On the Throughput of Feedbackless Segmented Network Coding in Delay Tolerant Networks

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***Abstract*—Epidemic routing using random linear network coding has been studied and proved as an efficient way for light data delivery in delay tolerant networks (DTNs). In this paper, we study bulk or stream-like data dissemination in DTNs. Segmented network coding is introduced to provide best-effort services and its performance in terms of sustained throughput is analyzed. To our best knowledge, we are the first to give the closed-form expression of the maximum sustained throughput using feedbackless segmented network coding in DTNs. A protocol is also proposed and simulation results show that it approaches the theoretical bound asymptotically.**

I. Introduction

Delay and Disruption-Tolerant Network (DTN) [1] is a class of networks with no guarantee for contemporaneous and continuous end-to-end connections. It emerges as a good complement to provide services to a variety of delay tolerant applications in some highly challenged environments. Some typical DTN examples are exotic media networks with long propagation delay, military ad-hoc networks with frequent node mobility, sensor networks with low duty cycle. Due to the inherent intermittent connectivity, epidemic routing, which is flooding-based in nature, has been proved an efficient way to tackle the routing issue in DTNs. Many epidemic routing protocols (e.g., PRoPHET [2], MaxProp [3], RAPID [4], Spray and Wait [5], etc.) have been proposed using the simple "store-and-forward" approach that a packet is replicated at each transmission opportunity in a hope that at least one copy will succeed in reaching its destination.

Recently, it has been found that the "encode-and-forward" approach by combining epidemic routing and random linear network coding (RLNC) can achieve improved performance for DTNs, especially under bandwidth and buffer constraints [6]. Using RLNC, a node just simply forwards a random linear combination of packets it has received so far at each transmission opportunity. It simplifies the buffer management such that the decisions at each relay node on which packet should be forwarded are not required. When the destination receives a sufficient number of linearly independent coded packets, the original packets can be recovered. For light data dissemination, it has been proven in [6] that RLNC achieves supreme performance by encoding all packets together. On the other hand, however, it is infeasible to encode the whole packets for bulk data dissemination due to high Gaussian elimination complexity at the destination as well as considerable overhead incurred by a long coefficient vector attached to each encoded packet. This approach is even impossible for stream-like data [7] transmission in DTNs.

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All these reasons motivate us to investigate the segmented network coding (SNC). In this paper, we investigate the behavior of epidemic routing using SNC in DTNs. The major contributions are summarized as follows. (1) To our best knowledge, we are the first to derive the maximum sustained throughput in a closed-form for data streaming using feedbackless segmented network coding in DTNs. (2) We provide two necessary conditions, i.e., the intra-segment coding condition and the inter-segment scheduling condition, for achieving this optimal performance. (3) Based on the understanding of these conditions, we then propose a practical SNC protocol that approaches the theoretical bound asymptotically.

The rest of this paper is organized as follows. Section II describes the network model and problem definition. Section III presents the theoretical analysis on the sustained throughput using SNC in DTNs. A feedbackless SNC-based protocol is then devised from our theoretical findings. The simulation results that verify our analysis are presented in Section IV. Section V concludes this work.

II. Network Model and Problem Formulation

In this paper, we consider a delay and disruption tolerant network with $N+1$ nodes. The encounter interval between any two nodes follows an exponential distribution with a rate λ. This model has been widely adopted in the recent literature, e.g., in [6], [8], and verified by both theoretical analysis [9] and real mobility traces [10].

We first consider the traditional RLNC that delivers a total K number of packets by a source node. Each coded packet P is a linear combination of K native packets $p_1, p_2, \cdots, p_K$ in a form of $P = \sum_{i=1}^{K} \alpha_i p_i$, where α_i, $i = 1, \ldots, K$, are coding coefficients randomly chosen from a Galois Filed (GF) The simulation studies in [6] have discovered that (1) RLNC under a single buffer can achieve almost the same performance as under multiple buffers and (2) each contact can increase the rank of the decoding matrix by one with high probability (w.h.p.). For this reason, we shall apply the above two assumptions in the rest of the paper. Under the single buffer model, at most one packet can be forwarded at each transmission opportunity.

Now, we investigate the segmented network coding for bulk data dissemination in DTNs. The streaming data is partitioned into segments with size K, i.e., each segment consists of K native packets. Note that at least K linearly independent coded packets should be generated by the source for each segment. Subsequently, it may disseminates additional redundant packets in order to improve the decoding probability before it moves to the next segment. Finally, after collecting K linearly independent coded packets belonging to the same segment, the destination node is able to retrieve the whole K native packets of this segment.

Due to the highly stochastic nature of DTNs, we consider a simple feedbackless SNC mechanism, i.e., no acknowledgment to be issued by a destination after its successful decoding of a segment data. Under such a circumstance, missing some segments is tolerable, but a high *sustained throughput* is desired. The sustained throughput is defined as

$$T_s \equiv \lim_{t\to\infty} \frac{K \cdot \sum_{i=1}^{s(t)} P_i(t, K)}{t}, \tag{1}$$

where $s(t)$ is the number of segments that has been sent out from the source node by time t and $P_i(t,k)$ is the probability that k linearly independent coded packets of segment-i have been received at the destination by time t. The major problem under investigation in this paper is to maximize the sustained throughput that can be achieved by segmented network coding in epidemic routing using segmented network coding.

III. Performance Analysis of Segmented Network Coding

We study the dissemination dynamics of a certain segment over a DTN. The theoretical result is first applied to the simplest case of single segment network coding and then extended to the general segmented network coding for streaming data.

Lemma 1. Let $X_i(t)$ be an arbitrary function that represents the expected number of nodes in the network with a coded packet of segment i at time t. The distribution of $P_i(t,k)$ is given by

$$P_i(t,k) = 1 - e^{-g_i(t)} \cdot \sum_{j=0}^{k-1} \frac{(g_i(t))^j}{j!}, \text{ where} \tag{2}$$

$$g_i(t) = \int_0^t \lambda \cdot X_i(t) dt. \tag{3}$$

Proof: Consider each encounter of the destination with another node which has a coded packet of segment i. As verified in [6], the destination node can obtain an innovative packet upon each of such contacts w.h.p. and thus $P_i(t,k)$ can be described by the following ordinary differential equations (ODEs):

$$\begin{aligned} &P_i(0,k) = 0, \quad k = 1, \dots, K \\ &\tfrac{\partial P_i(t,1)}{\partial t} = \lambda \cdot X_i(t) \cdot (1 - P_i(t,1)) \\ &\tfrac{\partial P_i(t,k)}{\partial t} = \lambda \cdot X_i(t) \cdot (P_i(t,k-1) - P_i(t,k)), 2 \le k \le K. \end{aligned} \tag{4}$$

Note that $\lambda \cdot X_i(t)$ in (4) represents the receiving rate of coded packets that can increase the rank of the decoding matrix for segment i at the destination node. The correctness of (4) is proved in [6]. Finally, we can verify that the result given in (2) is exactly the solution of ODEs (4). □

We shall see that the closed-form of $P_i(t,k)$ given in Lemma 1 is critical to the theoretical performance analysis of SNC in the following sections.

A. Single Segment Network Coding

While the theoretical performance of RLNC has been characterized by ODEs, e.g., in [6], an explicit solution in a closed-form has not been presented yet. Using the result of Lemma 1, this becomes possible as given in Theorem 1 below. Note that the subscript of notations is omitted since only one segment is considered in RLNC. For example, $X(t)$ denotes the expected number of nodes carrying coded packets at time t.

Theorem 1. In single segment network coding, the data reception probability $P(t,K)$ by time t is given by:

$$P(t,K) = 1 - \frac{N \cdot \sum_{j=0}^{K-1} \frac{1}{j!} \cdot \left(\ln \frac{N-1+e^{\lambda \cdot N \cdot t}}{N}\right)^j}{N - 1 + e^{\lambda \cdot N \cdot t}}. \tag{5}$$

Proof: Recalling that the encounter interval between any pair of nodes is exponentially distributed, we have the following ODEs to describe the dynamics of $X(t)$:

$$X(0) = 1, \quad X'(t) = \lambda \cdot X(t) \cdot (N - X(t)). \tag{6}$$

By solving (6) and (3), we obtain the expressions for $X(t)$ and $g(t)$ as follows:

$$X(t) = \frac{N \cdot e^{\lambda \cdot N \cdot t}}{N - 1 + e^{\lambda \cdot N \cdot t}} \tag{7}$$

$$g(t) = \ln(N - 1 + e^{\lambda \cdot N \cdot t}) - \ln N. \tag{8}$$

Finally, substituting (8) and $k = K$ into (2) leads to (5). □

B. Segmented Network Coding

When multiple segments coexist in the network, we have the following lemma for segmented network coding.

Lemma 2. Let $s(t)$ be the number of different segments that have been sent out from the source node by time t. Then $\sum_{i=1}^{s(t)} X_i(t) \le X(t)$ holds at any time instance t.

Proof: Recall that $X_i(t)$ is an arbitrary function that represents the expected number of nodes in the network with a coded packet of segment i at time t. Therefore, $\sum_{i=1}^{s(t)} X_i(t)$ presents the expected number of nodes with a coded packet for an arbitrary dissemination scheme using segmented network coding. Because epidemic routing scheme, characterized by ODE (6), is the fastest manner for data dissemination, its solution $X(t)$ is the maximum expected number of nodes with a coded packet, i.e., $\sum_{i=1}^{s(t)} X_i(t) \le X(t)$. □

Lemma 2 and (3) immediately lead to

$$\begin{aligned} \sum_{i=1}^{s(t)} g_i(t) &= \sum_{i=1}^{s(t)} \int_0^t \lambda \cdot X_i(t) dt \\ &= \int_0^t \lambda \cdot \sum_{i=1}^{s(t)} X_i(t) dt \le \int_0^t \lambda \cdot X(t) dt = g(t). \end{aligned} \tag{9}$$

Suppose the average number of coded packets disseminated by the source for each segment is $K' = \lim_{t\to\infty} \lambda N t / s(t)$, where $\lambda N t$ represents the expected total number of contacts between the source and other nodes. We use $T_s(K')$ to denote the corresponding sustained throughput achieved by an arbitrary SNC protocol under this condition. Note that $K' \geq K$ must be satisfied for a non-zero sustained throughput. The following theorem provides a theoretical upper bound of $T_s(K')$ for epidemic routing using SNC.

Theorem 2. In segmented network coding, the sustained throughput $T_s(K')$ is bounded by:

$$\begin{aligned} T_s(K') &\leq T_s^*(K') \\ &\equiv \frac{\lambda \cdot N \cdot K}{K'} \cdot \left(1 - e^{-K'} \cdot \sum_{j=0}^{K-1} \frac{(K')^j}{j!}\right). \end{aligned} \tag{10}$$

Proof: We construct a new function $F(x)$ such that $F(g_i(t)) = P_i(t, K)$, given by

$$F(x) = 1 - e^{-x} \cdot \sum_{j=0}^{K-1} \frac{x^j}{j!}.$$

Note that $F(x)$ is unconditionally concave when $K = 1$ because $F'(x) = e^{-x} > 0$ and $F''(x) = -e^{-x} < 0$. When $K \geq 2$, $F(x)$ is still concave if $x > K - 1$. This is because

$$\begin{cases} F'(x) = e^{-x} \cdot \frac{x^{K-1}}{(K-1)!} > 0, \\ F''(x) = e^{-x} \cdot \frac{x^{K-2}}{(K-2)!} \cdot \left(1 - \frac{x}{K-1}\right) < 0. \end{cases} \tag{11}$$

Under this condition, we have the following derivations using Jensen's inequality and (9) as

$$\begin{aligned} &\frac{1}{t} \cdot K \cdot \sum_{i=1}^{s(t)} P_i(t, K) = \frac{1}{t} \cdot K \cdot \sum_{i=1}^{s(t)} F(g_i(t)) \\ \leq &\frac{1}{t} \cdot s(t) \cdot K \cdot F\left(\frac{\sum_{i=1}^{s(t)} g_i(t)}{s(t)}\right) \leq \frac{1}{t} \cdot s(t) \cdot K \cdot F\left(\frac{g(t)}{s(t)}\right) \\ = &\frac{1}{t} \cdot s(t) \cdot K \cdot \left(1 - e^{-\frac{g(t)}{s(t)}} \cdot \sum_{j=0}^{K-1} \frac{1}{j!} \cdot \left(\frac{g(t)}{s(t)}\right)^j\right) \end{aligned} \tag{12}$$

where the equality will hold under the condition:

$$g_i(t) = \frac{g(t)}{s(t)} > K - 1, \forall i \geq 1. \tag{13}$$

In a long run, we have the following observations based on the definition of K':

$$\begin{aligned} \lim_{t\to\infty} \frac{s(t)}{t} &= \frac{\lambda \cdot N}{K'}, \\ \lim_{t\to\infty} \frac{g(t)}{s(t)} &= \lim_{t\to\infty} \frac{\ln(N - 1 + e^{\lambda \cdot N \cdot t}) - \ln N}{s(t)} \\ &= \lim_{t\to\infty} \frac{\lambda \cdot N \cdot t}{s(t)} = K'. \end{aligned}$$

When t approaches to infinity, (12) becomes

$$\begin{aligned} T_s(K') &= \lim_{t\to\infty} \frac{1}{t} \cdot K \cdot \sum_{i=1}^{s(t)} P_i(t, K) \\ &\leq \lim_{t\to\infty} \frac{1}{t} \cdot s(t) \cdot K \cdot \left(1 - e^{-\frac{g(t)}{s(t)}} \cdot \sum_{j=0}^{K-1} \frac{\left(\frac{g(t)}{s(t)}\right)^j}{j!}\right) \\ &= \frac{\lambda \cdot N \cdot K}{K'} \cdot \left(1 - e^{-K'} \cdot \sum_{j=0}^{K-1} \frac{(K')^j}{j!}\right). \quad \square \end{aligned}$$

Theorem 2 provides two necessary conditions for achieving the maximum sustained throughput T_s^* of SNC: (i) the inter-segment scheduling condition as required by (13) and (ii) the intra-segment coding condition $K' = K^*$, where

$$K^* = \arg\max_{K' \geq K} \frac{1}{K'} \cdot \left(1 - e^{-K'} \cdot \sum_{j=0}^{K-1} \frac{(K')^j}{j!}\right). \tag{14}$$

Therefore, the maximum sustained throughput can be expressed as

$$T_s^* = \max_{K' \geq K} T_s^*(K') = T_s^*(K^*). \tag{15}$$

C. A Segmented Network Coding Protocol

Based on the above two conditions, we propose a protocol for segmented network coding in this subsection. From (3), $g_i(t)$ represents the expected number of encounters happened till t between destination and relay nodes carrying coded packets of segment i. Similarly, $g(t)$ is the expected total number of contacts of the destination by time t. Therefore, the inter-segment scheduling condition (13) can be explained as: (i) for each segment i, the expected number of encounters between destination and relay nodes with packets belonging to segment i should exceed $K-1$ and (ii) the expected number of such encounters should be the same for all segments. The first requirement is obvious because otherwise the corresponding segment is impossible to be decoded. The second one suggests us to disseminate the same number of coded packets at the source for each segment.

To satisfy the intra-segment coding condition (14), we check the derivative of $T_s^*(K')$ given in (10) with respect to $K^{'}$

$$\begin{aligned} \frac{dT_s^*(K')}{dK'} =& \lambda \cdot N \cdot K \cdot (K')^{K-2} \cdot e^{-K'} \\ &\cdot \left(\frac{K-1}{K!} - \sum_{j=1}^{\infty} \frac{(K')^j}{(K+j)!}\right) \end{aligned} \tag{16}$$

and find that $T_s^*(K')$ is a decreasing function of K' when $K' \geq K^2 - 1$. In other words, the optimal expected number of coded packets sent by the source for each segment K^* should fall into $[K, K^2 - 1]$. In summary, Algorithm 1 specifies the protocol under the assumption that the source node has the knowledge of the N and λ values.

Algorithm 1

1: For each segment, the source keeps sending $K^* = \arg\max_{K \le K' \le K^2 - 1} T_s^*(K')$ coded packets, where $T_s^*(K')$ is defined in Theorem 2.
2: For intra-segment forwarding, RLNC is performed.
3: For inter-segment forwarding, a higher priority is given to a fresher segment. In other words, the buffer will be overwritten at the node with a packet belonging to an older segment.

IV. Validation and Performance Evaluation

In this section, we use simulations to verify the correctness and accuracy of our theoretical analysis. We have developed a discrete-event simulator with the implementation of both epidemic routing and network coding. The simulation environment, including both mobility model and transmission model, strictly follows what is described in Section II.

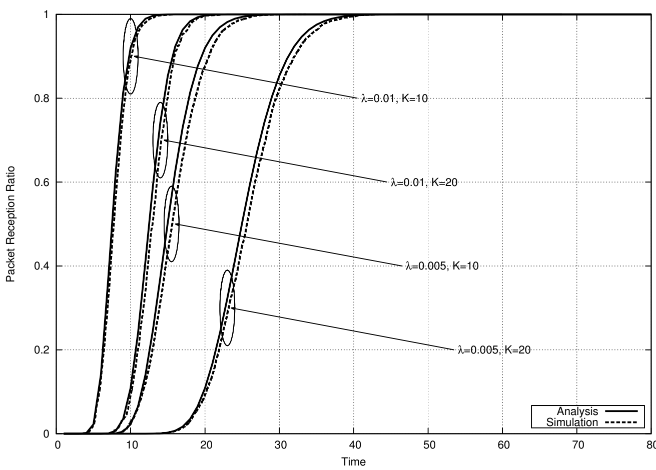

Fig. 1. Data Reception Probability

We first validate the accuracy of our analysis on the data reception probability (DRP) in single segment network coding. For each network setting with various values of K and λ, the empirical cumulative distribution of DRP is obtained from 4000 simulation instances with independent random seeds. These results are compared with the theoretical distribution given in Theorem 1 as demonstrated in Fig. 1. We observe that our derived analytical results are very close to but slightly outperform the simulation results under various network settings. This is because in our ODEs (4) we approximate the probability that the destination node obtains one innovative packet in a very short time interval $(t, t+\Delta t)$ by $\lambda \cdot X_i(t) \cdot \Delta t$, which is slightly larger than the exact probability $1 - e^{-\lambda \cdot X_i(t) \cdot \Delta t}$.

Based on Theorem 2, we propose a feedbackless SNC-based protocol Algorithm 1. We conduct two groups of experiments with various network sizes (N = 100, 200) and segment sizes ($K \le 100$). The experimental results are obtained by averaging the sustained throughput over 1000 simulation rounds under each network setting with $\lambda = 0.005$. Analytical results of maximum sustained throughput are calculated using (10) from Theorem 2. As illustrated in Fig. 2, Algorithm 1 approaches the asymptotical theoretical bound quickly. Furthermore, we observe that the maximum sustained throughput is an increasing function of segment size K, but the increasing rate degrades quickly when K is large. Even though the dissemination policy in Algorithm 1 is simple and intuitive, the numerical results demonstrate that it performs well.

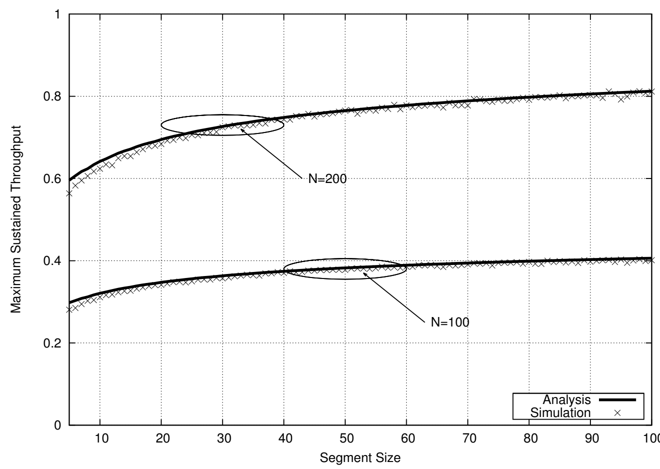

Fig. 2. Maximum Sustained Throughput

V. Conclusion

In this paper, we investigate the theoretical sustained throughput of segmented network coding in DTNs for bulk data dissemination. We analyze the dynamics of decoding process and derive the closed-form expression for the maximum sustained throughput. Simulation studies also validate our theoretical findings.

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