channel
c_k	effectiveness of data transmission at the k^{th} sensed channel
R_{s_k}	achievable transmission rate of the k^{th} sensed channel
λ_k	instantaneous reward for a secondary user pair at the k^{th} sensed channel
Λ_{k+1}	expected reward for a secondary user pair if skipping the k^{th} sensed channel
τ	time spent on channel sensing
T	duration of a timeslot
N	number of available channels
M	number of secondary user pairs
P_c	probability of packet transmission collision

following, we first propose a stopping rule and a channel exploitation approach for one secondary user pair. Then, we extend our investigation to a two-secondary user pair case and a multi-secondary user pair case in Section IV-B and Section IV-C, respectively.

A. Single-Secondary User Pair Scenario ($M = 1$)

1) Single-channel transmission: We first consider that a secondary user pair needs only one channel to transmit its data. Since the transmission rates of all the channels can be known via channel estimation, it is proposed that the source node of a secondary user pair senses the channels according to the descending order of their achievable rates. Therefore, for $i < j$, $R_{s_i} \geq R_{s_j}$, $\forall i, j$, where R_{s_k} is the achievable transmission rate of the k^{th} sensed channel with the secondary user pair using full transmit power. If the ${s_k}^{th}$ channel is sensed free, a secondary user pair can stop at the ${s_k}^{th}$ channel and transmit its data at the rate of R_{s_k}; otherwise, it proceeds to sense the ${s_{k+1}}^{th}$ channel. To characterize the dynamics of our channel exploitation, we employ the notion of rewards in the context of stopping rule. Denote λ_k as the instantaneous reward for a secondary user pair at the k^{th} sensed channel. Assuming there is no false alarm or missed detection, the instantaneous reward at the k^{th} sensed channel for $1 \leq k < N$ can be written as

$$\lambda_k = \begin{cases} c_k R_{s_k}, & \text{sensed free with probability } p\theta_{s_k} \\ \Lambda_{k+1}, & \text{otherwise with probability } (1 - p\theta_{s_k}) \end{cases} \quad (2)$$

and that for $k = N$ as

$$\lambda_N = \begin{cases} c_N R_{s_N}, & \text{sensed free with probability } p\theta_{s_N} \\ 0, & \text{otherwise with probability } (1 - p\theta_{s_N}) \end{cases} \quad (3)$$

where θ_{s_k} $(\in [0, 1])$ is the *primary-free probability*[2] of the k^{th} sensed channel and Λ_{k+1} is the expected reward if a secondary user pair skips the k^{th} sensed channel and uses one of the remaining $(N-k)$ channels. Notice that $p\theta_{s_k}$ represents the probability that the sensing attempt at the ${s_k}^{th}$ channel is accurate (with probability p) and the ${s_k}^{th}$ channel is free (with probability θ_{s_k}). The expected reward Λ_k is given by

$$\Lambda_k = \mathbb{E}[\lambda_k] = \begin{cases} p\theta_{s_k} c_k R_{s_k} + (1 - p\theta_{s_k})\Lambda_{k+1}, & 1 \leq k < N \\ p\theta_{s_N} c_N R_{s_N}, & k = N. \end{cases} \quad (4)$$

Notice that $c_k R_{s_k}$ represents the (effective) throughput obtained if a secondary user pair is active over the k^{th} sensed channel, and Λ_1 can be interpreted as the average throughput obtained by a secondary user pair and can be re-written as

$$\Lambda_1 = \sum_{k=1}^{N} \left[\prod_{i=1}^{k} \left(1 - p\theta_{s_{i-1}}\right) \right] p\theta_{s_k} c_k R_{s_k} \quad (5)$$

where we define $\theta_{s_0} = 0$. For the finite-horizon stopping problem, the stopping rule is completely specified by $\{\Lambda_k\}_{k=1}^{N}$, as the expected rewards can be obtained.

Proposed Stopping Rule – If a secondary user pair needs only one channel for data transmission, we propose that the secondary user pair should stop sensing at the first free channel. With the proposed channel sensing order, it can be proved that the secondary user pair achieves the maximal (expected) reward by stopping at the first sensed free channel and, hence, our stopping rule is optimal.

Proposition 1: Provided that the channels are sensed according to the descending order of their achievable transmission rates, a secondary user pair can achieve the maximal reward by stopping and transmitting at the first free channel.

According to Proposition 1, the proposed stopping rule is optimal. More importantly, the optimal stopping rule does not require *a priori* information of primary-free probabilities. The key implication is that, with the proposed sensing order, the estimation of θ_{s_k} is no longer necessary, while the emphasis can be put on improving the accuracy and efficiency of sensing capability (i.e., increasing the value of p and reducing the value of τ). On the other hand, if all the θ_{s_k}'s are equal, it is obvious that the proposed channel sensing order is throughput-optimal (i.e., optimal in terms of expected reward Λ_1).

Proposition 2: Provided that $\theta_{s_i} = \theta_{s_j}, \forall i, j$, the proposed sensing order is throughput-optimal.

2) Multi-channel transmission: When a secondary user pair can use multiple channels for data transmission, one conventional approach is that the secondary user pair should not exploit the first free channel immediately but explore it together with the rest of the available channels. This approach leads to a well-known tradeoff between channel exploration and channel exploitation [2,13]. As we will show in the following, blindly exploring channels can greatly degrade throughput performance. Our analysis also indicates that one critical condition of opportunistically employing multiple channels for data transmission is the knowledge of primary activities. Here, we formulate the multi-channel exploitation problem as a 1-stage look-ahead stopping problem, motivated by the BURGLAR problem in [21]. Let X_k denote the available/unavailable state of the k^{th} sensed channel, where

$$X_k = \begin{cases} 1, & {s_k}^{th} \text{ channel is sensed free with probability } p\theta_{s_k} \\ 0, & \text{otherwise with probability } (1 - p\theta_{s_k}) \end{cases} \quad (6)$$

[2] In today's wireless networks supporting mixed multimedia traffic, it is well-known that the distribution of idle/busy periods of a channel can hardly be predictable [15,22]. Thus, in this research work, we consider that θ_{s_k}'s are unknown to secondary users.

and Y_k denote the payoff (i.e., aggregate throughput) after k sensing attempts, where

$$Y_k = c_k \sum_{n=1}^{k} X_n R_{s_n}(v_n). \quad (7)$$

In (7), $R_{s_n}(v_n)$ is the achievable transmission rate of the ${s_n}^{th}$ channel with transmit power v_n, where $\sum_{n=1}^{k} v_n$ is upper-bounded by the maximum transmit power of the source node of a secondary user pair[3]. Let K_q^* denote the stopping channel index (i.e., the number of channels sensed) of a q-stage look-ahead stopping rule. Consider the following 1-stage look-ahead stopping rule

$$K_1^* = \min\{\min\{k \geq 1 : Y_k \geq E[Y_{k+1}|X_1 = x_1, X_2 = x_2, ..., X_k = x_k]\}, N\} \quad (8)$$

where x_k is an observation of X_k after sensing the ${s_k}^{th}$ channel. With (8), if its instantaneous payoff obtained at the k^{th} sensing attempt is at least as good as its expected payoff at the $(k+1)^{th}$ sensing attempt, the secondary user pair should stop after k sensing attempts. Regarding the expected payoff at the $(k+1)^{th}$ sensing attempt, we have

$$E[Y_{k+1}|X_1 = x_1, X_2 = x_2, ..., X_k = x_k] = E\left[c_{k+1} \sum_{n=1}^{k+1} X_n R_{s_n}(v_n) | X_1 = x_1, X_2 = x_2, ..., X_k = x_k\right] \quad (9)$$

$$= c_{k+1}\left(\sum_{n=1}^{k} x_n R_{s_n}(v_n) + R_{s_{k+1}}(v_{k+1}) E[X_{k+1}]\right) \quad (10)$$

$$= c_{k+1}\left(\sum_{n=1}^{k} x_n R_{s_n}(v_n) + R_{s_{k+1}}(v_{k+1}) p\theta_{s_{k+1}}\right). \quad (11)$$

Hence, the stopping channel index K_1^* of our 1-stage look-ahead rule is given by (12) shown at the top of the next page, where $\tilde{v}_n$ is the current power allocation solution at the k^{th} sensing and v_n the power allocation solution at the $(k+1)^{th}$ sensing. Notice that, in general, $\tilde{v}_n \neq v_n$. After stopping, a secondary user pair can employ a set of free channels, denoted by $\mathcal{N}_{K_1^*}$, for data transmission, where $\mathcal{N}_{K_1^*}$ is defined as $\mathcal{N}_{K_1^*} \subseteq \{i | X_i = 1, 1 \leq i \leq K_1^*\}$. By the same token, we can also obtain a general q-stage look-ahead stopping rule for $q > 0$, given by (13) shown at the top of the next page. Since our channel sensing problem is not monotonic (i.e., Y_k is not a non-decreasing function with k), to achieve (close to) optimal performance, we need to apply the 1-stage look-ahead stopping rule and stop at K_1^*, then apply the 2-stage look-ahead stopping rule and stop at K_2^*, and so on [21]. As such, finding an optimal stopping channel index can be computationally expensive. Even if we fix the power allocation (e.g., uniform power allocation), in order to compute K_q^*'s, we still need to acquire knowledge of the primary-free probabilities, i.e., $\theta_{s_k}, \forall k$. Thus, to achieve desired system throughput performance, it is not likely for a secondary user equipped with a simple transceiver to use multiple channels for its data transmission without knowing primary activities. As discussed, in today's wireless networks with users switching from one multimedia application to another from time to time, it is difficult for secondary users to have complete knowledge of θ_{s_k}'s [15]. Without knowledge of θ_{s_k}'s, naive channel exploration can lead to performance degradation. Consider a situation where the first channel is sensed free, but instead of transmitting its information immediately, a secondary user keeps searching for the next free channel. Suppose the remaining $(N-1)$ channels are not available. For $N = 512$ and $T/\tau = 5000$, the throughput loss is $\frac{c_1 - c_N}{c_1} = \frac{(N-1)\tau}{T-\tau} = 10.2\%$. Since we focus on efficient channel exploitation using simple devices, in this work, we consider the situation where primary-free probabilities are unknown to secondary users, and each secondary user can only employ one channel for data communications. In multi-channel transmissions, how to obtain a desired tradeoff between channel exploration and channel exploitation without any knowledge of primary activities is left for further work.

[3] Notice that, in multi-channel communications, power allocation is imperative since different transmit power levels lead to different achievable rates over different channels [3].

B. Two-Secondary User Pair Scenario ($M = 2$)

Consider two (active) secondary user pairs who are non-mobile and are 1-hop neighbors (i.e., no hidden terminal). Due to the random nature of multi-path propagation, the channel gains of the available channels for secondary user pair 1 are plausibly different from that for secondary user pair 2 (see Fig. 2) [4]. As such, it is very unlikely for the two secondary user pairs to have the same channel sensing sequence if the channels are sensed according to the descending order of their achievable rates. Here, it is proposed that each secondary user pair senses and accesses the channels based on its own channel sensing order. Without any coordination between the secondary user pairs, however, it is still possible that a particular channel is sensed free by these two secondary user pairs simultaneously, thereby leading to packet collisions. To avoid collisions, one approach is to exhaust all the possible combinations of their sensing sequences and find the combination that can maximize the total reward/throughput performance. This approach is undoubtedly computationally expensive, and the computational cost increases exponentially with the number of secondary user pairs. Such an exhaustive search also requires the knowledge of θ_{s_k}'s. Therefore, instead of devising optimal sensing sequences, we study the probability of transmission collision with respect to and investigate the reward performance of our proposed simple channel exploitation approach for two or more secondary user pairs. To derive this probability of collision, for simplicity, we assume that the channel sensing sequences are independent and equally likely, and the channels are randomly placed in a channel sensing sequence. Denote $\mathbf{s}^{(m)} = \left(s_1^{(m)}, s_2^{(m)}, ..., s_N^{(m)}\right)$ as the sensing sequence of the m^{th} secondary user pair, $m \in \{1, 2\}$, and $\mathcal{S}$ the set of all possible sensing sequences. Similar to the single-secondary user pair scenario discussed in Section IV-A, if the k^{th} channel in a sensing sequence is sensed free (i.e., no activities from primary and other secondary user pairs with a successful sensing attempt), the source node of the secondary user pair of interest transmits its information; otherwise, it proceeds to sense the next channel. Therefore, a collision occurs if the

$$\begin{aligned} K_1^* &= \min\left\{\min\left\{k \geq 1 : Y_k \geq c_{k+1}\left(\sum_{n=1}^{k} x_n R_{s_n}(v_n) + R_{s_{k+1}}(v_{k+1})\, p\theta_{s_{k+1}}\right)\right\}, N\right\} \\ &= \min\left\{\min\left\{k \geq 1 : c_k \sum_{n=1}^{k} x_n R_{s_n}(\tilde{v}_n) \geq c_{k+1}\left(\sum_{n=1}^{k} x_n R_{s_n}(v_n) + R_{s_{k+1}}(v_{k+1})\, p\theta_{s_{k+1}}\right)\right\}, N\right\} \\ &= \min\left\{\min\left\{k \geq 1 : \sum_{n=1}^{k} x_n R_{s_n}(\tilde{v}_n) \geq \left(\frac{c_{k+1}}{c_k}\right)\left(\sum_{n=1}^{k} x_n R_{s_n}(v_n) + R_{s_{k+1}}(v_{k+1})\, p\theta_{s_{k+1}}\right)\right\}, N\right\} \end{aligned} \tag{12}$$

$$K_q^* = \min\left\{\min\left\{k \geq 1 : \sum_{n=1}^{k} x_n R_{s_n}(\tilde{v}_n) \geq \left(\frac{c_{k+q}}{c_k}\right)\left(\sum_{n=1}^{k} x_n R_{s_n}(v_n) + \sum_{l=k+1}^{\min\{k+q,N\}} p\theta_{s_l} R_{s_l}(v_l)\right)\right\}, N\right\} \tag{13}$$

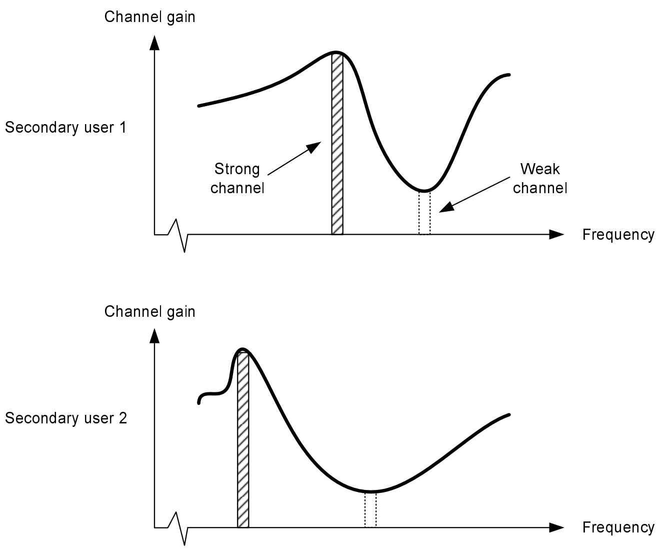

Fig. 2. An illustration of the channel gains of two secondary users.

two secondary user pairs are active over the same channel in the same timeslot[4]. Letting $P_{c|\mathbf{s}^{(1)}}$ denote the conditional probability of collision given $\mathbf{s}^{(1)}$, we have (15)-(16) shown at the top of the next page, where $\theta_{s_0^{(1)}} = 0$ by definition. Thus, the average probability of collision for a secondary user pair, denoted by P_c, is given by

$$P_c = \sum_{\mathbf{z} \in \mathcal{S}} P_{c|\mathbf{z}} P(\mathbf{s}^{(1)} = \mathbf{z}) \tag{14}$$

where $P(\mathbf{s}^{(1)} = \mathbf{z})$ is the probability that the first secondary user pair employs the channel sensing sequence $\mathbf{z}$. In the case of $p\theta_{s_k^{(1)}} = 1, \forall k > 0$, (14) corresponds to the probability of collision in conventional multi-channel MAC with two users employing random channel selection [23], given by $P_c = \frac{1}{N}$.

C. Multi-Secondary User Pair Scenario ($M > 2$)

Here, each secondary user pair senses and accesses the channels according to the descending order of the achievable rate. For M (> 2) secondary user pairs, a collision occurs when at least two secondary user pairs transmit over the same channel at the same time. At a large N, the probability of collision that three or more secondary user pairs are active over the same channel simultaneously is expected to be low[5]. Therefore, to simplify the derivation for P_c, we consider collisions due to two simultaneous transmissions only. Assuming that the sensing sequences are independent and equally likely, and the channels are randomly placed in a channel sensing sequence, we obtain the conditional probability of collision given $\mathbf{s}^{(1)}$, which can be approximated by $(M-1)P_{c|\mathbf{s}^{(1)}}$, where $P_{c|\mathbf{s}^{(1)}}$ is given in (16). Thus,

$$P_c \approx (M-1) \sum_{\mathbf{z} \in \mathcal{S}} P_{c|\mathbf{z}} P(\mathbf{s}^{(1)} = \mathbf{z}). \tag{17}$$

The probability of collision given in (17) should provide a good approximation when N is relatively larger than M. Notice that, given equal primary-free probabilities, the probability of transmission collision when multiple secondary user pairs employ a random sensing order can be approximated by (17).

V. Performance Evaluation

A. Simulation Environment

We consider secondary users randomly located in a 1km x 1km coverage area, and adopt the channel model suggested in [24] (i.e., hilly/moderate-to-heavy tree density). Packets are generated from a secondary user and transmitted to a destination located at the center of the coverage area. The maximum transmission rate of each channel is 200kb/s. The

[4]In general, there is another scenario that a packet collision can occur between two secondary user pairs: one secondary user pair transmits after a channel is sensed free with accurate sensing, while the other secondary user pair senses the same channel at a later time with a missed detection. In this work, however, we consider the case where p is close to 1, whereby the probability of such an event is very small and can be neglected.

[5]At a large N (e.g., $N \geq 128$), we observe that the probability of collision that two secondary user pairs transmit over the same channel simultaneously is less than 0.01 as shown in Table II (to be discussed in Section V); therefore, it is anticipated that the probability of collision that three or more secondary user pairs are active over the same channel at the same time is even lower than 0.01.

$$P_{c|\mathbf{s}^{(1)}} = P(s_1^{(1)} \text{ sensed free}, s_1^{(1)} = s_1^{(2)}|\mathbf{s}^{(1)}) + P(s_1^{(1)}, s_1^{(2)} \text{ sensed busy}, s_2^{(1)} = s_2^{(2)}, s_2^{(1)} \text{ sensed free}|\mathbf{s}^{(1)}) + \ldots + P(s_1^{(1)}, s_1^{(2)}, ..., s_{N-1}^{(1)}, s_{N-1}^{(2)} \text{ sensed busy}, s_N^{(1)} = s_N^{(2)}, s_N^{(1)} \text{ sensed free}|\mathbf{s}^{(1)}) \quad (15)$$

$$= \sum_{k=1}^{N} \frac{1}{N-k+1} \left(\prod_{i=1}^{k} \left(1 - p\theta_{s_{i-1}^{(1)}}\right)\right) \cdot \left(\frac{1}{\binom{N}{k-1}} + \sum_{j=2}^{\min(k,N-k+1)} \frac{\binom{k-1}{k-j}}{\binom{N}{k-1}} \sum_{i_1 > k} \sum_{i_2 > i_1} \cdots \sum_{i_{j-1} > i_{j-2}} \left(1 - p\theta_{s_{i_1}^{(1)}}\right)\left(1 - p\theta_{s_{i_2}^{(1)}}\right) \cdots \left(1 - p\theta_{s_{i_{j-1}}^{(1)}}\right)\right) p\theta_{s_k^{(1)}} \quad (16)$$

maximum transmit power constraint of a secondary user is 1mW. The background noise power level is assumed to be 10^{-12}W. Other system parameters are chosen as follows: $\tau = 1\mu$s and $T = 5$ms. The performance measurements are

- reward – the number of successful bits transmitted per second (i.e., throughput);
- channel utilization – the ratio of the throughput obtained by a secondary user pair on a channel to the maximum channel transmission rate;
- resource utilization – the fraction of system throughput contributed by secondary user pairs; and
- collision probability – the probability that two or more secondary user pairs initiate data transmission over the same channel simultaneously.

We perform the simulations for 100,000 runs, where each run sustains 50,000 timeslots. We then average different channel rates and channel availabilities over the timeslots, and different sensing sequences (which depend on user locations) over the simulation runs.

B. Simulation and Analytical Results

We evaluate the performance of the proposed spectrum exploitation approach with optimal stopping versus θ, N, and M. In the simulations, we consider two cases for θ: 1) equal θ, where $\theta_{s_k} = \theta, \forall k$ and $\theta = H$ with $H \in \{0.001, 0.005, 0.01, 0.03, 0.1, 0.3, 0.6, 0.9, 1\}$; and 2) unequal θ, where θ is uniformly distributed on $[0, H]$.

1) Effect of primary-free probability, θ: For $N = 512$, we study the impact of the value of θ on the reward performance for $p = 1$ (i.e., perfect sensing) and $p = 0.98$ (i.e., imperfect sensing). Fig. 3 depicts the simulation and analytical results for the reward performance versus the value of θ. The simulation and analytical results agree with each other. The rewards in both cases of equal θ and unequal θ increase with the value of primary-free probability. However, the rates of the reward performance improvement decrease with θ. The rationale for such a performance saturation is that, when the value of θ is small, a marginal increase in θ can greatly increase the transmission opportunities of a secondary user pair. In addition, with the increased primary-free probabilities, there are more free channels for a secondary user pair to choose from, resembling the case of multi-user diversity. Therefore, the reward performance increases sharply. At a large θ, however, a marginal increase in the value of θ only has a minimal impact on the channel availabilities and the channel gain variations across those free channels. Thus, the reward performance almost levels off from $\theta = 0.6$ onward. We also observe that perfect channel sensing (i.e., $p = 1$) outperforms imperfect channel sensing (i.e., $p = 0.98$) in terms of reward performance[6]. The channel utilization versus θ is shown in Fig. 4. The trend of channel utilization is the same as that of reward performance shown in Fig. 3. As observed in Fig. 4, our proposed approach with perfect sensing can achieve almost 100% channel utilization at a large θ, making efficient use of unoccupied channels. There is a performance gap between the curve with equal θ and that with unequal θ for the same value of p. The performance difference is mainly due to the variation of primary activities in the case of unequal θ. In other words, on average, more (less) channels are available in the case of equal (unequal) θ. On the other hand, for the same mean value of θ, we observe that the channel utilization for equal θ and that for unequal θ behave differently, depending on the mean value of θ. At a small θ, the curve for unequal θ outperforms that for equal θ. For example, at the mean value of 0.005 with $\theta = 0.01$ ($\theta = 0.005$) for the unequal (equal) θ case, the curve for unequal θ attains 79%, whereas the curve for equal θ attains only 75%. By contrast, at a large θ, the curve for equal θ achieves higher channel utilization than that for unequal θ. For example, at the mean value of 0.3 with $\theta = 0.6$ ($\theta = 0.3$) for the unequal (equal) θ case, the curves for equal θ and unequal θ attain 98% and 96%, respectively. These observations can be explained by the impact of increased transmission opportunities on the reward performance, discussed precedingly. The curves of channel utilization for $p = 0.98$ are also plotted for reference.

2) Effect of the number of channels, N: For $\theta = 0.3$ and $\theta \in [0, 0.3]$, Fig. 5 shows the impact of the number of available channels on the reward performance. The curves for the reward performance in both cases of equal θ and unequal θ rise with the value of N. Similar to the discussion in Section V-B1, the reward increase is due to the fact that the more the available channels, the more the variations in the channel gains across the (free) channels, realizing multi-user diversity. In Fig. 6, the channel utilization is also plotted for reference. It is observed that the channel utilization can be further improved at a larger N. In Figs. 5 and 6, it is seen that imperfect channel sensing (i.e., $p = 0.98$) causes performance degradation in terms of throughput and channel utilization. Nonetheless, our simulation results and analytical results closely match, validating our performance analysis.

[6]The effect of missed detections is not taken into account in this performance evaluation.

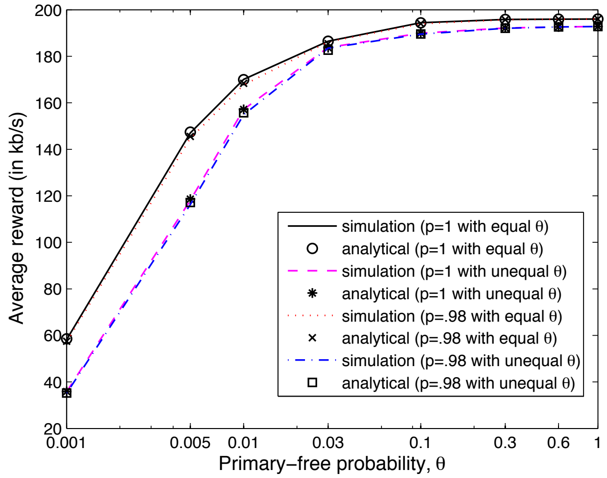

Fig. 3. Reward performance of the proposed channel sensing in a single-secondary user pair scenario vs. the value of θ (where $N = 512$).

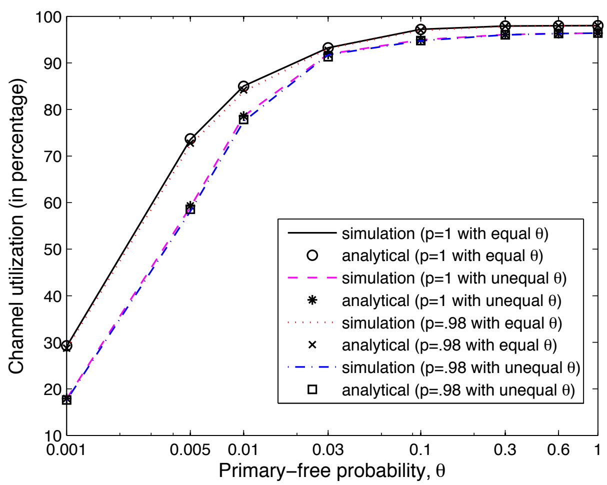

Fig. 4. Channel utilization of the proposed channel sensing in a single-secondary user pair scenario vs. the value of θ (where $N = 512$).

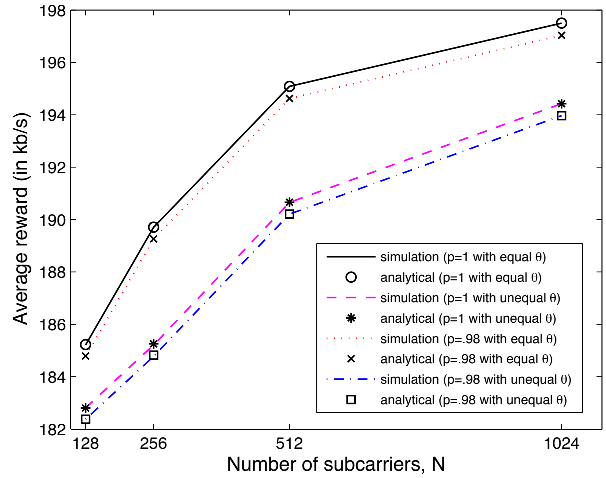

Fig. 5. Reward performance of the proposed channel sensing in a single-secondary user pair scenario vs. the value of N (where $\theta = 0.3$).

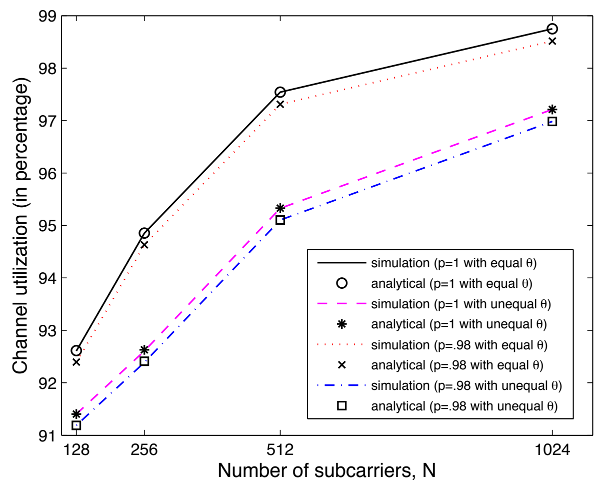

Fig. 6. Channel utilization of the proposed channel sensing in a single-secondary user pair scenario vs. the value of N (where $\theta = 0.3$).

3) Performance comparison: For $N = 512$ and $p = 1$, we compare our proposed channel exploitation approach with a random channel exploitation approach and the Jiang's intuitive sensing order suggested in [5]. In the random approach, the sensing sequence of a secondary user pair is a random permutation of the set $(1, 2, ..., N)$, whereas in the Jiang's approach, an intuitive sensing order is proposed such that a secondary user pair senses the channels according to the descending order of their primary-free probabilities. Notice that our proposed channel exploitation approach and the Jiang's approach are of similar computational complexity: instead of sorting the primary-free probabilities of the available channels as in the Jiang's approach [5], each secondary user pair in our approach sorts the achievable transmission rates of the available channels. In the performance comparison, we further consider two different settings, namely good resource utilization by primary users (e.g., $\theta = 0.005$ and $\theta \in [0, 0.005]$) and poor resource utilization by primary users (e.g., $\theta = 0.9$ and $\theta \in [0, 0.9]$). Four observations can be made from the channel utilization shown in Fig. 7.

First, the channel utilization for equal θ is generally better than that for unequal θ, as expected. Second, since the Jiang's approach is sensitive to the order of primary-free probabilities, it performs better than the random approach in the cases of unequal θ. On the other hand, the random approach and the Jiang's approach attain almost the same channel utilization in the equal θ case. Note that a secondary user pair employing the Jiang's intuitive sensing is to sense the channels according to their primary-free probabilities. As such, if all the primary-free probabilities are equal, the sensing sequence of a secondary user pair essentially becomes $(s_1, s_2, ..., s_N) = (1, 2, ..., N)$. Since the channel gains in our simulations are generated randomly, sensing the channels in a sequential manner is more or less the same as in a random manner. The newly proposed channel exploitation approach achieves the highest channel utilization in all the cases. The performance gains are due to the fact that our approach takes into account the unique characteristics of the channels. By sensing the channels according to the descending order of the achievable rate, a secondary user pair can take advantage of the channel variations, whereby the performance can be improved. Third, when the primary-free probabilities are low (i.e., $\theta = 0.005$ and $\theta \in [0, 0.005]$), all three channel exploitation approaches achieve similar channel utilization. The justification is that, since the number of free channels is very small and the transmission opportunities for a secondary user pair are low, the channels being sensed free in all of the three approaches are more or less the same. Our proposed approach performs slightly better than the others, as the benefits of channel variations can be exploited to a certain extent. Fourth, a large θ leads to higher channel utilization achieved by a secondary user pair. With more free channels and hence more channel variations, our proposed approach outperforms its two counterparts by at least 18%. Concerning the reward performance, we observe the same trend for the three approaches as in Fig. 7. Should imperfect sensing be considered (e.g., $p = 0.98$), performance degradation is observed in all of the three approaches.

4) Collision probability P_c in a two-secondary user pair case: Here, the two secondary user pairs employ the same proposed channel exploitation approach. In the case of equal θ and perfect sensing (i.e., $p = 1$), we study the impacts of θ and N values on the probability of transmission collision. Both simulation and analytical results are given in Table II. There are two main trends. First, for the same value of θ, P_c generally deceases with N. Second, for the same value of N, P_c generally increases with θ. As the number of channels increases, the chances for the two secondary user pairs to sense the same channel free simultaneously decrease, thereby leading to a smaller P_c. On one hand, the transmission opportunities for both secondary user pairs increase with the value of θ. With a fixed number of channels, the increased transmission opportunities cause a higher probability of collision, for the two secondary user pairs are more likely to sense the same channel free at the same time.

Similar performance trends are observed for the case of $p = 0.98$. On a different note, there is some discrepancy between the simulation results and the analytical results. In general, there is a positive correlation among neighboring channels. The rationale is that, if a channel is of good (bad) quality, its neighboring channels will possibly be of good (bad) quality as well. Thus, choosing which channel to sense next generally depends on the previously sensed channels. This phenomenon exacerbates as the number of channels decreases. Therefore, the assumptions that the channels are randomly placed in a sensing sequence and the sensing sequences are equally likely might be void. However, such a correlation diminishes as the separation distance between two frequency channels increases. As we will see in Section V-B5, this unique feature in our proposed channel sensing order can lead to improved system performance in a multi-secondary user pair scenario. On the other hand, we observe that the total reward obtained in a two-secondary user pair scenario is higher than that in a single-secondary user pair scenario given the same value of θ. Since the system is not saturated, a higher reward can be attained with more secondary user pairs in the system.

5) Collision probability and resource utilization in a multi-secondary user pair case: For the case of equal θ with $\theta = 0.9$ and $p = 1$, we study the impact of the number of secondary user pairs M (≥ 2) on the probability of collision and resource utilization. In specific, we gauge the system performance of our proposed sensing order and a random sensing order with different values of N and M. Simulation results for the collision probability are depicted in Fig. 8. Note that the probability of transmission collision for a random sensing approach can be approximated by (17) derived in Section IV-C. As expected, the probability of collision increases with M. We also notice that P_c rises more drastically with M when N is small, compared to the case when N is large. This phenomenon is ascribed to the elevated chances of simultaneous transmissions over the same sensed free channel(s) when the number of channels is small. Similar to the previous discussions, the probability of collision increases as N decreases. We observe that, when M approaches N, the proposed sensing approach begins to outperform a random sensing approach in terms of P_c. This phenomenon suggests that the sensing sequences with respect to the channel data rates be less susceptible to collisions when M is comparable to N. However, under what condition(s) this phenomenon holds needs more comprehensive investigation. On the other hand, when N is large compared to M, the proposed sensing approach and the random sensing approach achieve similar P_c performance, as expected. We also notice that the trend of P_c versus M for a small θ is similar to that shown in Fig. 8. Due to less transmission opportunities, a lower P_c is achieved in the case of a smaller θ. It is observed that the total reward generally increases (decreases) with M for $M \leq N$ ($M > N$). Since the system is not saturated, the more the secondary user pairs, the higher the total reward. However, when $M > N$, more packet collisions occur, degrading the total reward performance. We also observe that the rate of increment in the total reward increases with N due to a lower P_c and increased transmission opportunities (and hence multi-user diversity). On the contrary, the impact of N has the opposite effect on the resource utilization. Fig. 9 shows that the resource utilization of the proposed approach increases (drops) with M for $M \leq N$ ($M > N$), as expected. However, for $M \leq N$, given a value of M, the smaller the value of N, the higher the resource utilization. The difference in the resource utilization is due to the fact that a secondary user can transmit its data over a single channel only. Therefore, in a system with a large number of channels, a small number of secondary user pairs can utilize only a small portion of temporarily unoccupied channels, even if our throughput-optimal sensing approach is employed. By contrast, when the number of channels is small, the resources can be better utilized by the same number of secondary user pairs. Ideally, resource utilization can be improved when M approaches N yet at the cost of a high P_c. However, how to procure desired N and M values such that resource utilization and collision probability are optimized needs further investigation. Resource utilization of a random sensing approach is also plotted for comparison in Fig 9. By considering the intrinsic nature of multi-path channels, it can be observed that the newly proposed approach outperforms its random counterpart in terms of reward performance and resource utilization with a comparable P_c value.

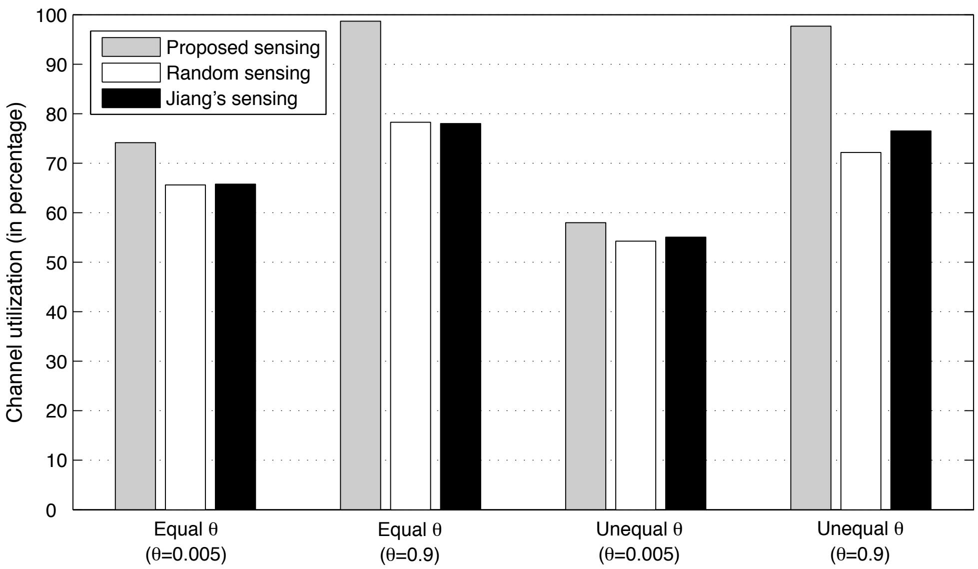

Fig. 7. Comparison of the channel utilization of our proposed sensing approach, a random sensing approach, and the Jiang's sensing approach [5] vs. the value of θ (where $N = 512$ and $p = 1$).

TABLE II
SIMULATION AND ANALYTICAL RESULTS FOR THE PROBABILITY OF COLLISION, P_c, IN A TWO-SECONDARY USER PAIR SCENARIO VS. THE NUMBER OF CHANNELS, N, FOR DIFFERENT VALUES OF θ (WHERE $p = 1$).

N		2	4	16	32	128	256	512	1024
$\theta = 0.005$	simulation	0.0055	0.0058	0.0046	0.0045	0.0034	0.0030	0.0016	0.0005
	analytical	0.0050	0.0050	0.0048	0.0045	0.0033	0.0023	0.0013	0.0006
$\theta = 0.01$	simulation	0.0103	0.0110	0.0100	0.0070	0.0045	0.0029	0.0012	0.0005
	analytical	0.0100	0.0098	0.0091	0.0082	0.0046	0.0025	0.0011	0.0005
$\theta = 0.1$	simulation	0.1092	0.1033	0.0439	0.0205	0.0049	0.0023	0.0012	0.0005
	analytical	0.0950	0.0831	0.0408	0.0202	0.0043	0.0021	0.0010	0.0005
$\theta = 0.3$	simulation	0.2603	0.1902	0.0404	0.0199	0.0059	0.0032	0.0014	0.0006
	analytical	0.2550	0.1721	0.0410	0.0192	0.0046	0.0023	0.0012	0.0006
$\theta = 0.6$	simulation	0.4212	0.2223	0.0480	0.0247	0.0069	0.0033	0.0015	0.0007
	analytical	0.4200	0.2100	0.0459	0.0226	0.0056	0.0028	0.0014	0.0007
$\theta = 0.9$	simulation	0.5033	0.2652	0.0588	0.0306	0.0082	0.0037	0.0020	0.0009
	analytical	0.4950	0.2351	0.0572	0.0285	0.0071	0.0036	0.0018	0.0009
$\theta = 1$	simulation	0.5061	0.2621	0.0672	0.0312	0.0076	0.0042	0.0021	0.0009
	analytical	0.5000	0.2500	0.0625	0.0313	0.0078	0.0039	0.0020	0.0010

VI. DISCUSSION

In practice, channel bundling is often employed in legacy wireless networks (i.e., primary user networks) so as to reduce signalling overhead and facilitate efficient resource allocation. The idea is that, instead of allocating the channels one by one, we group and allocate a set of channels, referred to as a channel bundle, to users, thereby reducing computational cost. Concerning a secondary user network, since all the channels in the same bundle are either available or busy simultaneously, a secondary user can sense each bundle of channels in lieu of each channel individually. For example, every 8 channels can be grouped as a bundle, and the 4^{th} channel can be viewed as its representative. As such, with respect to our proposed sensing order, a secondary user essentially senses the channel bundles according to the descending order of the achievable rates of their 4^{th} channels. Therefore, if the 4^{th} channel of a bundle is sensed free, a secondary user transmits its information over the channels in that bundle, or proceeds to sense the 4^{th} channel of the next bundle otherwise. With the channel bundling in place, the system throughput can be increased, for more free channels can be utilized by each secondary user per timeslot. In general, however, the best 4^{th} channel is not tantamount to the best channel bundle. For instance, some channel bundle can achieve the maximum total achievable rate among the channel bundles, but the rate of its representative (e.g., 4^{th} channel) may not be the maximum among all the individual channels. As a result, in the presence of channel bundling, the optimality of our proposed stopping

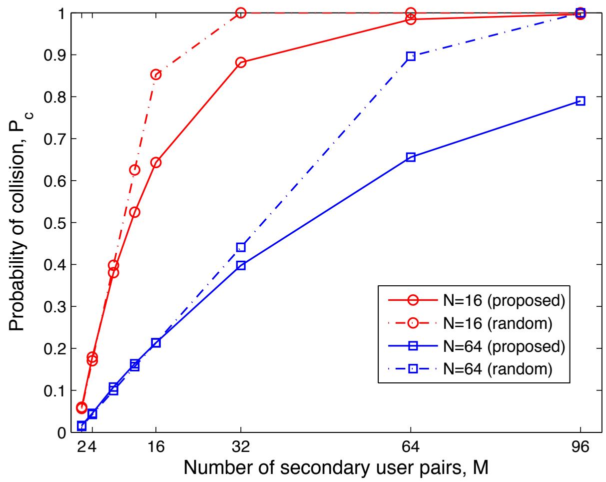

Fig. 8. Collision probability, P_c, of the proposed channel sensing order and a random sensing order in a multi-secondary user pair scenario vs. the number of secondary user pairs, M, for different values of N (where $\theta = 0.9$ and $p = 1$).

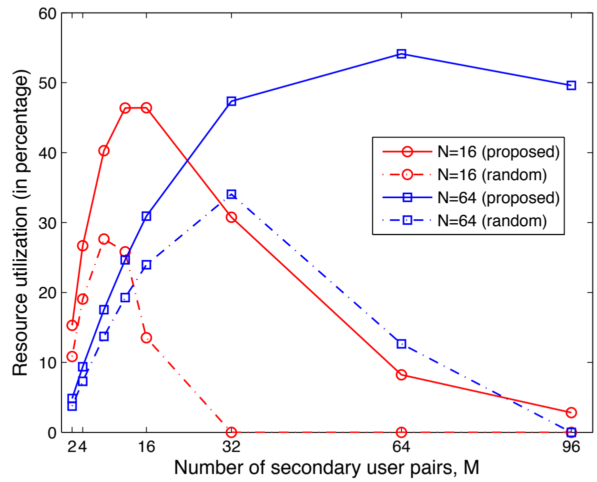

Fig. 9. Resource utilization of the proposed channel sensing approach and a random channel sensing approach vs. the value of M, for different values of N (where $\theta = 0.9$ and $p = 1$).

rule can be void. All in all, optimal channel bundling is strongly desired for the sake of system performance improvement and computational complexity reduction; addressing this issue, however, is left for further work.

In this work, we consider a simple sensing order for secondary user pairs in a general case where θ_{s_k}'s are unknown. If θ_{s_k}'s are available to the secondary user pairs, simple channel sensing according to the descending order of R_{s_k} (i.e., the proposed approach), θ_{s_k} (i.e., intuitive channel sensing [5]), or $\theta_{s_k} R_{s_k}$ cannot attain the optimal throughput. For example, for $p = 1$ and $c_k = 1 - 0.0002k$, consider two channels with rates and primary-free probabilities $R_1 = 20$ and $R_2 = 21$, and $\theta_1 = 0.1$ and $\theta_2 = 0.09$, respectively. Suppose we sense the channels according to the descending order of $\theta_{s_k} R_{s_k}$. As $\theta_1 R_1 > \theta_2 R_2$, the sensing sequence is $(1, 2)$ with average throughput $\Lambda_1 = 3.6999$. The intuitive sensing order is also $(1, 2)$ with $\Lambda_1 = 3.6999$ as $\theta_1 > \theta_2$. However, it is easy to check that we can achieve higher average throughput by using the sensing sequence $(2, 1)$ with $\Lambda_1' = 3.7089$. Thus, channel sensing with respect to $\theta_{s_k} R_{s_k}$'s or θ_{s_k}'s is not an optimal strategy. Consider another example with $R_1 = 100$, $R_2 = 99$, $\theta_1 = 0.001$, and $\theta_2 = 0.99$. The sensing sequence $(2, 1)$ gives higher average throughput than the sensing sequence $(1, 2)$. As such, channel sensing with respect to R_{s_k}'s is not an optimal strategy either.

In fact, an exhaustive search is generally needed to obtain the best throughput performance, for the optimal channel sensing order is now contingent on R_{s_k} and θ_{s_k}, and stopping at the first sensed free channel may no longer be optimal if multiple channels are employed (as discussed in Section IV-A2). The complexity of such an approach can be up to $O(NN!)$, since there are $N!$ possible sensing sequences and, for each sequence, there are N possible stops. For practical implementation, low-complexity yet effective approaches are desired; however, procuring a near-optimal sensing order with the consideration of θ_{s_k}'s is left for further investigation.

Regarding spectrum exploration, a key concern is the accuracy of channel sensing. In this work, we focus on the situation where $p \approx 1$, meaning that channel sensing is considered very accurate. If the channel sensing is not very accurate, however, both false alarms (i.e., a channel is sensed busy without any primary activity) and missed detections (i.e., a channel is sensed free but with primary activities) should be taken into account. The former with probability $(1 - p)\theta_{s_k}$ results in poor resource utilization, whereas the latter with probability $(1 - p)(1 - \theta_{s_k})$ causes the throughput loss of a primary user's transmission and a waste of radio resources. One way to deal with false alarms and missed detections is via the notion of trusting [5]. For example, a sensing result is trusted if a channel is sensed busy. On the other hand, if a channel is sensed free, a sensing result is trusted only to a certain degree. For the sake of desired system performance, channel sensing and spectrum exploitation should be jointly taken into consideration.

VII. Conclusions

In this paper, a simple channel sensing order with optimal stopping is proposed for secondary user pairs without *a priori* knowledge of primary-free probabilities. We show that, by taking unique multi-path propagation channel characteristics into account, the proposed channel exploitation approach is promising, outperforming its two counterparts in a single-secondary user pair scenario. In the case of multiple secondary user pairs, our study shows that, when the number of secondary user pairs approaches the number of channels, resource utilization can be further improved at the expense of a higher collision probability. Our simulation and analytical results also show that imperfect channel sensing can lead to performance degradation in terms of throughput and channel utilization. Further, our investigation reveals that, when secondary user pairs can employ multiple channels for data transmission, we not only require complete knowledge of primary-free probabilities, but also need to jointly consider channel sensing orders, stopping rules, and power allocation for the sake of optimality.

APPENDIX

A. Proof of Proposition 1

To prove Proposition 1 is equivalent to show $\Lambda_1 > \Lambda_2 > ... > \Lambda_N$. We prove $\Lambda_1 > \Lambda_2 > ... > \Lambda_N$ by (backward) mathematical induction. For $k = N - 1$,

$$\begin{aligned}\Lambda_{N-1} &= p\theta_{s_{N-1}} c_{N-1} R_{s_{N-1}} + (1 - p\theta_{s_{N-1}})\Lambda_N \\ &= p\theta_{s_{N-1}}(c_{N-1} R_{s_{N-1}} - p\theta_{s_N} c_N R_{s_N}) + \Lambda_N \\ &> \Lambda_N \quad (\because p\theta_{s_N} \leq 1,\ c_{N-1} > c_N, \text{ and } R_{s_{N-1}} \geq R_{s_N}).\end{aligned}$$

Therefore, the statement is true for $k = N - 1$. Assuming that the statement is true for $k = i$, i.e., $\Lambda_i > \Lambda_{i+1}$ (and hence $c_i R_{s_i} > \Lambda_{i+1}$). For $k = i - 1$, we have

$$\begin{aligned}\Lambda_{i-1} - \Lambda_i &= p\theta_{s_{i-1}} \left(c_{i-1} R_{s_{i-1}} - p\theta_{s_i} c_i R_{s_i} - (1 - p\theta_{s_i})\Lambda_{i+1}\right) \\ &> p\theta_{s_{i-1}} \left(c_i R_{s_i} - p\theta_{s_i} c_i R_{s_i} - (1 - p\theta_{s_i})\Lambda_{i+1}\right) \\ &\qquad\qquad (\because c_{i-1} > c_i \text{ and } R_{s_{i-1}} \geq R_{s_i}) \\ &= p\theta_{s_{i-1}}(1 - p\theta_{s_i})(c_i R_{s_i} - \Lambda_{i+1}) \\ &> 0 \quad (\because \text{by the assumption}).\end{aligned}$$

Therefore, the statement is also true for $k = i - 1$.

By mathematical induction, the statement $\Lambda_i > \Lambda_{i+1}$ is true for $1 \leq i < N$ and, therefore, $\Lambda_1 > \Lambda_2 > ... > \Lambda_N$. Given that the channels are sensed according to the descending order of their achievable rates, a secondary user pair can achieve the maximal reward by stopping at the first sensed free channel. ■

B. Proof of Proposition 2

We prove this proposition by contradiction. Let $(\tilde{s}_1, ..., \tilde{s}_i, ..., \tilde{s}_j, ..., \tilde{s}_N)$ be the proposed sensing order with the expected reward $\tilde{\Lambda}_1$. Suppose there is another sensing order $(\hat{s}_1, ..., \hat{s}_i, ..., \hat{s}_j, ..., \hat{s}_N)$ with the expected reward $\hat{\Lambda}_1$, where $\tilde{s}_k = \hat{s}_k, \forall k \neq i, j$, $\tilde{s}_i = \hat{s}_j$, $\tilde{s}_j = \hat{s}_i$, and $\hat{\Lambda}_1 > \tilde{\Lambda}_1$. Let $\theta_{s_k} = \theta, \forall k > 0$. Here, consider $\hat{\Lambda}_1 - \tilde{\Lambda}_1$. We have (B-1) shown at the top of the next page. Since $\hat{\Lambda}_1 > \tilde{\Lambda}_1$, $0 \leq p\theta \leq 1$, and $c_i > c_j$, we have $R_{\tilde{s}_i} < R_{\tilde{s}_j}$ for $i < j$. However, it contradicts to the proposed sensing order that the channels are sensed according to the descending order of their achievable transmission rates. Therefore, no such a sensing order $(\hat{s}_1, ..., \hat{s}_i, ..., \hat{s}_j, ..., \hat{s}_N)$ with the expected reward $\hat{\Lambda}_1$ exists. In other words, the throughput or expected reward $\tilde{\Lambda}_1$ procured according to the proposed sensing order is indeed maximal. Therefore, our proposed channel sensing order achieves optimality in terms of throughput. ■

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$$\begin{aligned}\hat{\Lambda}_1 - \tilde{\Lambda}_1 &= \sum_{k=1}^{N}\left[\prod_{m=1}^{k}\left(1-p\theta_{\hat{s}_{m-1}}\right)\right]p\theta_{\hat{s}_k}c_k R_{\hat{s}_k} - \sum_{k=1}^{N}\left[\prod_{m=1}^{k}\left(1-p\theta_{\tilde{s}_{m-1}}\right)\right]p\theta_{\tilde{s}_k}c_k R_{\tilde{s}_k}\\ &= \left[\prod_{m=1}^{i}(1-p\theta)\right]p\theta c_i\left(R_{\hat{s}_i}-R_{\tilde{s}_i}\right) + \left[\prod_{m=1}^{j}(1-p\theta)\right]p\theta c_j\left(R_{\hat{s}_j}-R_{\tilde{s}_j}\right)\\ &= (1-p\theta)^i p\theta c_i\left(R_{\tilde{s}_j}-R_{\tilde{s}_i}\right) + (1-p\theta)^j p\theta c_j\left(R_{\tilde{s}_i}-R_{\tilde{s}_j}\right)\\ &= (1-p\theta)^i p\theta\left(R_{\tilde{s}_i}-R_{\tilde{s}_j}\right)\left((1-p\theta)^{j-i}c_j - c_i\right)\end{aligned} \tag{B-1}$$

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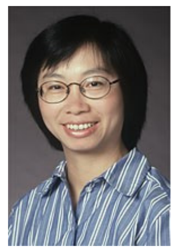

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