$H_\infty$ Constrained Pareto Suboptimal Strategy for Stochastic LPV Time-Delay Systems

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Not only in control problems, but also in dynamic games, several sources of performance degradation, such as model variation, deterministic and stochastic uncertainties and state delays, need to be considered. In this paper, we present an $H_\infty$ constrained Pareto suboptimal strategy for stochastic linear parameter-varying (LPV) time-delay systems involving multiple decision makers. The goal of developing the $H_\infty$ constrained Pareto suboptimal strategy set is to construct a memoryless state feedback strategy set, so that the closed-loop stochastic LPV system is stochastically mean-square stable. In the paper, the existence condition of the extended bounded real lemma is first established via linear matrix inequalities (LMIs). Then, a quadratic cost bound for cost performance is derived. Based on these preliminary results, sufficient conditions for the existence of such a strategy set under the $H_\infty$ constraint are derived by using cross-coupled bilinear matrix inequalities (BMIs). To determine the strategy set, a viscosity iterative scheme based on the LMIs is established to avoid the processing of BMIs. Finally, two numerical examples are presented to demonstrate the reliability and usefulness of the proposed method.

Keywords: Gain-scheduled control; Pareto suboptimal strategy; stochastic linear parameter varying (LPV) system; cross-coupled matrix inequalities (CCMIs); $H_\infty$-constraint.

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1. Introduction

In robust controller design, deterministic and stochastic uncertainties should be addressed, which are caused by the linearization and unmodeled dynamics of the original systems, as well as external uncertainties including stochastic system noise and disturbance. The linear parameter-varying (LPV) system is a reliable mathematical model to capture system variations that are arbitrarily smooth or continuous. It is well known that LPV systems can be accurately represented by using many parameter variations Apkarian et al. [1995]; Briat [2013]. Gain scheduling (GS) control techniques are often used to control LPV systems because they can effectively compensate for parameter variations. In particular, an $H_{\infty}$ GS control problem for stochastic LPV systems is first discussed in [Ku and Wu, 2015] to recover the degradation of stability margin caused by external disturbances. Moreover, the delay-dependent $H_{\infty}$ control problem of deterministic LPV systems with time-varying state delay has been addressed Zope et al. [2012]. The stabilization control problem of the milling process using a state feedback strategy as a practical delay system has been solved. In recent years, with advances in GS control technology, dynamic game problems of stochastic LPV systems involving multiple decision makers have been investigated to guarantee stochastical mean-square stability Mukaidani et al. [2018]; Mukaidani and Xu [2018]. However, no time delay in the system model and control is considered in the existing studies. It is well known that delay is a main cause of degraded stability performance for optimized cost values. Therefore, further research is needed to maintain the robust stability in the presence of delay and multiple decision makers.

Consequently, in this paper, we investigate the $H_{\infty}$ constrained Pareto suboptimal control problem of a stochastic LPV time-delay system involving multiple decision makers. This study is based on the $H_2/H_{\infty}$ control approach Chen and Zhang [2004], and the Pareto strategy, to ensure the bound weighted sum of linear quadratic costs Engwerda [2005] under stochastically mean-square stable and $H_{\infty}$ performance. Although the $H_{\infty}$ constrained Pareto suboptimal strategy for stochastic LPV systems has been studied Mukaidani et al. [2018], research on the $H_{\infty}$ constrained Pareto suboptimal strategy for time-delay systems remains open. There are few results on the $H_2/H_{\infty}$ control problems for stochastic LPV time-delay systems involving multiple decision makers. The contributions of this paper are as follows: First, an extended existence condition of the existing bounded real lemma for delay LPV systems Ku and Wu [2015] is investigated using the LMI approach. In addition, the existence condition of the quadratic cost bound for each player is established based on the guaranteed cost control technique Mohimani and Petersen [1996]; Rotondo et al. [2015]; second, by using these preliminary results, sufficient conditions for the existence of an $H_{\infty}$ constrained Pareto suboptimal control strategy set are obtained in terms of the cross-coupled bilinear matrix inequalities (BMIs). The $H_{\infty}$ constrained condition for delay stochastic LPV systems is derived for the first time; third, to solve the cross-coupled BMIs, a viscosity iterative scheme
based on isolated LMIs is proposed to obtain an LMI solution set cor-
responding to the strategy set. Furthermore, the strong convergence property
of the proposed iterative method can be attained successfully; finally, to demonstrate
the effectiveness of the proposed algorithm and the reliability and usefulness of the
proposed strategy set, two numerical examples are presented.

Notation: The notations used in this paper are fairly standard: $I_n$ denotes the $n \times n$
identity matrix; $\| \cdot \|$ denotes the Euclidean norm of a matrix; $L^2_F([0, \infty), \mathbb{R}^k)$
denotes the space of nonanticipative stochastic processes $\phi(t) \in \mathbb{R}^k$ with respect to an
increasing $\sigma$-algebras $F_t$, $t \geq 0$, satisfying $E[\int_0^\infty \|\phi(t)\|^2 dt] < \infty$; $C([-h, 0]; \mathbb{R}^n)$, $h > 0$,
denotes the family of continuous functions $\phi$ from $[-h, 0]$ to $\mathbb{R}^n$ with norm $\|\phi\| = \sup_{-h \leq \theta \leq 0} \|\phi(\theta)\|$; $\lambda_{\text{max}}[\cdot]$ and $\lambda_{\text{min}}[\cdot]$ denote its largest and smallest
eigenvalue, respectively.

2. Preliminary Results

Consider the following stochastic LPV time-delay system:

$$dx(t) = [A(\theta)x(t) + A_h(\theta)x(t - h) + D(\theta)\nu(t)]dt + A_p(\theta)x(t)dw(t),$$  \tag{1a}

$$x(t) = \phi(t), \quad t \in [-h, 0],$$  \tag{1b}

$$z(t) = E(\theta)x(t),$$  \tag{1c}

where $x(t) \in \mathbb{R}^n$ denotes the state vector, $\nu(t) \in \mathbb{R}^{n_r}$ denotes the external dis-
turbance, $z(t) \in \mathbb{R}^{n_z}$ denotes the controlled output, $w(t) \in \mathbb{R}$ denotes a one-
dimensional standard Wiener process defined in the filtered probability space,
$\theta(t) \in \mathbb{R}^r$ denotes the time-varying parameters, and $r$ denotes the number of
time-varying parameters.

In [1], $h \in (0, \infty)$ is the time delay of the stochastic LPV time-delay system,
and $\phi(t)$ is a real-valued initial function. It is assumed that, for all $\delta \in [-h, 0]$,
there exists scalar $\varepsilon > 0$ such that $\|x(t + \delta)\| \leq \varepsilon \|x(t)\|$ [Cao and Lam 2000].

The coefficient matrices in the stochastic LPV time-delay system are parameter-
dependent matrices, and can be expressed as

$$[A(\theta)A_h(\theta)A_p(\theta)] = \sum_{k=1}^M \alpha_k(t)[A_k A_{hk} A_{pk}],$$  \tag{2a}

$$D(\theta) = \sum_{k=1}^M \alpha_k(t)D_k, \quad E(\theta) = \sum_{k=1}^M \alpha_k(t)E_k,$$  \tag{2b}

where $\alpha_k(t) \geq 0$, $\sum_{k=1}^M \alpha_k(t) = 1$, $M = 2^r$.

As an extension of our previous results in [Mukaidani et al. 2018], the following
theorem can be derived.

Theorem 1. Consider the stochastic LPV time-delay system in (1). Given an
attenuation performance level, $\gamma > 0$, suppose that there exist matrices $Z = Z^T > 0$
H. Mukaidani, H. Xu & W. Zhuang

and $U = U^T > 0$ satisfying the following LMIs:

$$M_k^0(Z, U) < 0,$$

$$(3a)$$

$$M_{kl}^0(Z, U) < 0,$$

$$(3b)$$

where

$$M_k^0(Z, U) := \begin{bmatrix} \Xi^0_k & ZA_{hk} & ZD_k & A_{pk}^T Z & E_k^T \\ A_{hk}^T Z & -U & 0 & 0 & 0 \\ D_k^T Z & 0 & -\gamma^2 I_{n_u} & 0 & 0 \\ ZA_{pk} & 0 & 0 & -Z & 0 \\ E_k & 0 & 0 & 0 & -I_{n_z} \end{bmatrix},$$

$$M_{kl}^0(Z, U) := \begin{bmatrix} \Xi_{kl}^0 & ZA_{hk} & ZD_k & A_{pk}^T Z & E_k^T \\ A_{hk}^T Z & -2U & 0 & 0 & 0 \\ D_k^T Z & 0 & -2\gamma^2 I_{n_u} & 0 & 0 \\ ZA_{pk} & 0 & 0 & -2Z & 0 \\ E_{kl} & 0 & 0 & 0 & -2I_{n_z} \end{bmatrix},$$

$k < \ell, \ k = 1, \ldots, M,$

$$\Xi_k^0 = \Xi_k^0(Z, U) :=ZA_k + A_k^T Z + U,$$

$$\Xi_{kl}^0 = \Xi_{kl}^0(Z, U) :=ZA_{kl} + A_{kl}^T Z + U,$$

$$A_{kl} := A_k + A_\ell, \ A_{hk} := A_{hk} + A_{hl}, \ D_{kl} := D_k + D_\ell,$$

$$E_{kl} := E_k + E_\ell, \ A_{pk} := A_{pk} + A_{pl}.$$

Then, we have the following results:

(i) The stochastic LPV time-delay system in \(1\) is stochastically mean-square stable with $v(t) \equiv 0$;

(ii) The following inequality holds:

$$\|z\|^2 \leq \gamma^2 \|v\|^2 + F(Z, U),$$

$$(4)$$

where

$$\|z\|^2 := \mathbb{E} \left[ \int_0^\infty \|z(t)\|^2 dt \right], \quad \|v\|^2 := \mathbb{E} \left[ \int_0^\infty \|v(t)\|^2 dt \right],$$

$$F(Z, U) := \mathbb{E}[x^T(0)Zx(0)] + \mathbb{E} \left[ \int_{-h}^0 \phi^T(s)U\phi(s)ds \right];$$
(iii) The worst-case disturbance is given by
\[ v^*(t) = F^*_v(\theta)x(t) = \gamma^{-2}D^T(\theta)Zx(t). \] (5)

**Proof.** First, define the following Lyapunov–Krasovskii function:
\[ V_v(x,t) := x^T(t)Zx(t) + \int_{t-h}^{t} x^T(s)Ux(s)ds, \] (6)

where \( Z = Z^T > 0 \) and \( U = U^T > 0 \).

By using Itô formula with infinitesimal generator \( \mathcal{L} \), the stochastic differential equation can be obtained as
\[
\mathcal{L}V_v(x,t) + \|z(t)\|^2 - \gamma^2\|v(t)\|^2 \\
= \xi^T(t)M(Z,U,\theta)\xi(t) - \gamma^2(v(t) - \gamma^{-2}D^T(\theta)Zx(t))^T \\
\times (v(t) - \gamma^{-2}D^T(\theta)Zx(t)),
\] (7)

where
\[
\mathcal{L}V_v(x,t) = x^T(t)Ux(t) - x^T(t-h)Ux(t-h) \\
+ 2x^T(t)Z[A(\theta)x(t) + A_h(\theta)x(t-h) + D(\theta)v(t)] \\
+ x^T(t)A_p^T(\theta)ZA_p(\theta)x(t),
\]

\[
M(Z,U,\theta) = \begin{bmatrix} \Phi(Z,U,\theta) & ZA_h(\theta) \\ A_p^T(\theta)Z & -U \end{bmatrix},
\]

\[
\xi(t) = \begin{bmatrix} x(t) \\ x(t-h) \end{bmatrix},
\]

\[
\Phi(Z,U,\theta) := ZA(\theta) + A^T(\theta)Z + U + A^T_p(\theta)ZA_p(\theta) \\
+ \gamma^{-2}ZD(\theta)D^T(\theta)Z + E^T(\theta)E(\theta).
\]

Hence, if \( v(t) = v^*(t) \), we have
\[
\mathcal{L}V_v(x,t) + \|z(t)\|^2 - \gamma^2\|v(t)\|^2 \leq \xi^T(t)M(Z,U,\theta)\xi(t),
\] (8)

On the other hand, using the Schur complement, \( M(Z,U,\theta) < 0 \) is equivalent to the following inequality:
\[
\begin{bmatrix}
ZA(\theta) + A^T(\theta)Z + U & ZA_h(\theta) & ZD(\theta) & A_p^T(\theta)Z & E^T(\theta) \\
A_p^T(\theta)Z & -U & 0 & 0 & 0 \\
D^T(\theta)Z & 0 & -\gamma^2I_n & 0 & 0 \\
ZA_p(\theta) & 0 & 0 & -Z & 0 \\
E(\theta) & 0 & 0 & 0 & -I_{n_z}
\end{bmatrix} < 0.
\] (9)

Hence, \( v^*(t) = \gamma^{-2}D^T(\theta)Zx(t) \) in (5) is obtained.
Furthermore, inequality (9) can be re-written in the following format:

\[
\sum_{k=1}^{M} \alpha_k^2 M_k^0(Z, U) + \sum_{k=1}^{M-1} \sum_{\ell=k+1}^{M} \alpha_k \alpha_\ell M_{k\ell}^0(Z, U) < 0.
\] (10)

Thus, by using a similar technique to that given in Apkarian et al. [1995] and Ku and Wu [2015], the equivalence of \( M(Z, U, \theta) < 0 \) and (3) can be proved. Therefore, if inequality (3) holds, (11) holds. That is, if \( \nu(t) \equiv 0 \), \( \mathcal{L} V_\nu(x, t) \leq 0 \) holds. Therefore, by using the technique similar to the proof in [Cao and Lam, 2000], the following inequality can be obtained:

\[
\lim_{t_f \to \infty} \mathbb{E} \left[ \int_{0}^{t_f} x^T(t, \phi)x(t, \phi)dt \right] \leq x^T(0) \Lambda x(0),
\] (12)

where

\[
\Lambda = \frac{\lambda_{\max}[Z] + \epsilon^2 h \lambda_{\max}[U]}{\nu \lambda_{\min}[Z]} I_n, \quad \nu := \min_{\theta} \left\{ \frac{\lambda_{\min}[-M(Z, U, \theta)]}{\lambda_{\max}[Z] + \epsilon^2 h \lambda_{\max}[U]} \right\}.
\]

As a result, the stochastic LPV time-delay system (11) is stochastically mean-square stable. Furthermore, using Dynkin’s formula, we obtain

\[
J_\nu(x(0), t_f) \leq -\mathbb{E} \left[ \int_{0}^{t_f} \mathcal{L} V_\nu(x, t)dt \right] = \mathbb{E}[V_\nu(x(0), 0)] - \mathbb{E}[V_\nu(x(t_f), t_f)] \leq F(Z, U),
\] (13)

where

\[
J_\nu(x(0), t_f) := \mathbb{E} \left[ \int_{0}^{t_f} \|z(t)\|^2 - \gamma^2 \|v(t)\|^2 dt \right].
\]

Considering that the stochastic LPV time-delay system is stochastically mean-square stable when \( t_f \) approaches infinity, inequality (11) holds.

Next, the stochastical mean-square stability and the quadratic cost bound for stochastic LPV time-delay system (11) are investigated. Consider the following stochastic LPV time-delay system:

\[
\dot{x}(t) = [A(\theta)x(t) + A_h(\theta)x(t-h)]dt + A_p(\theta)x(t)dw(t),
\] (14a)

\[
x(t) = \phi(t), \quad t \in [-h, 0].
\] (14b)

Define the quadratic cost function as

\[
J(x^0) = \mathbb{E} \left[ \int_{0}^{\infty} x^T(t)Q(\theta)x(t)dt \right],
\] (15)

where

\[
Q(\theta) = Q^T(\theta) = \sum_{k=1}^{M} \alpha_k(\theta)Q_k > 0.
\]
Theorem 2. Assume that there exist matrices $P = P^T > 0$ and $S = S^T > 0$ satisfying the following LMIs:

$$\begin{align*}
\mathbf{L}_0^k(P, S) &< 0, \tag{16a} \\
\mathbf{L}_{k\ell}^0(P, S) &< 0, \tag{16b}
\end{align*}$$

where

$$\begin{align*}
\mathbf{L}_0^k(P, S) &:= \begin{bmatrix}
\Upsilon_k^0 & PA_hk & A_{pk}^T P \\
A_{hk}^T P & -S & 0 \\
PA_{pk} & 0 & -P
\end{bmatrix}, \\
\mathbf{L}_{k\ell}^0(P, S) &:= \begin{bmatrix}
\Upsilon_{k\ell}^0 & PA_{k\ell} & A_{p\ell}^T P \\
A_{hk}^T P & -2S & 0 \\
PA_{p\ell} & 0 & -2P
\end{bmatrix},
\end{align*}$$

$k < \ell, \ k = 1, \ldots, M,$

$$\Upsilon_k^0 = \Upsilon_k^0(P, S) := PA_k + A_k^T P + S + Q_k,$$  

$$\Upsilon_{k\ell}^0 = \Upsilon_{k\ell}^0(P, S) := PA_{k\ell} + A_{k\ell}^T P + S + Q_k + Q_\ell.$$  

Then, we have

$$J(x^0) < \mathcal{F}(P, S),$$

where

$$\mathcal{F}(P, S) := \mathbb{E}[x^T(0)Px(0)] + \mathbb{E} \left[ \int_{-h}^0 \phi^T(s)S\phi(s)ds \right].$$

Proof. We introduce the following parameter independent Lyapunov–Krasovskii function

$$\mathcal{V}_u(x, t) = x^T(t)Px(t) + \int_{t-h}^t x^T(s)Sx(s)ds,$$  

where $P = P^T > 0$ and $S = S^T > 0.$

Following similar steps as in the previous discussion, suppose that there exist $P > 0$ and $S > 0$ such that

$$\mathcal{L}\mathcal{V}_u(x, t) + x^T(t)Q(x(t)) = \xi^T(t)\mathbf{L}(P, S, \theta)\xi(t),$$

where

$$\mathcal{L}\mathcal{V}_u(x, t) = x^T(t)Sx(t) - x^T(t-h)Sx(t-h)$$

$$+ 2x^T(t)P[A(\theta)x(t) + A_h(\theta)x(t-h)] + x^T(t)A_{p\ell}(\theta)PA_{p\ell}(\theta)x(t),$$

$$\mathbf{L}(P, S, \theta) = \begin{bmatrix}
\Psi(P, S, \theta) & PA_h(\theta) \\
A_h(\theta)P & -S
\end{bmatrix},$$

$$\Psi(P, S, \theta) := PA(\theta) + A^T(\theta)P + S + A_{p\ell}(\theta)PA_{p\ell}(\theta) + Q(\theta).$$
Then, \( L(P, S, \theta) < 0 \) holds due to \( Q(\theta) > 0 \). Therefore, the stochastic LPV time-delay system in (14) is mean-square stable. Furthermore, using the result of the stochastic stability of (14), we have
\[
J(x^0) < V_u(x, 0) = F(P, S).
\]

By re-writing inequality \( L(P, S, \theta) < 0 \),
\[
\sum_{k=1}^{M} \alpha_k^2 L^0_k(P, S) + \sum_{k=1}^{M-1} \sum_{\ell=k+1}^{M} \alpha_k \alpha_\ell L^0_{k\ell}(P, S) < 0.
\]

Thus, if both inequalities in (16) are satisfied, the robust stability and the quadratic cost bound (17) are obtained.

3. Problem Formulation

Consider a stochastic LPV time-delay system with multiple decision makers defined by
\[
dx(t) = \left[ A(\theta)x(t) + A_h(\theta)x(t - h) + \sum_{j=1}^{N} B_j(\theta)u_j(t) + D(\theta)v(t) \right] dt + A_p(\theta)x(t)dw(t),
\]
\[
x(t) = \phi(t), \quad t \in [-h, 0],
\]
\[
z(t) = \begin{bmatrix}
E(\theta)x(t) \\
H_1 u_1(t) \\
\vdots \\
H_N u_N(t)
\end{bmatrix}
\]
where \( u_i(t) \in \mathbb{R}^{m_i}, i = 1, \ldots, N \), denotes the \( i \)th decision maker’s control input. The other variables are defined by stochastic equation (1). The coefficient matrices \( B_i(\theta) \), \( i = 1, \ldots, N \) in (22) can be expressed as
\[
B_i(\theta) = \sum_{k=1}^{M} \alpha_k(t) B_{ik}.
\]

Furthermore, note that \( H_i \) does not depend on a time-varying parameter, because the controlled output can be chosen by the controller designer. Hence, without loss of generality, it can be assumed that \( H_i \) is a constant matrix.

**Assumption 1.** \( H_i^TH_i = I_{m_i}, i = 1, \ldots, N \), \( H_i \in \mathbb{R}^{m_i \times m_i} \). Furthermore, without loss of generality, to remove the dependence on \( x(0) \), assume that \( x(0) \) is a zero mean random variable satisfying \( \mathbb{E}[x(0)x^T(0)] = I_n \).
The cost performance functions are defined by

\[ J_v(u_1, \ldots, u_N, v, x^0) = \mathbb{E} \left[ \int_0^\infty \left[ \gamma^2 \|v(t)\|^2 - \|z(t)\|^2 \right] dt \right], \]

(24a)

\[ J_i(u_i, v, x^0) = \mathbb{E} \left[ \int_0^\infty \left\{ x^T(t)Q_i x(t) + u^T_i(t)R_i u_i(t) \right\} dt \right], \]

(24b)

where \( Q_i = Q_i^T > 0, R_i = R_i^T > 0, i = 1, \ldots, N. \)

The problem of \( H_\infty \) constrained Pareto strategy [Engwerda, 2005] for stochastic LPV system (22) is stated as follows.

**Problem 1.** For any given positive parameter \( \gamma, v(t) = v(t) \in L^2_F([0, \infty), \mathbb{R}^{m_v}), \)

find a Pareto suboptimal state feedback memoryless strategy set

\[ u_i(t) = u^*_i(t) = K_i(\theta)x(t) = \sum_{k=1}^M \alpha_k K_{ik} x(t) \]

(25)

such that

(i) \( u_i(t) = u^*_i(t), \) \( i = 1, \ldots, N, \) make stochastic system LPV (22) stochastically mean-square stable when \( v(t) = 0 \) and the following inequality holds:

\[ \|z\|^2 \leq \gamma^2 \|v\|^2 + \mathcal{F}(Z, U); \]

(ii) When \( v(t) = v^*(t) = F^*_i x(t) \) is applied, consider a weighted sum of the cost function of the followers, given by

\[ J_\rho(u_1, \ldots, u_N, v^*, x^0) := \sum_{i=1}^N \rho_i J_i(u_i, v^*, x^0), \]

(27)

\[ \sum_{i=1}^N \rho_i = 1, \quad 0 < \rho_i < 1, \quad i = 1, \ldots, N. \]

A Pareto suboptimal strategy set \((u_1, \ldots, u_N)\) minimizes the upper bound defined by the following cost function:

\[ \bar{J}_\rho(u_1, \ldots, u_N, v^*, x^0) = \sup_{\theta(t)} J_\rho(u_1, \ldots, u_N, v^*, x^0). \]

(28)

In the following section, the existence conditions of the \( H_\infty \) constrained Pareto suboptimal strategy set are established in terms of the preliminary results.

**4. Main Results**

The main result related to the quadratic cost bound under the \( H_\infty \) constraint condition is given in the following.

**Theorem 3.** Consider stochastic LPV time-delay system (22) with multiple decision makers \( u_i(t) = K_i(\theta)x(t), \) \( i = 1, \ldots, N, \) and deterministic disturbance
v(t) = F_c(\theta)x(t). Given attenuation performance level \gamma, assume that there exists a strategy set for the real symmetric matrices \(X = X^T > 0, W = W^T > 0, Z = Z^T > 0, U = U^T > 0\) and \(Y_k\) such that the following cross-coupled BMIIs are satisfied:

\[
L_k(X, W, Y_k, F_{\gamma_k}) < 0, \quad (29a)
\]

\[
L_{k\ell}(X, W, Y_k, Y_\ell, F_{\gamma_k}, F_{\gamma_\ell}) < 0, \quad (29b)
\]

\[
M_k(Z, U, K_k) < 0, \quad (29c)
\]

\[
M_{k\ell}(Z, U, K_k, K_\ell) < 0, \quad (29d)
\]

where \(k < \ell, k = 1, \ldots, M\),

\[
L_k(X, W, Y_k, F_{\gamma_k}) = 
\begin{bmatrix}
\Upsilon_k & XA_{pk}^T & X & X & Y_k^T \\
A_{pk}X & -X & 0 & 0 & 0 \\
X & 0 & -Q_\rho^{-1} & 0 & 0 \\
X & 0 & 0 & -W & 0 \\
Y_k & 0 & 0 & 0 & -R_\rho^{-1}
\end{bmatrix},
\]

\[
L_{k\ell}(X, W, Y_k, Y_\ell, F_{\gamma_k}, F_{\gamma_\ell}) := 
\begin{bmatrix}
\Upsilon_{k\ell} & XA_{pk\ell}^T & X & X & Y_{k\ell}^T \\
A_{pk\ell}X & -2X & 0 & 0 & 0 \\
X & 0 & -Q_\rho^{-1} & 0 & 0 \\
X & 0 & 0 & -W & 0 \\
Y_{k\ell} & 0 & 0 & 0 & -2R_\rho^{-1}
\end{bmatrix},
\]

\[
M_k(Z, U, K_k) := 
\begin{bmatrix}
\Xi_k & ZA_{hk} & ZD_k & A_{pk}^T & Z & E_{Kk}^T \\
A_{hk}^T & -U & 0 & 0 & 0 \\
D_k^T & Z & -\gamma^2I_{n_v} & 0 & 0 \\
ZA_{pk} & 0 & 0 & -Z & 0 \\
E_{Kk} & 0 & 0 & 0 & -I_{n_z}
\end{bmatrix},
\]

\[
M_{k\ell}(Z, U, K_k, K_\ell) := 
\begin{bmatrix}
\Xi_{k\ell} & ZA_{hk\ell} & ZD_{k\ell} & A_{pk\ell}^T & Z & E_{Kk\ell}^T \\
A_{hk\ell}^T & -2U & 0 & 0 & 0 \\
D_{k\ell}^T & Z & -2\gamma^2I_{n_v} & 0 & 0 \\
ZA_{pk\ell} & 0 & 0 & -2Z & 0 \\
E_{Kk\ell} & 0 & 0 & 0 & -2I_{n_z}
\end{bmatrix},
\]

\[
\Upsilon_k = \Upsilon_k(X, W, Y_k, F_{\gamma_k}) := X(A_k + D_kF_{\gamma_k})^T + (A_k + D_kF_{\gamma_k})X + B_{ck}Y_k + Y_k^TB_{ck}^T + A_{hk}W A_{hk}^T,
\]
Then, the $H_\infty$ constrained Pareto suboptimal strategy set under consideration is given by

$$u_i^*(t) = K_i(\theta)x(t) = \sum_{k=1}^{M} \alpha_k K_{ik} x(t). \quad (30)$$

Furthermore, the optimal cost bounds are given by

$$J_\rho(u_1, \ldots, u_N, v^*, x^0) < \bar{J}_\rho(u_1, \ldots, u_N, v^*, x^0) = \mathcal{F}(P, S), \quad (31)$$

with the worst case disturbance

$$v(t) = v^*(t) = F_\gamma(\theta)x(t) = \gamma^{-2} D^T(\theta)Zx(t). \quad (32)$$

**Proof.** First, the $H_\infty$ constraint condition is derived. By applying Pareto suboptimal strategy set (30) to the original stochastic LPV time-delay system in (22), we
H. Mukaidani, H. Xu & W. Zhuang

have the following closed-loop stochastic LPV time-delay system:

\[
dx(t) = \left[ \begin{array}{c} (A(\theta) + \sum_{j=1}^{N} B_j(\theta) K_j(\theta)) x(t) + A_h(\theta)x(t-h) \\ + D(\theta)x(t) \end{array} \right] dt + A_p(\theta)x(t)dw(t),
\]

\[
x(t) = \phi(t), \quad t \in [-h, 0],
\]

\[
z(t) = E_K(\theta)x(t).
\]

By termwise comparison between (1) and (33), we have

\[
B(\theta)K(\theta) \leftarrow \sum_{j=1}^{N} B_j(\theta) K_j(\theta) = B_c(\theta)K_c(\theta)
\]

\[
= \sum_{k=1}^{M} \alpha_k^2 B_{ck}K_k + \sum_{k=1}^{M-1} \sum_{\ell=k+1}^{M} \alpha_k \alpha_\ell (B_{ck}K_\ell + B_{c\ell}K_k),
\]

\[
E(\theta) \leftarrow E_K(\theta) = \sum_{k=1}^{M} \alpha_k E_{K_k}.
\]

Thus, by applying Theorem 1 to this problem, the conditions (29c) and (29d) can be obtained.

Second, the existence condition of the Pareto suboptimal strategy set is derived. Consider the stochastic LPV time-delay system with the following cost function:

\[
J_\rho(K_1(\theta)x, \ldots, K_M(\theta)x, u^*, x^0)
\]

\[
= \mathbb{E} \left[ \int_{0}^{\infty} x^T(t) [Q_\rho + K_c(\theta)^T R_\rho K_c(\theta)] x(t) dt \right],
\]

such that

\[
dx(t) = [[A(\theta) + B_c(\theta)K_c(\theta) + D(\theta)F_\gamma(\theta)] x(t) \\
+ A_h(\theta)x(t-h)] dt + A_p(\theta)x(t)dw(t),
\]

\[
\begin{bmatrix}
u_1 \\
\vdots \\
u_M
\end{bmatrix} = \sum_{k=1}^{M} \alpha_k \begin{bmatrix} K_{1k} \\
\vdots \\
K_{Mk}
\end{bmatrix} x(t) = \sum_{k=1}^{M} \alpha_k K_k x(t).
\]

Following steps similar to those in the previous problem and by termwise comparison between (14), (15) and (35) with

\[
A(\theta) \leftarrow A(\theta) + B_c(\theta)K_c(\theta) + D(\theta)F_\gamma(\theta),
\]

\[
Q \leftarrow Q_\rho + K_c(\theta)^T R_\rho K_c(\theta),
\]
we have the following matrix inequalities using Theorem 2:

\[
\bar{L}_k(P, S, K_k, F_{\gamma_k}) < 0, \tag{36a}
\]

\[
\bar{L}_{k\ell}(P, S, K_k, K_{\ell}, F_{\gamma_k}, F_{\gamma_{\ell}}) < 0, \tag{36b}
\]

where \(k < \ell, \ k = 1, \ldots, M\),

\[
\bar{L}_k(P, S, K_k, F_{\gamma_k}) = \begin{bmatrix}
\Delta_k & PA_{hk} & A_{pdk}^TP \\
A_{hdk}^TP & -S & 0 \\
PA_{pk} & 0 & -P
\end{bmatrix},
\]

\[
\bar{L}_{k\ell}(P, S, K_k, K_{\ell}, F_{\gamma_k}, F_{\gamma_{\ell}}) := \begin{bmatrix}
\Delta_{k\ell} & PA_{hk\ell} & A_{pdk\ell}^TP \\
A_{hdk\ell}^TP & -2S & 0 \\
PA_{pk\ell} & 0 & -2P
\end{bmatrix},
\]

\[
\Delta_k = \Delta_k(P, S, K_k, F_{\gamma_k})
= P(A_k + B_{ck}K_k + D_kF_{\gamma_k})
+ (A_k + B_{ck}K_k + D_kF_{\gamma_k})^TP
+ S + Q_\rho + K_{k}^TP_\rho K_k,
\]

\[
\Delta_{k\ell} = \Delta_{k\ell}(P, S, K_k, K_{\ell}, F_{\gamma_k}, F_{\gamma_{\ell}})
= P(A_{k\ell} + B_{ck\ell}K_{\ell} + D_kF_{\gamma_{\ell}} + B_{c\ell}K_k + D_{\ell}F_{\gamma_k})
+ (A_{k\ell} + B_{ck\ell}K_{\ell} + D_kF_{\gamma_{\ell}} + B_{c\ell}K_k + D_{\ell}F_{\gamma_k})^TP
+ S + Q_\rho + K_{k\ell}^TP_\rho K_{k\ell}.
\]

Applying the Schur complement lemma to inequalities \ref{ineq} and multiplying the following matrix

\[
\text{block diag}(P^{-1} I_n I_n I_m), \quad m := \sum_{i=1}^{N} m_i
\]

on both sides, LMIs \ref{ineq} and \ref{ineq} can be obtained. In addition, the quadratic cost bound of \ref{ineq} can be derived as \ref{ineq}.

It should be noted that the optimization problem related to the upper bound of the cost \(J_p(u_1, \ldots, u_N, v^*, x^0)\) in \ref{ineq} is tackled in the following subsection in the proposed iterative algorithm.

### 4.1. Iterative Algorithm

In order to obtain the \(H_\infty\) constrained Pareto suboptimal strategy set of \ref{ineq}, we need to solve the cross-coupled BMIs \ref{ineq}. In particular, the following optimization
should be solved:

$$\min_{X,W,Y} J_p(u_1, \ldots, u_N, v^*, x^0) = \min_{X,W,Y} (\text{Trace}[X^{-1}] + \text{Trace}[LLTW^{-1}]),$$  \hspace{1cm} (37a)

s.t. (29a), \hspace{1cm} (29b), \hspace{1cm} (37b),

where $LL^T = \int_0^{-h} \phi(s)\phi^T(s)ds$.

In the following, an iterative algorithm including the optimization problem (37) by means of combining LMI s with the viscosity iterative scheme \cite{2004} is established:

**Step 1.** Set the initial values: choose an appropriate $\gamma_k^{(0)}$.

**Step 2.** Solve the following optimization problem based on the LMI for $X^{(n+1)}$, $W^{(n+1)}$, $Y^{(n+1)}$:

$$\min_{x^{(n+1)}} \text{Tr}[\Gamma^{(n+1)}] + \text{Tr}[\tau^{(n+1)}],$$  \hspace{1cm} (38a)

$$x^{(n+1)} := (X^{(n+1)}, W^{(n+1)}, Y_{1}^{(n+1)}, \ldots, Y_{M}^{(n+1)}, \Gamma^{(n+1)}, \tau^{(n+1)}),$$

s.t. $L_k(X^{(n+1)}, W^{(n+1)}, Y_k^{(n)}), F^{(n)}_k < 0$, \hspace{1cm} (38b)

$$L_{kl}(X^{(n+1)}, W^{(n+1)}, Y_{k}^{(n+1)}, Y_{l}^{(n+1)}, F^{(n)}_k, F^{(n)}_l) < 0,$$  \hspace{1cm} (38c)

$$\begin{bmatrix} -\Gamma^{(n+1)} & I_n \\ I_n & -X^{(n+1)} \end{bmatrix} < 0,$$  \hspace{1cm} (38d)

$$\begin{bmatrix} -\tau^{(n+1)} & L^T \\ L & -W^{(n+1)} \end{bmatrix} < 0.$$  \hspace{1cm} (38e)

**Step 3.** Compute the strategy set:

$$K_k^{(n+1)} = Y^{(n+1)}_k [X^{(n+1)}]^{-1}, \hspace{1cm} k = 1, \ldots, M.$$  \hspace{1cm} (39)

**Step 4.** Solve the following optimization problem for $Z^{(n+1)}$ and $U^{(n+1)}$:

$$\min_{Z^{(n+1)}, U^{(n+1)}} \text{Tr}[Z^{(n+1)}] + \text{Tr}[U^{(n+1)}], \hspace{1cm} \text{s.t.,}$$

$$M_k(Z^{(n+1)}, U^{(n+1)}, K^{(n+1)}_k) < 0,$$  \hspace{1cm} (40b)

$$M_{kl}(Z^{(n+1)}, U^{(n+1)}, K^{(n+1)}_k, K^{(n+1)}_l) < 0,$$  \hspace{1cm} (40c)

where

$$E^{(n+1)}_{\gamma_k} := \gamma^{-2} D_k^T Z^{(n+1)} , \hspace{1cm} E^{(n+1)}_{kK} := \begin{bmatrix} E_k \\ L_1 K^{(n+1)}_{1k} C_1 \\ \vdots \\ L_N K^{(n+1)}_{Nk} C_N \end{bmatrix}.$$
**Step 5.** For any appropriate fixed value of $\eta \in (0, 1)$, set
\[
\mathbf{z}^{(n+1)} \leftarrow \eta (1 - \beta_n) \mathbf{z}^{(n)} + \beta_n \mathbf{z}^{(n+1)},
\]
where $\mathcal{F}$ is the mapping from Steps 2–4 such that
\[
\mathbf{z}^{(n+1)} = \mathcal{F}(\mathbf{z}^{(n)}),
\]
\[
\mathbf{z}^{(n)} := \begin{bmatrix}
\text{vec}[X^{(n)}] & \text{vec}[W^{(n)}] & \text{vec}[Y_1^{(n)}] & \cdots & \text{vec}[Y_M^{(n)}] & \text{vec}[Z^{(n)}] & \text{vec}[U^{(n)}]
\end{bmatrix}.
\]
Furthermore, parameter $\{\beta_n\} \in [0, 1]$ satisfies the following conditions:
\[
\lim_{n \to \infty} \beta_n = 0,
\]
\[
\sum_{n=0}^{\infty} \beta_n = \infty,
\]
\[
\sum_{n=0}^{\infty} |\beta_{n+1} - \beta_n| < \infty.
\]

**Step 6.** If the iterative algorithm consisting of Steps 2–5 converges, we have obtained solutions for the cross-coupled BMIs \([20]\); otherwise, if the number of iterations reaches a preset threshold, declare that there is no strategy set. Stop.

The fact theorem indicates the norm-convergence of the proposed iterative scheme that is known as the strong convergence property that has been proved in \([Xu, 2004]\).

**Fact 1.** If $\mathcal{F}$ is a monotone nonexpansive mapping with a fixed point, then the viscosity iterative scheme $\{\mathbf{z}^{(n)}\}$ converges strongly to a fixed point in uniformly smooth Banach space.

5. Numerical Examples

In order to show the effectiveness of the proposed $H_\infty$ constrained Pareto suboptimal strategy, two numerical examples are presented.

5.1. **Academic example**

Consider the following stochastic LPV time-delay system with two decision makers, modified from \([Wu \text{ and Grigoriadis}, 2001]\):
\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 + 0.2 \sin t \\ -2 & -3 + 0.1 \sin t \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \sin t & 0.1 \\ -0.2 + 0.1 \sin t & -0.3 \end{bmatrix} x(t-h)
+ \begin{bmatrix} 0.2 \sin t \\ 0.1 + 0.1 \sin t \end{bmatrix} u_1(t) + \begin{bmatrix} 0.1 + 0.1 \sin t \\ 0.2 \sin t \end{bmatrix} u_2(t) + \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix} v(t),
\]
\[
z(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_2(t).
\]
Note that the stochastic LPV with multiple control inputs is considered, different from that in [Wu and Grigoriadis [2001]].

It is assumed that the state-dependent noise $A_p(\theta)x(t)dw(t)$ has 15% perturbation based on state matrix $A(\theta)$. Using the fact that $\sin t = \cos^2(\pi/4 - t/2) - \sin^2(\pi/4 - t/2)$, the system matrices of (22) are given by

$$A_1 = \begin{bmatrix} 0 & 1.2 \\ -2 & -2.9 \end{bmatrix}, \quad A_h = \begin{bmatrix} 0.2 & 0.1 \\ -0.1 & -0.3 \end{bmatrix}, \quad A_{p1} = 0.15A_1,$$

$$A_2 = \begin{bmatrix} 0 & 0.8 \\ -2 & -3.1 \end{bmatrix}, \quad A_h = \begin{bmatrix} -0.2 & 0.1 \\ -0.3 & -0.3 \end{bmatrix}, \quad A_{p2} = 0.15A_2,$$

$$B_{11} = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \quad B_{21} = \begin{bmatrix} -0.2 \\ 0 \end{bmatrix}, \quad B_{12} = \begin{bmatrix} -0.2 \\ 0 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},$$

$$D_1 = D_2 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}, \quad h = 0.1, \quad \phi(t) = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}, \quad -h \leq t \leq 0,$$

$$Q_1 = \text{diag}(0.9, 1.9), \quad Q_2 = \text{diag}(1.2, 1.2),$$

$$R_1 = 0.9, \quad R_2 = 2.4, \quad \rho_1 = \rho_2 = 0.5.$$

The disturbance attenuation level is set as $\gamma = 3.1$. The cross-coupled BMIs in (20) are solved by using the proposed viscosity iterative scheme. The computed strategy set in (30) and the worst case disturbance in (32) with the related solution matrices are given by

$$P = X^{-1} = \begin{bmatrix} 2.5632 & 4.5954 \times 10^{-1} \\ 4.5954 \times 10^{-1} & 5.2412 \times 10^{-1} \end{bmatrix},$$

$$S = W^{-1} = \begin{bmatrix} 4.3870 \times 10^{-1} & -8.5127 \times 10^{-2} \\ -8.5127 \times 10^{-2} & 1.4887 \times 10^{-1} \end{bmatrix},$$

$$K_{11} = Y_{11}X^{-1} = \begin{bmatrix} -4.8206 \times 10^{-1} \\ -8.9643 \times 10^{-1} \end{bmatrix},$$

$$K_{21} = Y_{21}X^{-1} = \begin{bmatrix} 1.0418 \times 10^{-1} \\ 2.4881 \times 10^{-1} \end{bmatrix},$$

$$K_{12} = Y_{12}X^{-1} = \begin{bmatrix} 6.0283 \times 10^{-1} \\ -6.1613 \times 10^{-1} \end{bmatrix},$$

$$K_{22} = Y_{22}X^{-1} = \begin{bmatrix} -3.0265 \times 10^{-1} \\ 1.4370 \times 10^{-1} \end{bmatrix},$$

$$Z = \begin{bmatrix} 1.6635 & 2.3750 \times 10^{-1} \\ 2.3750 \times 10^{-1} & 4.6005 \times 10^{-1} \end{bmatrix},$$

$$U = \begin{bmatrix} 2.3961 \times 10^{-1} & -7.2524 \times 10^{-2} \\ -7.2524 \times 10^{-2} & 1.3598 \times 10^{-1} \end{bmatrix},$$

$$F_{\gamma 1} = F_{\gamma 2} = \begin{bmatrix} 3.9564 \times 10^{-2} \\ 1.4517 \times 10^{-2} \end{bmatrix}.$$
Next, the proposed viscosity iterative scheme to obtain the converged solutions and the strategies is verified. The initial value is set as $F^{(0)}(0) = 2^2D_{T}^1 I_n$ for $k = 1, 2$. In Step 5 of the proposed algorithm, the value of $\eta$ is set to 0.99 and $\beta_n = \frac{1}{n}$. The proposed algorithm converges after 691 iterations with an accuracy of $\psi(n) := \|z^{(n+1)} - z^{(n)}\| < 10^{-6}$.

Finally, the robust stability of the stochastic LPV time-delay system is confirmed. Figure 1 shows that all the states attain the mean-square stability, in the presence of state delay and multiple decision makers.

5.2. Williams–Otto process

Second, we discuss a practical numerical example based on the Williams–Otto process Williams and Otto [1960]; Ross [1971]; Ahmadizadeh et al. [2014], which is well known as a nonisothermal continuous stirred-tank reactor (CSTR). It should be noted that it is widely known as a typical chemical process adopted in the control engineering literature. In this example, stochastic LPV delay system representations of the nonlinear model of the CSTR are developed to adjust the system nonlinearities in the LPV scheduling variables Puna and Bakošová M [2007]; Rahme et al. [2013]. Consider the linearized delay stochastic system of the CSTR was obtained in the following form:

$$dx(t) = \left[ A(\theta)x(t) + A_h(\theta)x(t - h) + \sum_{j=1}^{2} B_j(\theta)u_j(t) + D(\theta)v(t) \right] dt + A_p(\theta)x(t)dw(t),$$

$$x(t) = \phi(t), \quad t \in [-1, 0],$$

$$z(t) = \begin{bmatrix} E(\theta)x(t) \\ H_1 u_1(t) \\ H_2 u_2(t) \end{bmatrix},$$

![Fig. 1. State trajectories.](image-url)
where

\[
x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}, \quad \phi(t) = \begin{bmatrix} 0.4 \\ 1.5 \\ 2.5 \\ 0.5 \end{bmatrix}.
\]

Due to the variation of the reaction rate in nonlinear dynamics, the linearized mathematical models \( M = 2 \) are assumed and suppose that 5\% of the perturbation exists from the nominal value of the state matrix of \( A \). Therefore, the coefficient matrices are defined as follows:

\[
A = \begin{bmatrix}
-4.93 & -1.01 & 0 & 0 \\
-3.20 & -5.30 & -12.8 & 0 \\
6.40 & 0.347 & -32.5 & -1.04 \\
0 & 0.833 & 11.0 & -3.96
\end{bmatrix},
\]

\[
A_1 = A + 0.05A, \quad A_2 = A - 0.05A,
\]

\[
A_{hk} = \text{diag}(1.92, 1.92, 1.87, 0.724), \quad A_{pk} = 0.05A_k.
\]

\[
B_{1k} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_{2k} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad D_k = \begin{bmatrix} 0.1 \\ -0.2 \\ -1 \\ 0.5 \end{bmatrix},
\]

\[
E_k = \begin{bmatrix} 0.1 & 0.1 & 0 & 0 \end{bmatrix}, \quad H_i = [1], \quad i = 1, 2, \quad k = 1, 2.
\]

These systems represent vertices of the uncertain polytopic system. On the other hand, we also consider that 5\% of the state coefficient can be represented by a Wiener process due to stochastic perturbations.

The weight matrices of cost functions and the \( \rho_i \) are given by

\[
Q_1 = 4I_4, \quad Q_2 = 2I_4, \quad R_1 = 1, \quad R_2 = 2, \quad \rho_1 = 1 = \rho_2 = 0.5.
\]

Next, we select \( \gamma = 3 \). Using the proposed iterative algorithms in Sec. 4.1 with the initial conditions

\[
F_{\gamma k}^{(0)} = \gamma^{-2}D_k^T I_4, \quad k = 1, 2,
\]

we obtain the following \( H_\infty \) constrained Pareto suboptimal strategy set in \( 8 \):

\[
K_{11} = \begin{bmatrix} -2.1731 & 8.4985 \times 10^{-1} & -4.3998 \times 10^{-1} & -1.2814 \times 10^{-1} \end{bmatrix},
\]

\[
K_{21} = \begin{bmatrix} 4.1397 \times 10^{-1} & -5.6098 \times 10^{-1} & 1.5535 \times 10^{-1} & -1.4498 \times 10^{-1} \end{bmatrix}
\]
H∞ Constrained Pareto Suboptimal Strategy

It should be noted that the proposed iterative algorithm converges after 52 iterations with an accuracy of 10⁻⁴, and the strategy set was computed.

Second, Fig. 2 shows how the system states with time. In the case that the deterministic time-varying uncertainty is \( \alpha_1(t) = e^{-t} \), \( \alpha_2(t) = 1 - e^{-t} \) as a special case, it is observed that all the states are stable even though the delay and the stochastic noise exist.

6. Conclusion

In this paper, we have studied an H∞ constrained Pareto suboptimal strategy for stochastic LPV systems with constant state delay. The challenge is to achieve stochastic mean-square stability even when several unsatisfactory factors exist such as the time delay and disturbances. As a main contribution, the existence conditions of the H∞ constrained Pareto suboptimal strategy set are derived via cross-coupled BMIs. In addition, a computational framework based on isolated LMIs using the viscosity iterative scheme [Xu 2004] is proposed to avoid directly processing BMIs. Finally, two numerical examples are solved to demonstrate the effectiveness and convergence of the proposed algorithm.

References


H. Mukaidani, H. Xu & W. Zhuang


