Access Point Association in Uplink Two-Hop Cellular IoT Networks with Data Aggregators

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Abstract—Node clustering and data aggregation help extend the coverage of cellular networks and increase the number of supported devices, while meeting the various service quality requirements and reducing energy consumption, making them suitable for enabling future massive cellular IoT applications. Consequently, we propose to overlay the cellular network with a layer of data aggregators (DAs) to act as relays. DAs use cellular back haunting, and thus, the network provides both single and two-hop routes; however, DAs share radio resources with single-hop devices, creating dependency between the two routes. Thus, the proper design of the DA enabled network becomes critical for cost effectiveness and efficient radio resource utilization. In this work, we formulate a joint access point association, resources utilization, and energy efficient communication optimization problem that takes into account various networking factors such as number of devices, number of DAs, number of available resource units, interference, transmission power limitation of the devices, DA transmission performance, and channel conditions. The objective is to show the usefulness of data aggregation and shed light on the importance of network design when the number of devices is massive. We propose a coalition game theory based algorithm, PAUSE, to transform the optimization problem into a simpler form that can be successfully solved in polynomial time. Different network scenarios are simulated to showcase the effectiveness of PAUSE and to draw observations on cost effective network design with DAs.

Index Terms—Data aggregation, resource allocation, user association, coalition game theory, Internet of Things (IoT).

I. INTRODUCTION

Internet of Things (IoT) is a networking paradigm where a massive number of simple devices gain internet access through the cellular network to facilitate every day life activities. Nonetheless, providing access to such a massive number of devices requires non-traditional networking solutions. Many IoT applications rely on a massive number of battery powered devices that produce small data packets with a wide range of service quality requirements. Given the small size of the generated data packets, running out of resources is not the main concern. The challenge rather lies in how to use the available resources effectively, while avoiding congestion of the random access channel (RACH) of the cellular network due to massive concurrent access requests, as the congestion leads to longer delays and higher energy consumption [1].

Recently, data aggregation has seen a growing interest as a better suited and cheaper approach for providing channel access to many cellular IoT applications [2]–[5]. In this approach, the cellular network is overlaid with a layer of data aggregators (DAs) which function as data collection nodes and relays. In the network, devices have the choice of directly connecting with BSs or going through DAs to transmit their data in a two-hop fashion. Data packets from DA associated devices are first aggregated into larger data packets and then transmitted from the DA to a nearby BS. Data aggregation has the advantage of providing energy efficient communication as the DAs are often at a closer proximity to the devices than the cellular BSs. It also improves resource utilization with increased payload in each larger packet. This approach presents a scalable and cost effective alternative to small cell heterogeneous networks (SCHNs).

As the devices have the option of choosing between single- and two-hop transmissions, and each provides different network performance, deciding to which AP devices connect is of critical importance and should be carefully designed such that resource utilization is improved, while reducing the energy consumption of the devices. This problem is known as the access point (AP) association (APA) problem. APA is similar to the traditional radio access technology (RAT) association problem in SCHNs, where a device has the option of connecting to a small cell, a macro cell, or a WiFi access point [6]. However, different from RAT association, in APA, devices have the choice of using both single or multi-hop transmission paradigm. The difference introduces dependency between different APs in terms of resource utilization, since DAs are cellular users in this case and share the cellular resources with directly connected devices [7]. In this paper, we aim at designing an APA method that jointly enhance network performance and reduce energy consumption of the devices, while catering for their minimum throughput requirements. We use a centralized approach to formulate the APA problem as a mixed integer non-liner programming problem and develop a game theory based heuristic algorithm to solve it. To the best of our knowledge, besides the very recent work by Ibrahim et al. [7], the APA problem is still new and is yet to be studied in depth. The main contributions of this work are:

• **DA infused cellular network:** Motivated by the findings from our previous studies [1], [8], that data aggregation can alleviate congestion of the RACH channel in cellular IoT applications with massive number of devices, we propose to use data aggregation as a cost effective way to improve resource utilization and reduce energy consumption, while catering for the different QoS requirements. The cellular network is overlaid with DA nodes that help...
off-load some data traffic from the devices, leading to less congestion at the RACH channel. With proper user association, power control, and resource allocation, the DA infused network can be optimized to provide energy efficient access to a massive number of IoT devices;

- **Novel joint problem formulation**: We consider joint optimization of AP association, efficient resource utilization, and device transmission power control in a DA infused cellular network. The objective is to maximize the number of QoS satisfied devices, while minimizing their total transmission power. We consider the limitations on the maximum allowable transmit power of the devices and the heterogeneity of different APs in terms of available resources;

- **Problem simplification**: The original multi-objective joint optimization problem is mixed-integer nonlinear and non-convex in nature, requiring discrete device-AP associations as well as discrete channel assignments, while the transmission power of the devices is continuous within a range. We propose a novel algorithm, referred to as PAUSE (Power control, resource Allocation, User association, QoS Satisfaction and Energy consumption optimization) algorithm, to effectively solve the optimization problem based on coalition formation game and difference between two concave functions programming (D.C. programming) optimization theories to solve the problem in polynomial time;

- **Case studies**: Results show the advantages of the proposed Algorithm in terms of maximizing the number of successfully supported devices and minimizing the overall energy consumption.

The rest of this paper is organized as follows. System model is presented in Section II. Section III details the problem formulation and presents the general multi-objective optimization problem. In Sections IV and V, we describe the proposed heuristic algorithm to efficiently solve the joint optimization problem, and discuss its convergence and complexity. In Section VI, we evaluate the performance of the algorithm by means of simulations. We conclude this study in Section VII.

II. SYSTEM MODEL

A. Physical network

Consider an uplink network made up of a layer of BSs (Figure 1). BS locations form a point Poisson process (PPP) \( \Psi_b = b_1, b_2, ... \) with density \( \lambda_b \), where \( b_i \) denotes the location of the \( i^{th} \) BS in the network. Let \( B = |\Psi_b| \) denote the cardinality of set \( \Psi_b \), i.e., the number of BSs in the network. The network is overlaid with a layer of DAs uniformly distributed over the network coverage. DA locations form a PPP \( \Psi_a = a_1, a_2, ... \) with density \( \lambda_a > \lambda_b \), where \( a_i \) denotes the location of the \( i^{th} \) DA. Let \( A = |\Psi_a| \) denote the cardinality of set \( \Psi_a \). Coverage area of a DA is much smaller than that of a BS. We approximate the coverage area of a DA by a disk of radius \( \Lambda_a \). In the following, we use AP to denote both BS and DA wherever there is no ambiguity.

The network supports a large number, \( D \), of low mobility and battery powered IoT devices. Let \( \Psi_d = \{d_1, d_2, ..., d_D\} \) denote the location set of the devices, where \( d_i \) denotes the location of the \( i^{th} \) device. A device can associate with only one AP at a time. A device may be located in a single covered area (i.e., area covered only by a BS), or in a double covered area (i.e., area covered a BS and one or more DA). A device located in a double covered area has the choice of associating with the BS or the DA that best meets its QoS requirements and minimizes its transmission power.

Denote the set of devices associated with BS \( j \) as \( \Psi_{b_j} = \{d_1, d_2, ..., d_k\} \), where \( d_i \) denotes the location of the \( i^{th} \) device associated with BS \( b_j \). Similarly, let \( \Psi_{a_h} = \{d_1, d_2, ..., d_k\} \) denote the location set of the devices associated with the \( h^{th} \) DA, where \( d_i \) denote the location of the \( i^{th} \) device associated with DA \( a_h \). Finally, let \( \Psi_b = \{b_1, b_2, ..., b_g\} \) and \( \Psi_a = \{a_1, a_2, ..., a_h\} \) denote the super location sets of the assigned devices to all BSs and DAs respectively. Notice that a device may not be associated in a transmission frame, thus \( |\Psi_b| + |\Psi_a| \leq D \). Also, as a device is allowed to associate with only a single AP at a time, \( \Psi_k \cap \Psi_j = \emptyset, \) for \( k \neq j \in \{\Psi_b, \Psi_a\} \).

We use binary indication variables \( x_{d_i,b_j} \) and \( y_{d_i,a_h} \) to indicate the association status of the \( i^{th} \) device with the \( j^{th} \) BS and the \( h^{th} \) DA respectively. The variables equal 1 if the device is associated with that particular AP, and 0 otherwise. A device cannot have both \( x_{d_i,b_j} \) and \( y_{d_i,a_h} \) equal to 1 as the device can only associate with one AP at a time. A device is considered associated to an AP if and only if the AP is able to provide the necessary QoS level required by the device.

![Fig. 1: An illustration of BS, DA, and IoT device locations as well as the Voronoi tessellation formed by the coverage of the BSs and the circular coverage of the DAs.](image)
network in which all RUs at each BS has a DA scheduled and that DAs do not change their RU assignments. Sub-band \( C_d^1 \) is divided into, \( K = \{1, 2, \ldots, K\} \), equal bandwidth, \( \omega_b \), channels referred to as resource blocks (RBs). Depending on the QoS requirements, interference level, location, and transmission power, a single device may be allocated one or more RBs to accommodate its needs. Each device connected to a BS can be scheduled on one RB at a time; however, multiple devices can be scheduled on the same RB while connecting to different BSs. Thus, directly connected devices may experience inter-cellular interference.

Devices that transmit in two-hop fashion via DAs, share a device-DA sub-band, denoted by \( C_d^2 \), that is orthogonal to both bands \( C_a \) and \( C_d^1 \). Sub-band \( C_d^2 \) consists of \( N = \{1, 2, \ldots, N\} \) resource channels (RCs) of equal bandwidth \( \omega_c \). Similar to BS-directly connected devices on the RBs, a device connected to a DA on \( C_d^2 \) may be assigned one or more RCs depending on its QoS requirements, signal to interference ratio (SIR), and resource availability. An RC can only be used by one device connecting to the same DA, yet devices can be assigned the same RC at different DAs. Thus, two-hop connecting devices experience only inter-cluster interference.

Signal transmissions experience propagation attenuation according to a general power-law path-loss model. The signal power decays at rate \( D^{-\alpha} \), where \( D \) is the distance and \( \alpha \) is the path-loss exponent. All transmissions suffer from Rayleigh fading that introduces random instantaneous power gain, \( g \), following an exponential distribution with unity mean (i.e. \( g \sim \exp\{1\} \)). The channel power gains are distance independent, independent of each other and identically distributed (i.i.d.). The network capacity is interference limited.

DAs are assumed to have access to infinite power supply and hence they employ full power inversion to compensate for path-loss attenuation. Let \( q_{a} \) denote the nominal transmission power of any DA in the network. We focus on a DA located at \( a \) and associated with the BS located at \( b \). Accordingly, the initial transmission power of the DA after full power inversion to compensate for the path-loss attenuation is given by \( q_a = q_{a}||a|-|b||^{\alpha} \). Consequently, the received power at the BS from the DA is given by \( g_{a} = q_{a}g \), which eliminates the impact of path-loss attenuation. This assumption is made for simplicity of analysis as it reduces the complexity due to distance-based path-loss attenuation. In contrast, devices are assumed to be battery powered and thus there is an upper limit on the maximum transmission power that can be used by any device. The transmission power of each device is to be allocated based on the optimization problem that best provides the required QoS satisfaction at the minimum energy consumption. Accordingly, let \( q_{d_i} = \{q_{d_i}^k, b_j, q_{d_i}^n, a_k\} \) denote the transmit power of the \( i^{th} \) device when transmitting to AP \( \in \{b_j, a_k\} \) on the uplink scheduled resource \( x \in \{k, n\}, \) for \( b_j \in \Psi_b, a_h \in \Psi_a, k \in K \) and \( n \in N \). Let the maximum transmit power of a device be denoted by \( Q_{\text{max}} \). The achievable data rate by the \( i^{th} \) device on the \( k^{th} \) RB between the \( i^{th} \) device (associated with the \( j^{th} \) BS) and the \( k^{th} \) BS will be denoted by \( g_{d_i,j,b_k}^k \). Also, let \( q_{d_i,b} \) be \( 0 \) and \( s_{d_i,b} \in \{0, 1\} \) be the transmission power level and the binary RB assignment variables of the \( i^{th} \) device from the \( j^{th} \) BS on the \( k^{th} \) RB. As a device can be assigned multiple RBs at the same time, \( Q_{d_i,b} = \{q_{d_i,b}^k, \ldots, q_{d_i,b}^n\} \) and \( S_{d_i,b} = \{s_{d_i,b}^k, \ldots, s_{d_i,b}^n\} \) are vectors of the overall transmission power and RB assignment vectors of the \( i^{th} \) device from the \( j^{th} \) BS on all \( K \) RBs. Accordingly, we have super sets \( Q_{b} = \{Q_{d_i,b}, Q_{d_j,b}, \ldots, Q_{d_I,b}, \ldots, Q_{d_{N_b},b}\} \) and \( S = \{S_{d_i,b}, S_{d_j,b}, \ldots, S_{d_I,b}, \ldots, S_{d_{N_b},b}\} \) as the collection of the overall transmission power and RB assignment vectors for all BS-connected devices. The achievable data rate by the \( i^{th} \) device on the \( j^{th} \) BS on the \( k^{th} \) RB, using Shannon’s channel capacity formula, is

\[
R_{d_i,b}^k = \omega_b \log_2 \left( 1 + \frac{q_{d_i,b}^k ||d_i - b_j||^{-\alpha g_{d_i,j,b_j}^k} }{ \sum_{b_h \in \Psi_b, d_m \in \chi_b} s_{d_m,b_h}^k q_{d_m,b_h}^n ||d_m - b_j||^{-\alpha g_{d_m,b_h}^k} } \right)
\]

where \( X / x_i \) means all items in set \( X \) excluding item \( x_i \).

D. Two-hop instantaneous data rate

Different from the one-hop transmission, in the two-hop route, a device first transmits data to its serving DA which then relays the data to the serving BS. The achievable data rate using the two-hop route is the minimum of the achievable rates of the two links. For the first link (device to DA), the same analysis from Sub-section II-C can be applied with notational variation as follows. As devices connected to DAs share \( N = \{1, 2, \ldots, N\} \) RCs, where \( n \in N \) indicates the RC index, let the channel power gain on the \( n^{th} \) RC from the \( j^{th} \) DA between the \( i^{th} \) device and the \( h^{th} \) DA be denoted by \( g_{d_i,j,h}^n \). Also, let \( q_{a}^n, a_j \geq 0 \) and \( v_{a,i}^n, d_i \in \{0, 1\} \) be the transmission power level and the binary RC assignment variables of the \( i^{th} \) device from the \( j^{th} \) DA on the \( n^{th} \) RC. Further, vectors \( Q_{d_i,a_j} = \{q_{d_i,a_j}^1, \ldots, q_{d_i,a_j}^N\} \) and \( V_{d_i,a_j} = \{v_{d_i,a_j}^1, \ldots, v_{d_i,a_j}^N\} \) indicates the overall transmission power and RC assignments of the \( i^{th} \) device from the \( j^{th} \) DA on all \( N \) RCs. Accordingly, we have super sets \( Q_a = \{Q_{d_i,a_j}, Q_{d_j,a_j}, \ldots, Q_{d_I,a_j}, \ldots, Q_{d_{N_b},a_j}\} \) and \( V = \{V_{d_i,a_j}, V_{d_j,a_j}, \ldots, V_{d_I,a_j}, \ldots, V_{d_{N_b},a_j}\} \) as the collection of the overall transmission power and RC assignment vectors for all DA-connected devices in the network. The achievable data rate by the \( i^{th} \) device on the \( n^{th} \) RC from the \( j^{th} \) DA, using Shannon’s channel capacity formula, is

\[
R_{d_i,a_j}^n = \omega_c \log_2 \left( 1 + \frac{q_{a}^n ||d_i - a_j||^{-\alpha g_{a}^n} }{ \sum_{a_h \in \Psi_a,a_j} s_{a_h,a_j}^n q_{a_h,a_j}^n ||d_m - a_j||^{-\alpha g_{a_h,a_j}^n} } \right)
\]

As for the second link (DA to BS), DAs share \( L = \{1,2,\ldots,L\} \) of RUs when relaying their aggregated data packets. As we consider a fully loaded network of DAs such that there is a DA scheduled on every RU at every BS, let
\( \Phi_d^j = \{ a_0^j, a_1^j, a_2^j, ..., a_l^j \} \) denote the location set of inter-cell interfering DAs on the \( l \)th RU, where \( a_l^j \) is the location of the DA scheduled on the \( l \)th RU of the \( j \)th BS, for \( j \in \Psi_b \). Further, let \( g_{a_j,k} \) denote the channel power gain on the \( l \)th RU from the \( j \)th BS to the \( k \)th BS. We focus on a typical DA located at \( a_0^j \) and associated with the BS located at the origin. Using Shannon’s channel capacity formula, the achievable data rate is given by

\[
R_{an}^l = \omega_n \log_2 \left( 1 + \sum_{a_l'^j \in \Phi_d^j} q_{a_l'^j} g_{a_l'^j,0} \right)
\]

As a result, the achievable data rate by the \( i \)th device from the \( h \)th BS via two-hop route, going through the DA located at \( a_h \) towards the serving BS, is

\[
R_{d,2} = \min \{ R_{an}, \sum_n v_{d,ah}^n R_{d,1,aj}^n \}
\]

III. Problem formulation

Devices have QoS requirements and want to minimize their energy consumption as they are battery powered. Due to the limited radio resources at the APs, maximizing the resource utilization means maximizing the number of satisfied devices. In what follows, we formulate an optimization problem to maximize the utility of the network, defined as the number of successfully served devices given their data rate requirements, while minimizing the energy consumption of the devices via the minimization of the total transmit power of all served devices [9]. The outcome of the optimization problem is a set of device-AP association matrices, a vector containing the transmission power assignments of all served devices, and a set of RB and RC assignment matrices for resource allocation. For brevity, we write \( X = [x_{d,1}] \), \( Y = [y_{d,1}] \), \( V = [v_{d,1,n}] \), \( N = [n_{d,1,n}] \), \( S = [s_{d,1}] \), \( Q_b = [q^b_{d,1}] \), and \( Q_a = [q^a_{d,1}] \) as the BS association matrix, DA association matrix, RC assignment matrix, RB assignment matrix, and the transmit power vectors to BSs and DAs respectively. We refer to this problem as the joint AP association, power control, and resource allocation problem, formulated as

\[
\begin{align*}
\text{PI-1:} & \quad U^* = \max_{X,Y,V,S,Q_b,Q_a} \left( \sum_{b_j \in \Phi_p} \sum_{d_i \in \Psi_d} x_{d,1,b_j} + \sum_{a_n \in \Psi_a} \sum_{d_i \in \Psi_d} y_{d,1,a_n} \right) \\
\text{s.t.} & \quad x_{d,1,b_j} \sum_{k=1}^K s_{d,1,b_j}^k R_{d,1,2,1} + y_{d,1,a_n} \geq R_{d,1,2} \geq R_{\min} \quad \forall m \in \Psi_d \quad (C1) \\
& \quad \sum_{b_j \in \Phi_p} x_{d,1,b_j} + \sum_{a_n \in \Psi_a} y_{d,1,a_n} \leq 1 \quad \forall d_i \in \Psi_d \quad (C2) \\
& \quad \begin{cases} x_{d,1,b_j} \in \{0,1\}, \quad & \text{if } d_i \in \theta_{b_j} \forall b_j \in \Psi_p \\
& \text{otherwise} \end{cases} \quad (C3) \\
& \quad \begin{cases} y_{d,1,a_n} \in \{0,1\}, \quad & \text{if } d_i \in \theta_{a_n} \forall a_n \in \Psi_a \\
& \text{otherwise} \end{cases} \quad (C4) \\
& \quad \sum_{d_i \in \Psi_d} s_{d,1,b_j}^k \leq 1, \quad \forall b_j, & k \quad (C5) \\
& \quad \sum_{d_i \in \Psi_d} v_{d,1,a_n}^n \leq 1, \quad \forall a_n, & n \quad (C6) \\
& \quad s_{d,1,b_j}^k = [0,x_{d,1,b_j}], \quad \forall d_i, b_j, & k \quad (C7) \\
& \quad v_{d,1,a_n}^n = [0,y_{d,1,a_n}], \quad \forall d_i, a_n, & n \quad (C8) \\
& \quad 0 \leq x_{d,1,b_j} \sum_{k=1}^K s_{d,1,b_j}^k R_{d,1,2,1} + y_{d,1,a_n} \sum_{n=1}^N v_{d,1,a_n}^n q_{d,1,a_n} \leq Q_{\max} \quad (C9)
\end{align*}
\]

\[
\begin{align*}
\text{PI-2:} & \quad \min_{X,Y,V,S,Q_b,Q_a} \sum_{b_j \in \Phi_p} \sum_{d_i \in \Psi_d} x_{d,1,b_j} \sum_{k=1}^K s_{d,1,b_j}^k R_{d,1,2,1} + y_{d,1,a_n} \sum_{n=1}^N v_{d,1,a_n}^n q_{d,1,a_n} \\
\text{s.t.} & \quad \begin{align*}
& \quad + \sum_{a_n \in \Psi_a} \sum_{d_i \in \Psi_d} y_{d,1,a_n} \sum_{n=1}^N v_{d,1,a_n}^n q_{d,1,a_n} \quad (C10) \\
& \quad \max_{X,Y,V,S,Q_b,Q_a} \left( \sum_{b_j \in \Phi_p} \sum_{d_i \in \Psi_d} x_{d,1,b_j} + \sum_{a_n \in \Psi_a} \sum_{d_i \in \Psi_d} y_{d,1,a_n} \right) = U^* \quad (C11)
\end{align*}
\end{align*}
\]

(C1) - (C9).
In P1, (C1) is to guarantee that a device is assigned service if it is able to obtain an average date rate no less than its minimum average rate requirement $R_{\text{min}}$. Constraint (C2) is to ensure that a device is associate only with one AP at a time. In constraints (C3) and (C4), $d_i \in \theta_z$ means that the statement is valid if and only if the $i^{th}$ device is in the coverage of the $z$ AP, for $z \in \{b_j, \theta_k\}$. Constraints (C5) and (C6) are to ensure that an RB or RC is occupied only by at most one device from the same AP’s set of associated device. Constraints (C7) and (C8) draw the relationship between resource allocation and user association dynamics, where a device is only allowed to occupy an RB of a BS or RC of a DA if and only if it is associated with that particular AP. Constraint (C9) ensures that any device is not allowed to transmit at a power level higher than $Q_{\text{max}}$. The second-stage sub-problem (C10) means that every served device should be associated with the right AP with the minimum total transmit power when the system utility in the first-stage sub-problem is maximized, which is reinforced by the constraint given in (C11). The second-stage sub-problem is also subject to the same constraints defined in (C1) through (C9), represented by constraint (C12).

Problem P1 is a complex multi-objective mixed integer non-convex optimization problem, as it contains discrete indicative binary variables as well as a continuous variable representing the transmission power of the devices, and has a non-linear constraint presented in the logarithmic form of the achievable data rate. Furthermore, as the optimization objectives are part of the constraints of each other, this creates a duality that adds another dimension of complexity to the problem, forbidding the use of simple convex optimization arguments to solve P1. One way of simplifying the problem is to formulate it as a single objective optimization problem which can be done via the method of weighted sum [10], given as

\[
P_2: \min_{{x, y, v, q, Q, g}} \epsilon \left( \sum_{b_j \in \Psi_b} \sum_{d_i \in \Theta_{b_j}} x_{d_i, b_j} q_{d_i}^{b_j} + \sum_{a_h \in \Psi_a} \sum_{d_i \in \Theta_{a_h}} y_{d_i, a_h} q_{d_i}^{a_h} \right) - (1 - \epsilon) \left( \sum_{b_j \in \Psi_b} \sum_{d_i \in \Theta_{b_j}} x_{d_i, b_j} + \sum_{a_h \in \Psi_a} \sum_{d_i \in \Theta_{a_h}} y_{d_i, a_h} \right)
\]

s.t. \hspace{10pt} (C1) – (C9)

where $q_{d_i}^{b_j} = \sum_{k=1}^{K} s_{d_i, b_j} q_{d_i, b_j}$ is the total transmission power of the $i^{th}$ device from the $j^{th}$ BS. $q_{d_i}^{a_h} = \sum_{n=1}^{N} y_{d_i, a_h} q_{d_i, a_h}$ is the total transmission power of the $i^{th}$ device from the $h^{th}$ DA, and $\epsilon$ is a constant that satisfies the following inequality for P2 to be equivalent to P1 (Proof in Appendix)

\[
0 \leq \epsilon \leq \frac{1}{1 + \sum_{a_h \in \Psi_a} Q_{\text{max}}}. \hspace{10pt} (2)
\]

Although P2 is a single objective optimization problem and is simpler than yet equivalent to P1, it is still complex and combinatorial in nature due to the nested dependency between the objectives and constraints. Furthermore, problem P2 can be classified as mixed-integer non-convex optimization problem due to the multiplication between discrete and continuous variables. Thus, traditional convex approaches cannot be applied. In what follows, we propose a novel approach to solve problem P2, based on the theory of coalition formation games, where the problem is divided into multiple simpler sub-problems that can be tackled one at a time.

IV. USER ASSOCIATION, RESOURCE ALLOCATION, AND POWER CONTROL ALGORITHM

In this section, we present a novel Power control, resource Allocation, User association, QoS Satisfaction and Energy consumption optimization (PAUSE) algorithm, to solve P2. The key idea is to divide P2 into simpler sub-problems by means of Coalition formation game theory and use the appropriate optimization techniques to solve them.

A. Preliminaries on Coalition formation games

In coalition formation games, players attempt to cluster such that the benefit gained by each player from the cooperative clustering decisions is maximized, over the gains when they selfishly maximize their individual utilities. A coalition formation game is defined by means of three attributes: i) the set of all players (i.e., the set of devices, $D$) participating in the game to form cooperative clusters; ii) the partition of these players into clusters, denoted by $P = \{C_1, ..., C_T\}$, which is a collection of $T$ coalitions; and iii) the coalition value, $\mathcal{V}(C_j, P)$, which quantifies the gain of the $j^{th}$ coalition, $C_j$, given partition $P$. The objective of a coalition formation game is to find a partition that maximizes the gain value of all of its coalitions. Let $P_1$ and $P_2$ be two possible partitions of the devices in our considered DA infused network. In this context, a partition is essentially equivalent to determining which devices associate with which AP. Accordingly, to determine if the set of device associations corresponding to partition $P_1$ is better than the set of device associations corresponding to partition $P_2$, the following condition must be met:

\[
\sum_{j \in \Psi_b, \Psi_a} \mathcal{V}(C_j, P_1) < \sum_{j \in \Psi_b, \Psi_a} \mathcal{V}(C_j, P_2) \hspace{10pt} (3)
\]

where the “less than” inequality is to find the partition that minimizes the objective function in the context of P2.

B. Coalition formation game for P2

When applying the framework of coalition formation games to solving P2, the objective is to find the best user association decisions such that a user is associated with the AP which satisfies its data rate requirement while minimizing its energy consumption. As we have two tiers of APs, namely BSs and DAs, we create a super set of access points $C = \{C_1, C_2, ..., C_B, C_D, ..., C_D+\alpha\}$, such that $C_j$ denotes the $j^{th}$ BS in the network, for $0 < j \leq B$, and the $j^{th}$ DA in the network, for $B < j \leq B+\alpha$. For ease of notation, let
$J = \mathbb{B} + \mathcal{A}$. Accordingly, let $\mathcal{P}_i = \{C_{1,i}, C_{2,i}, \ldots, C_{J,i}, C_{J+1,i}\}$ denote the $i^{th}$ possible partition of the device set, $\mathcal{D}$, where $C_{j,i}$ denotes the set of devices associated with the $j^{th}$ AP, and $C_{J+1,i}$ is an extra coalition in which devices that are not yet associated with any AP are placed. Since constraint (C2) in P2 enforces that a device is associated with only one AP at a time, we have a non-overlapping coalition formation game where $C_{j,i} \cap C_{j',i} = \emptyset$ for all $j \neq j'$, and $\sum_{j=1}^{J+1} C_{j,i} = \mathcal{D}$. Note that coalition $C_{J+1}$ include devices that cannot be served at the required data rate, given the limitation on maximum transmission power of devices and limited resources at APs.

Define the gain value function for problem P2, $\mathcal{V}(C_{j,i}, \mathcal{P}_i)$, for the $j^{th}$ coalition $C_{j,i}$ given the $i^{th}$ partition $\mathcal{P}_i$, as

$$\mathcal{V}(C_{j,i}, \mathcal{P}_i) = \begin{cases} \epsilon \sum_{d_{k,j} \in C_{j,i}} \tilde{q}_{d_{k,j}} - (1 - \epsilon)|C_{j,i}|, & \text{if } 0 \leq j < J \\ \epsilon \sum_{d_{k,j} \in C_{j,i}} \tilde{q}_{d_{k,j}} - (1 - \epsilon)|C_{j,i}|, & \text{if } j = J + 1 \end{cases}$$

where $\tilde{q}_{d_{k,j}}$ is the optimal transmission power of the $k^{th}$ device from the $j^{th}$ AP and $|C_{j,i}|$ is the cardinality of the $j^{th}$ coalition given the $i^{th}$ partition (i.e., the number of satisfied devices associated with the $j^{th}$ AP in partition $\mathcal{P}_i$).

C. Resource allocation and transmission power

The outcome of the coalition formation game is a set of coalitions, $\{C_{1}, \ldots, C_{J+1}\}$, which determines the AP association of the devices. Accordingly, given partition $\mathcal{P} = \{C_{1}, C_{2}, \ldots, C_{J+1}\}$, $x_{d_i,b_j} = 1$ if $d_i \in C_j$ and zero otherwise, for $< j \leq \mathcal{B}$, and $y_{d_i,a_h} = 1$ if $d_i \in C_j$ and zero otherwise, for $\mathcal{B} < j \leq \mathcal{J}$. Also, both $x_{d_i,b_j}$ and $y_{d_i,a_h}$ are zero if $d_i \notin C_{J+1}$. To test the optimality of the association matrices corresponding to a particular partition against the objective function of P2, we need to prove that the partition minimizes the value gain function in (4) among all the coalitions. As the first part of the value function is the summation of transmission power of all devices in a coalition, for the given partition, we need to find the corresponding optimal resource allocations and minimum power assignment decisions that satisfy the data rate requirements of the devices. Mathematically, given partition $\mathcal{P}_h$, the optimal transmission power of the devices and RBs and RCs assignments can be written as a minimization problem, given by

$$\begin{align*}
\text{P3:} \quad & \min_{\mathcal{S}_h, \mathcal{Q}_h, \mathcal{Q}_0} \sum_{d_i \in \mathcal{C}_{h}} \sum_{b_1}^{K} \sum_{b_2}^{K} s_{d_i,b_1,b_2}^k \tilde{q}_{d_i,b_1,b_2}^k \\
& \quad + \sum_{d_i \in \mathcal{C}_{J+1}} \sum_{n=1}^{N} v_{d_i,c_j}^{n} q_{d_i,c_j}^{n} \\
\text{s.t.} \quad & \sum_{k=1}^{K} s_{d_i,c_j}^{k} R_{d_i,c_j}^{k} \geq \mathcal{R}_{\text{min}}, \quad \text{if } j \leq \mathcal{B} \\
& \min \left( \mathcal{R}_{d_j} \right) \geq \mathcal{R}_{\text{min}}, \quad \text{if } \mathcal{B} < j \leq \mathcal{J} \\
& 0, \quad \text{if } j = \mathcal{J} + 1 \\
& \sum_{d_i \in \mathcal{C}_{j+1}} s_{d_i,c_j}^{k} \leq 1, \quad \text{for } j \neq \mathcal{J} + 1 \\
& \sum_{d_i \in \mathcal{C}_{j+1}} v_{d_i,c_j}^{n} \leq 1, \quad \text{for } j \neq \mathcal{J} + 1
\end{align*}$$

where $\mathcal{R}_{d_j} = \max \left( \sum_{d_i \in \mathcal{C}_{j+1}} v_{d_i,c_j}^{n} q_{d_i,c_j}^{n} \right)$, $\mathcal{R}_{\text{min}} = \min_{\mathcal{S}_h, \mathcal{Q}_h, \mathcal{Q}_0} \sum_{d_i \in \mathcal{C}_{j+1}} \sum_{b_1}^{K} \sum_{b_2}^{K} s_{d_i,b_1,b_2}^k \tilde{q}_{d_i,b_1,b_2}^k$, and $\mathcal{C}_{J+1}$ is the set of devices not associated with any AP.

P4-1: $\min_{\mathcal{S}_h, \mathcal{Q}_h, \mathcal{Q}_0} \sum_{d_i \in \mathcal{C}_{h}} \sum_{b_1}^{K} \sum_{b_2}^{K} s_{d_i,b_1,b_2}^k \tilde{q}_{d_i,b_1,b_2}^k$

s.t. $\sum_{k=1}^{K} s_{d_i,c_j}^{k} R_{d_i,c_j}^{k} \geq \mathcal{R}_{\text{min}}, \quad \text{if } j \leq \mathcal{B}$

$\sum_{d_i \in \mathcal{C}_{J+1}} s_{d_i,c_j}^{k} \leq 1, \quad \text{for } j \neq \mathcal{J} + 1$

$\sum_{k=1}^{K} s_{d_i,c_j}^{k} \tilde{q}_{d_i,c_j} \geq \mathcal{R}_{\text{min}}, \quad \text{if } j \leq \mathcal{B}$

$\sum_{k=1}^{K} s_{d_i,c_j}^{k} \tilde{q}_{d_i,c_j} \geq \mathcal{R}_{\text{min}}, \quad \text{if } \mathcal{B} < j \leq \mathcal{J}$

As sub-bands $C_1$ and $C_2$ are orthogonal, transmissions from devices to DAs and those directly to BSs do not interfere. There is no dependency between the RB and RC assignments or the uplink transmission power of directly connected devices and that of those transmitting towards DAs. This independence permits the division of problem P3 into two independent yet similar optimization problems, one for BS-directly connected devices and the other for devices connected to DAs. The optimization problems can be written as (given partition $\mathcal{P}_h$)

$$\begin{align*}
\text{P4-2:} \quad & \min_{\mathcal{S}_h, \mathcal{Q}_h, \mathcal{Q}_0} \sum_{d_i \in \mathcal{C}_{h}} \sum_{b_1}^{K} \sum_{b_2}^{K} s_{d_i,b_1,b_2}^k \tilde{q}_{d_i,b_1,b_2}^k \\
\text{s.t.} \quad & \sum_{k=1}^{K} s_{d_i,c_j}^{k} R_{d_i,c_j}^{k} \geq \mathcal{R}_{\text{min}}, \quad \text{if } j \leq \mathcal{B} \\
& \sum_{d_i \in \mathcal{C}_{J+1}} s_{d_i,c_j}^{k} \leq 1, \quad \text{for } j \neq \mathcal{J} + 1 \\
& \sum_{k=1}^{K} s_{d_i,c_j}^{k} \tilde{q}_{d_i,c_j} \geq \mathcal{R}_{\text{min}}, \quad \text{if } j \leq \mathcal{B} \\
& \sum_{k=1}^{K} s_{d_i,c_j}^{k} \tilde{q}_{d_i,c_j} \geq \mathcal{R}_{\text{min}}, \quad \text{if } \mathcal{B} < j \leq \mathcal{J}
\end{align*}$$
0 \leq \sum_{n=1}^{N} q_{d_i,C_{i,j},h}^n q_{d_i,C_{i,j},h}^n \leq Q_{\text{max}}, \text{ for } B < j \leq J. \quad (C8 - b)

However, due to the non-convex nature of the joint power minimization and resource allocation problem and the fact that it is an MILP, solving it optimally might not be doable in polynomial time. Thus, in what follows, we propose a sub-optimal resource allocation and power minimization algorithm to solve P4-1 and P4-2. We start by reformulating the problem as a difference of two concave functions programming (D.C. programming). Although there is no theoretical analysis, many results from the literature demonstrate that, with a proper choice of the starting point, D.C. programming algorithms can find a local optimal point that often yields the global optimum. A number of regularization and starting-point selection methods have been proposed and can be used in this work to find the global optimum [13].

D. Sub-optimal reformulation as D.C. programming

Both P4-1 and P4-2 are highly non-convex due to constraints (C1 - b) and (C5 - b). They are also mixed integer non-linear programming problems due to the presence of both continuous and discrete variables. As both $s_{d_i,C_{i,j},h}$ and $v_{d_i,C_{i,j},h}^n$ are binary variables, we have the following equalities

$$s_{d_i,C_{i,j},h} = \omega_p \log_2 \left(1 + \frac{s_{d_i,C_{i,j},h} C_{i,j,h} |d_{i,j}|^{-a} g_{d_i,C_{i,j},h}^k}{c_{m,h} \sum_{k=1}^{K} s_{d_i,C_{i,j},h} C_{i,j,h} |d_{i,j}|^{-a} g_{d_i,C_{i,j},h}^k} \right), \text{ for } 0 < j & m \leq b$$

$$v_{d_i,C_{i,j},h}^n = \omega_c \log_2 \left(1 + \frac{v_{d_i,C_{i,j},h}^n C_{i,j,h} |d_{i,j}|^{-a} g_{d_i,C_{i,j},h}^n}{c_{m,h} \sum_{k=1}^{K} v_{d_i,C_{i,j},h}^n C_{i,j,h} |d_{i,j}|^{-a} g_{d_i,C_{i,j},h}^n} \right), \text{ for } B < j & m \leq J.$$

One way of simplifying problems P4-1 and P4-2 is to redefine constraints (C3 - b) and (C7 - b) with an equivalent continuous representation while enforcing variables $s_{d_i,C_{i,j},h}$ and $v_{d_i,C_{i,j},h}^n$ to take binary values. To do so, we first represent constraints (C3 - b) and (C7 - b) as the intersection of the following regions:

$$(C3 - b) \Rightarrow \begin{cases} R_{i,j,k}^1: 0 \leq s_{d_i,C_{i,j},h}^j \leq 1, & \forall i, j, k \\ R_{i,j,k}^2: \sum_{i} \sum_{j} [s_{d_i,C_{i,j},h}^j - (s_{d_i,C_{i,j},h})^2] \leq 0 \end{cases} \quad (5)$$

$$(C7 - b) \Rightarrow \begin{cases} R_{i,j,k}^1: 0 \leq v_{d_i,C_{i,j},h}^n \leq 1, & \forall i, j, n \\ R_{i,j,k}^2: \sum_{i} \sum_{n} [v_{d_i,C_{i,j},h}^n - (v_{d_i,C_{i,j},h})^2] \leq 0. \end{cases} \quad (6)$$

Second, to deal with the mixed integers constraints (C1 - b) and (C5 - b), we simply relax them and reformulate the integer variable to be continuous over their domains. Utilizing the reformulated constraints (C3 - b) and (C7 - b) given in (5) and (6), and with the help of Lagrangian methodology [14], [15], P4-1 and P4-2 can be rewritten as

**P5-1:**

$$\begin{align*}
\min_{s_{d_i,C_{i,j},h}} & \sum_{i} \sum_{j} \sum_{k=1}^{K} q_{d_i,C_{i,j},h}^n + \mu_1 R_{i,j,k}^2 \\
\text{s.t.} & \sum_{i} \sum_{j} \sum_{k=1}^{K} R_{i,j,k}^2 \geq R_{\text{min}}, \text{ for } j \leq B \\
& \sum_{i} \sum_{j} s_{d_i,C_{i,j},h}^j \leq 1, \text{ for } j \neq J + 1
\end{align*} \quad (C1 - c)$$

**P5-2:**

$$\begin{align*}
\min_{v_{d_i,C_{i,j},h}, Q_{d_i,C_{i,j},h}} & \sum_{i} \sum_{j} \sum_{n=1}^{N} q_{d_i,C_{i,j},h}^n + \mu_2 R_{i,j,k}^2 \\
\text{s.t.} & \sum_{i} \sum_{j} \sum_{n=1}^{N} R_{i,j,k}^2 \geq R_{\text{min}}, \\
& \sum_{i} q_{d_i,C_{i,j},h}^n \leq Q_{\text{max}}, \text{ for } B < j \leq J \\
& \sum_{i} v_{d_i,C_{i,j},h}^n \leq 1, \text{ for } j \neq J + 1 \\
& \sum_{i} Q_{d_i,C_{i,j},h}^n \leq Q_{\text{max}}, \text{ for } B < j \leq J
\end{align*} \quad (C9 - c)$$

where $\mu_1 \gg 1$ and $\mu_2 \gg 1$ are Lagrangian multipliers which define the penalties when variables $s_{d_i,C_{i,j},h}$ and $v_{d_i,C_{i,j},h}^n$ are set to values other than 0 or 1, ensuring that problems P5-1 and P5-2 are equivalent to problems P4-1 and P4-2 respectively. For compactness, let $W_1$ and $W_2$ denote the set of constraints (C2 - c) - (C5 - c) and (C7 - c) - (C10 - c)
Our proposed method to solve P7-1 and P7-2 is given in Algorithm 1. As both problems have the same structure, for brevity, Algorithm 1 is given in the notations of problem P7-1, in which $|| \cdot ||$ is the 1-norm of the argument which is the sum of the absolute values of the columns of the argument matrix (Manhattan norm). Note that, with inequalities (7) and (8), problems P7-1 and P7-2 represent an upper bound to problems P6-1 and P6-2 respectively. The iterative method in Algorithm 1 tightens the upper bound depending on the stopping condition. In the algorithm, we start with an initial point $\Gamma_0$ to solve P7-1 and P7-2 for an updated temporal solution $\Gamma$. The updated temporal solution is then recursively used to find another temporal solution. The update process continues until the algorithm converges. Algorithm 1 results in a series of solutions to problems P7-1 and P7-2 by solving their equivalent convex counterparts. The work presented in [18] can be used to show that Algorithm 1 converges locally to a stationary point with polynomial time.

E. Iterative algorithm for the sub-optimal formulation

The following analysis is applicable for both problems P6-1 and P6-2. However, for brevity, we focus only on P6-1. To solve P6-1, the concavity introduced by the logarithm and square functions can be transformed into an equivalent convex form by recognizing that both $u(s)$ and $\zeta_b(Q_b)$ are differentiable. Thus, using their first order Taylor expansion, the following global underestimators, shown by the inequalities, hold for any feasible solution point $(Q^f_b, S^f)$:

$$ u(S) \geq u(S') + \nabla u(S')(S - S') \quad (7) $$
$$ \zeta_b(Q_b) \geq \zeta_b(Q^f_b) + \nabla Q_b \zeta_b(Q^f_b)(Q_b - Q^f_b) \quad (8) $$

where $\nabla_X$ denotes the partial derivatives with respect to vector $X$, and $Q^f_b$ and $S^f$ are the feasible solutions of problem P6-1 at the $t^{th}$ iteration of the algorithm. By taking the differentials of the logarithmic constraints and the squared portion of the objective functions of P6-1 and P6-2, we can re-write them in a convex manner as P7-1 and P7-2 given by

P6-1: \[ \min_{Q_b, \eta_b} \quad \Omega_1(Q_b) + \mu_1 \sum_{i} \sum_{n} \left( s_{i,n}^n - (s_{i,n}^n)^2 \right)^2 \]
\[ \text{s.t.} \quad W_1, \quad \omega_b(\eta_b(Q_b) - \zeta_b(Q_b)) \geq \mathcal{R}_{\min}, \]

P7-1: \[ \min_{Q_b, \eta_b} \quad \Omega_1(Q_b) + \mu_1 \sum_{i} \sum_{n} \left( s_{i,n}^n - (s_{i,n}^n)^2 - 2(s_{i,n}^n)^3 \right)^2 \]
\[ \text{s.t.} \quad W_1, \quad \omega_b(\eta_b(Q_b) - \zeta_b(Q_b)) \geq \mathcal{R}_{\min}, \quad \text{for } j \leq \mathcal{B} \]

P7-2: \[ \min_{Q_b, \eta_b} \quad \Omega_2(Q_a) + \mu_2 \sum_{i} \sum_{n} \left( v_{i,n}^n - (v_{i,n}^n)^2 - 2(v_{i,n}^n)^3 \right)^2 \]
\[ \text{s.t.} \quad W_2, \quad \omega_c(\eta_a(Q_a) - \zeta_a(Q_a)) \geq \mathcal{R}_{\min}, \quad \text{if } \mathcal{R}_{\mathcal{A}} > \mathcal{R}_{\min}, \quad \text{for } \mathcal{B} < j \leq \mathcal{J} \]
Algorithm 1 Iterative D.C. programming algorithm for problem P7-1 (P7-2)

1: Initialize the stopping criterion tolerance $\epsilon > 0$, the maximum number if iterations $T_{\text{max}}$, $t = 0$, penalty factor $\mu_1 > 0$, and feasible matrices $Q_h^0$ and $S^0$;
2: Iteration $t$: For a given point $\Gamma^{t} = (Q_h^{t}, S^{t})$, execute the following:
3: Step 1: Compute $\nabla Q_h \Gamma_h(Q_h^{t})$ and $\nabla S \Gamma(S^{t})$;
4: Step 2: Solve the convex problem P7-1 (P7-2) to find the temporal solution $\Gamma = (Q_h, S)$:
5: Step 3: If $||Q_h||_1 + ||Q_h||_1 \leq \epsilon$, then stop solving problem P7-1 (P7-2). Set $Q = Q_h$ and $S = S$.
Otherwise, $t = t + 1$ and go back to step 1; = 0.

Table I summarizes the step-by-step significant change applied at each step to transform problem PI to problem P7.

<table>
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<th>Problem ID</th>
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<td>Original multi-objective problem formulation</td>
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<td>P2</td>
<td>Transformation of P1 to single objective optimization problem via weighted sum</td>
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<td>P3</td>
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<td>P4</td>
<td>Decomposition of P3 into its disjoint components; P4-1 for optimizing BS associated devices, and P4-2 for optimizing for DA associated ones</td>
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<td>P5</td>
<td>Problems P4-1 and P4-2 are transformed into their continuous counter-parts and Lagrangian is applied</td>
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<td>The final form of the problem. Taylor expansion is used to transform concave terms into their convex equivalent.</td>
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V. THE PAUSE ALGORITHM

A. Algorithm Description

The algorithm consists of two main stages: the initial partition creation stage and the partition update stage.

Initialization: First, we need to choose an initial feasible partition, $\mathcal{P}_0 = \{C_{1,0}, ..., C_{j+1,0}\}$. Initially, all devices are assumed to be in $C_{j+1,0}$; i.e., $C_{j,0} = \emptyset$ for $0 < j \leq J$ and $C_{j+1,0} = \mathcal{D}$. Sequentially, each device from $C_{j+1,0}$ is randomly placed into one of the other $J$ coalitions, given it is in coverage of the corresponding AP (i.e., $d_i \in \theta_j$ for $j \in \{\Psi_h, \Psi_a\}$). The partition, $\mathcal{P}_0$, is updated and the corresponding association matrices, $X_0$ and $Y_0$ are generated and checked for feasibility using Algorithm 1. If a solution is attainable, then the corresponding placement of the $i^{th}$ device into the $j^{th}$ coalition is approved, for $j \leq J$; otherwise, the device movement is rejected and the partition update is reverted. Convergence happens if $C_{j+1,0} = \emptyset$, or when $\rho = 1 - \exp(-\kappa \theta) \leq \varrho$, where $0 < \kappa < 1$ is a decay speed tune parameter, $\theta$ is the number of times the re-execution of the feasibility check after the $D^{th}$ time happens, and $0 \leq \varrho \leq 1$ is a design threshold parameter for terminating the feasibility check stage (see Algorithm 2).

Partition update: At the $h^{th}$ iteration of the partition update, a mutated partition, $\mathcal{P}_{h-1}$, of $\mathcal{P}_{h-1}$ is created by randomly choosing a device from two different coalitions $C_{j,0}$ and $C_{j',0}$, for $j \neq j'$ and $0 < j$ & $j' \leq J + 1$, and swapping them. The corresponding association metrics are passed to Algorithm 1 to check for feasibility. If not feasible, the mutation is rejected and $\mathcal{P}_h = \mathcal{P}_{h-1}$. If $\mathcal{P}_{h-1}$ is feasible, the value functions of $\mathcal{P}_{h-1}$ and of $\mathcal{P}_{h-1}$ are computed based on the definition in (4) and compared. If $V(\mathcal{P}_{h-1}) < V(\mathcal{P}_{h-1})$, then $\mathcal{P}_h = \mathcal{P}_{h-1}$ with probability 1. If $V(\mathcal{P}_{h-1}) < V(\mathcal{P}_{h-1})$, then $\mathcal{P}_h = \mathcal{P}_{h-1}$ with probability 1 - $\sigma_h$, where $\sigma_h = \exp(\delta / H)$, and $\mathcal{P}_h = \mathcal{P}_{h-1}$ with probability 1 - $\sigma_h$, where $\delta = |V(\mathcal{P}_{h-1}) - V(\mathcal{P}_{h-1})|$ and $H = \frac{H^0}{\log(h)}$ is a design parameter corresponding to the number of times the partition update stage has been executed, where $H^0$ is the rate of decay. Let $\eta_0$ be the tolerance such that the PAUSE algorithm terminates if $H < \eta_0$ (see Algorithm 3).

Algorithm 2 Create the initial feasible partition $\mathcal{P}_0$

1: Initialize $\varrho$, $\theta = 1$, the partition $\mathcal{P}_0 = \{C_{1,0}, ..., C_{j+1,0}\}$ such that $C_{j,0} = \emptyset$, for $j \leq J$, and $C_{j+1,0} = \mathcal{D}$. Initialize the association matrices $X_0 = [0]$, $Y_0 = [0]$, $S_0 = [0]$, $V_0 = [0]$, $Q_{b,0} = [0]$, $Q_{a,0} = [0]$;
2: Repeat:
3: Let $i = 1$ ;
4: Temp = $||C_{j+1,0}||$ ;
5: while $i \leq$ Temp do:
6: Move the $i^{th}$ device from $C_{j+1,0}$ to a randomly chosen coalition, $C_{j,0}$;
7: Generate a corresponding association matrices $X_{i,0,\theta}$ and $Y_{i,0,\theta}$ and check feasibility using Algorithm 1;
8: if $\mathcal{P}_{i,0,\theta} = \emptyset$ then
9: $\mathcal{P}_{i,0,\theta} = \mathcal{P}_0$, $i = i + 1$;
10: end if
11: end while
12: $\theta = \theta + 1$;
13: Compute: $\rho = 1 - \exp(-\kappa \theta)$;
14: if $C_{j+1,0} = \emptyset$ or $\rho < \rho$ then
15: Exit;
16: end if
17: $\rho = 2$ and $i = 1$
18: Go to Step 4 ;
19: end if
20: $\mathcal{P}_0$, $X_0$, and $Y_0 = 0$.

B. Convergence and Complexity analysis

In the initialization stage of PAUSE, devices are sequentially placed into one of their feasible APs. The association matrices corresponding to the placement are generated and checked for feasibility. In the feasibility check, P7-1 and P7-2 are solved based on D.C. programming the complexity of which can be evaluated using methods such as those presented in [12], [19]. Here, we use the interior point method as an example to solve the optimization problems. Let $D_{b}$ and $D_{a}$ denote the maximum number of devices associated with a BS and
Algorithm 3 The PAUSE algorithm for solving problem P2

1: Initialization: Create the initial feasible partition \( \mathcal{P}_0 = \{C_{1,0}, \ldots, C_{T+1,0}\} \) using Algorithm 2. Initialize the matrices \( X_h = [0], Y_h = [0], S_h = [0], V_h = [0], Q_{h,h} = [0], Q_{a,h} = [0] \). Initialize \( H^0 \) and \( h = 1; \)
2: Repeat;
3: Swap two devices, compute corresponding association matrices \( X_{h-1}, Y_{h-1}, X_h, Y_h \) and check feasibility using Algorithm 1;
4: if \( \mathcal{P}_{h-1} \) is feasible then
5: Compute corresponding value function for both \( \mathcal{P}_{h-1} \) and \( \mathcal{P}_h \);
6: if \( |\mathcal{V}(\mathcal{P}_{h-1}) - \mathcal{V}(\mathcal{P}_h)| > 0 \) then
7: if \( \mathcal{V}(\mathcal{P}_{h-1}) < \mathcal{V}(\mathcal{P}_h) \) then
8: \( \mathcal{P}_h = \mathcal{P}_{h-1}; \)
9: else \( \mathcal{P}_h = \mathcal{P}_{h-1} \) with probability \( \sigma_h = \exp(\delta / H) \), and \( \mathcal{P}_h = \mathcal{P}_{h-1} \) with probability \( 1 - \sigma_h \);
10: end if
11: else \( \mathcal{P}_h = \mathcal{P}_{h-1} \) with probability 0.5, and \( \mathcal{P}_h = \mathcal{P}_{h-1} \) with probability 0.5
12: Exit;
13: end if
14: end if
15: end if
16: 4: Save the value function, user association, resource allocation, and power assignment matrices of the updated partition;
17: if \( H < \eta_0 \) then
18: Exit;
19: end if
20: 5: \( h = h + 1; \)
21: 6: The optimal solution to problem P2 and hence problem P1 is equal to \( X_{h-1}, Y_{h-1}, S_{h-1}, V_{h-1}, Q_{h,h-1}, Q_{a,h-1} = 0 \)

a DA respectively, i.e., \( D_b \triangleq \max_{j \in \mathcal{P}_b} C_j \) and \( D_a \triangleq \max_{h \in \mathcal{P}_a} C_h \). Thus, for P7-1 and P7-2, we have a total of \( 2D_b K \mathcal{B} + D_b \) and \( 2D_a N \mathcal{A} + D_a \), respectively. Furthermore, there are \( G_b = K B + 2D_b K \mathcal{B} + 2D_b \mathcal{B} \) and \( G_a = N \mathcal{A} + 2D_a N \mathcal{A} + 2D_a \mathcal{A} \), convex and linear constraints for problems P7-1 and P7-2 respectively. Thus, every iteration to check the feasibility has a computational complexity of \( O((2D_b K \mathcal{B} + 2D_a N \mathcal{A} + D_a + D_b)(\tilde{G}_b + \tilde{G}_a)) \), where \( O(\cdot) \) is the big-O notation. There are at least \( \mathcal{D} \) iterations and \( \theta = \frac{\log(\frac{1}{\epsilon})}{\kappa} \) subsequent re-executions. So, the computational complexity of the initialization stage in iterations is \((\theta + D)(O((2D_b K \mathcal{B} + 2D_a N \mathcal{A} + D_a + D_b)(\tilde{G}_b + \tilde{G}_a)))\).

In the partition update stage, mutations of the initial partition are generated sequentially and their feasibility is checked. The number of iterations is a function of decay parameter \( H = \frac{H^0}{\text{end of time}} \). The update stage completes as \( H \) approaches zero (i.e., \( H < \eta_0 \)), leading to the computational complexity of \( (\exp(H^0) \cdot O((2D_b K \mathcal{B} + 2D_a N \mathcal{A} + D_a + D_b)(\tilde{G}_b + \tilde{G}_a))) \) iterations. Thus, the total computational complexity of PAUSE in number of iterations is \((\theta + D + \exp(H^0)) \cdot O((2D_b K \mathcal{B} + 2D_a N \mathcal{A} + D_a + D_b)(\tilde{G}_b + \tilde{G}_a)))\).

VI. SIMULATION RESULTS AND DISCUSSIONS

A. Algorithm performance examination

We evaluate the effectiveness of PAUSE via computer simulations with parameters listed in Table II. Three different study cases are used to examine the impact of \( \epsilon \), DA density \( \lambda_a \), and the number of RCSs \( N \) on the performance of PAUSE. Studies are conducted on a square area of \( 800 \times 800 \) m. Two BSs, each of circular coverage area with radius \( \Lambda_b = 200 \) m, are located at \((200m, 200m)\) and \((600m, 600m)\) from the origin. For the initialization phase of the PAUSE algorithm, the devices are associated based on proximity and coverage. That is, a device can be associated with an AP if both of the following conditions are met: i) the physical distance between the device and the AP is less than the radius of coverage of that AP, and 2) the maximum transmission range of the device on any of the RBs/sub-channels of that AP is greater than or equal to the AP's coverage radius. As we consider a Rayleigh fading channel and path-loss propagation attenuation, the maximum transmission ranges of the \( i \)th device located at \( d_i \) towards the \( j \)th BS, located at \( b_j \), or towards the \( h \)th DA, located at \( a_h \), are respectively given by

\[
\begin{align*}
D_{i,j} & = Q_{max} ||d_i - b_j||^{-\alpha} g_{i,j} \\
D_{i,h} & = Q_{max} ||d_i - a_h||^{-\alpha} g_{i,h}
\end{align*}
\]

where \( g_{i,j} \) and \( g_{i,h} \) are the channel gain between the device and the AP.

It should be noted that, the objective from using this method of association in the initialization stage is to mimic the traditional max-RSS association mechanism. However, since in the initialization stage we use Algorithm 1 to optimize power consumption as well as resource utilization, the outcome of the initialization phase can be thought of as an optimized version of the max-RSS based user association. This serves as a baseline to compare the performance of the proposed PAUSE algorithm to as will be shown.

<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>( w_b )</th>
<th>50 kHz</th>
<th>( \mu_1 = \mu_2 )</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_c )</td>
<td>100 kHz</td>
<td>1</td>
<td>( \mathcal{K}_{min} )</td>
<td>1 kbps</td>
<td>1e-3</td>
</tr>
<tr>
<td>( w_a )</td>
<td>150 kHz</td>
<td>1</td>
<td>( \epsilon )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>4</td>
<td>4</td>
<td>( \Lambda_a )</td>
<td>200 m</td>
<td>0.5</td>
</tr>
<tr>
<td>( \Lambda_b )</td>
<td>40 m</td>
<td>1</td>
<td>( \eta_0 )</td>
<td>500 m</td>
<td>0.5</td>
</tr>
<tr>
<td>( Q_{max} )</td>
<td>500 mW</td>
<td>1</td>
<td>( \xi )</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>

1) Case Study I: The main objective is to study the impact of \( \epsilon \) on the behavior the proposed PAUSE algorithm when solving problem P2. Recall that, as \( \epsilon \) decreases, more weight emphasis is placed on maximizing the number of supported devices, while the minimization of the transmission power becomes of less importance. To study the impact of \( \epsilon \) on the behavior of PAUSE when solving P2, \( \epsilon \in \{0.0008, 0.001, 0.0014, 0.002, 0.004\} \), are chosen such that the condition set in (2) is met. The network is made up of ten DAs, each of radius \( \Lambda_a = 40 \) m, randomly deployed across the 2D plane within the simulate square area. Each DA has \( N = 10 \) RCs with \( \omega_c = 100 \) kHz. There are \( D = 240 \) devices

```markdown
<table>
<thead>
<tr>
<th>Name</th>
<th>Value</th>
<th>( w_b )</th>
<th>50 kHz</th>
<th>( \mu_1 = \mu_2 )</th>
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</tr>
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<td>1 kbps</td>
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</tr>
<tr>
<td>( w_a )</td>
<td>150 kHz</td>
<td>1</td>
<td>( \epsilon )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>4</td>
<td>4</td>
<td>( \Lambda_a )</td>
<td>200 m</td>
<td>0.5</td>
</tr>
<tr>
<td>( \Lambda_b )</td>
<td>40 m</td>
<td>1</td>
<td>( \eta_0 )</td>
<td>500 m</td>
<td>0.5</td>
</tr>
<tr>
<td>( Q_{max} )</td>
<td>500 mW</td>
<td>1</td>
<td>( \xi )</td>
<td>0.5</td>
<td></td>
</tr>
</tbody>
</table>
```
randomly distributed over the region. The initial partition successfully serves 84 devices with an average per device transmission power of 94.7626 mW for all $\epsilon$ values. Starting from this point, the partition update stage of the PAUSE algorithm is used to optimize the performance for different $\epsilon$ values.

The results after convergence for different $\epsilon$ values are given in Table III. It is observed that increasing $\epsilon$ prioritizes reducing power consumption over increasing the number of supported devices. For instance, at $\epsilon = 0.0008$, the algorithm successfully serves 9 more devices compared to the initial point, while the per device average transmission power increases from 94.7626 mW to 113.9413 mW. On the other hand, at $\epsilon = 0.0041$, only 2 more devices are served compared to the initial point, however, the transmission power decreases to 77.9303 mW. From problem P2, it can be seen that, as $\epsilon$ approaches zero, the problem simplifies to a maximization problem of the number of supported devices and ignores the minimization of transmission power. This is supported by the results presented in Figure 2, where we set $\epsilon = 0$ and the algorithm converges to the absolute maximum possible number of 95 devices. Note that, for all $\epsilon$ values, not all the devices can be accommodated due to the interference which limits the possibility of supporting more devices at the required minimum data rate.

![Fig. 2: The maximum possible number of devices that can be supported at $\epsilon = 0$.](image)

2) Case Study II: To study the impact of DA density as a design parameter, we test the network when it has 4, 8, 16, and 32 DAs that are randomly deployed each with coverage area of radius $R = 40$ m. Each DA has 10 RCs each of BW 100 kHz. A total of 280 devices are randomly deployed. For all DA densities, $\epsilon = 0.0014$. Table IV shows the total number of supported devices and their distribution between BSs and DAs (the 3rd column), the average transmission power per device (the 4th column) after the initialization phase, and the resultant values (in the 5th and 6th columns) after the PAUSE algorithm converges. The following observations can be made:

- Increasing the number of DAs increases the amount of available resources and hence the network can support more devices, however, the increase is non-linear due to increased interference; ii) as the DA number increases, more devices are associated with them, which reduces the load on the BSs;
- iii) there is an optimal number of DAs that results in the best overall network coverage. This is evident when the number of DAs increases from 4 to 8 to 16 which results in an increase in the per device average transmission power compared to the initial values. This is because PAUSE is associating devices that may be far away leading to an increase in the average transmission power. However, when 32 DAs are used, network coverage is maximized and PAUSE now optimizes power by including the devices that best maximize the number of satisfied users at the minimum transmission power possible; and
- iv) more DAs leads to an increase in the number of supported devices, yet the network is unable to support all the 280 devices. As the number of DAs increases, the amount of resources allocated for every DA decreases, leading to a lower average achievable data rate per DA. As the QoS requirements of the devices using two-hop routes depends on the link rates of the two-hop path, lower DA uplink data rate limits the number of devices a DA can support. This result highlights the dependency between the two-hop route and the available resources; there is an optimal number of DAs for maximal cost effectiveness.

3) Case Study III: The network has 20 randomly deployed DAs. To study the impact of varying the number of available RCs at each DA, we set the number of RCs, $N$, to 10, 15, 20, and 30 each of BW 100 kHz. Accordingly, the total number of available uplink resources (RBs+RCs) is in 240, 340, 440, and 640 respectively. A total of 350 devices are randomly deployed. Results of this case study are shown in Table V, with $\epsilon = 0.0014$. Based on the results in Table V, few observations can be made: i) As $N$ increases, the number of devices that can be successfully supported increases, and the average per device transmission power decreases. It is expected, as more RUs are available for the devices and the devices are more dispersed across the available RUs, leading to a decrease in interference and in the transmission power needed to achieve the required QoS; ii) a DA can only serve the devices located within its vicinity; hence, increasing the number of DAs does not always lead to an increase in the number of supported devices. For example, increasing the RCs from 20 to 30 does not lead to an increase in the maximum number of supported devices after convergence; iii) increasing the number of RCs might lead to an improvement in the average transmission power of the devices. This is because increasing the number of RCs provides more options for the disperse of the devices across the RCs. Using PAUSE algorithm, devices are assigned the RCs that best improve their transmission power; iv) the increase in the number of supported devices is non-linear with respect to the increase in the number of RCs due to interference and competition; and
- v) increasing the number of RCs may lead to a slight increase in the per average transmission power of the devices which can be avoided by choosing different $\epsilon$ values to put emphasis on the desired objective. In conclusion, the number of DAs and the number of RCs are important network design factors that should be jointly optimized to support the largest number of devices at the lowest transmission power.
TABLE III. Impact of $\epsilon$ on the behaviour of the PAUSE algorithm

<table>
<thead>
<tr>
<th>Epsilon ($\epsilon$)</th>
<th>Value of partition</th>
<th>No. of associated devices</th>
<th>Average trans. power (mW)</th>
<th>Value of optimized partition</th>
<th>Value difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0041</td>
<td>44.8029</td>
<td>86</td>
<td>77.9303</td>
<td>58.1692</td>
<td>13.3663</td>
</tr>
<tr>
<td>0.0020</td>
<td>64.8795</td>
<td>88</td>
<td>93.7671</td>
<td>71.3210</td>
<td>6.4415</td>
</tr>
<tr>
<td>0.0014</td>
<td>70.6156</td>
<td>89</td>
<td>108.1003</td>
<td>74.9077</td>
<td>4.2921</td>
</tr>
<tr>
<td>0.0010</td>
<td>74.4397</td>
<td>90</td>
<td>112.1331</td>
<td>79.8180</td>
<td>5.3783</td>
</tr>
<tr>
<td>0.0008</td>
<td>76.3518</td>
<td>93</td>
<td>113.9413</td>
<td>84.4484</td>
<td>8.0966</td>
</tr>
</tbody>
</table>

TABLE IV. Impact of DA density

<table>
<thead>
<tr>
<th>Number of DA density</th>
<th>Total available resources RB + RC</th>
<th>Number of supported Resource allocation (BS, DA, total)</th>
<th>Ave. trans. power (mW)</th>
<th>Opt. Resource allocation (BS, DA, total)</th>
<th>Opt. Device’s ave. trans. power (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>40+40</td>
<td>(31, 35, 66)</td>
<td>113.5642</td>
<td>(30, 36, 66)</td>
<td>113.2334</td>
</tr>
<tr>
<td>8</td>
<td>40+80</td>
<td>(28, 55, 83)</td>
<td>109.3373</td>
<td>(29, 64, 93)</td>
<td>131.7634</td>
</tr>
<tr>
<td>16</td>
<td>40+160</td>
<td>(25, 98, 123)</td>
<td>87.1789</td>
<td>(25, 105, 130)</td>
<td>97.9106</td>
</tr>
<tr>
<td>32</td>
<td>40+320</td>
<td>(15, 260, 175)</td>
<td>70.1782</td>
<td>(16, 264, 190)</td>
<td>67.2939</td>
</tr>
</tbody>
</table>

TABLE V. Impact of varying $N$ on the behaviour of the PAUSE algorithm

<table>
<thead>
<tr>
<th>No. of RCs</th>
<th>No. of Served Devices</th>
<th>Per device ave. trans. power (mW)</th>
<th>Opt. No. of Served Devices</th>
<th>Per device ave. trans. power (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>56</td>
<td>135.4565</td>
<td>66</td>
<td>135.1226</td>
</tr>
<tr>
<td>15</td>
<td>102</td>
<td>105.7782</td>
<td>114</td>
<td>122.7326</td>
</tr>
<tr>
<td>20</td>
<td>255</td>
<td>92.498</td>
<td>308</td>
<td>82.2356</td>
</tr>
<tr>
<td>30</td>
<td>275</td>
<td>76.9939</td>
<td>308</td>
<td>72.449</td>
</tr>
</tbody>
</table>

B. Performance comparison

For performance comparison, we consider three main baseline schemes, namely: random AP association scheme, max-RSS AP association scheme [20], and optimized max-RSS AP association scheme. In the random AP association scheme, each device is first randomly associated with one of its reachable APs. Algorithm 1 is then used to find the optimal resource assignments and power allocations for the current associations. In the max-RSS scheme, each device is associated with the AP to which it has the best channel conditions, however, resource assignment and power allocation are done randomly. On the other hand, the optimized max-RSS based scheme works in a way similar to the max-RSS scheme, except once the devices are associated, Algorithm 1 is then used to find the optimal resource assignments and power allocations for the current partition.

1) Performance comparison at different DA densities: For the same 800 m x 800 m square area, we simulate a network that contains the two BSs, each with $K = 20$ RBs, and a total of 280 devices to be served. We vary the number of DAs in the network from 4 to 32 DAs that are randomly deployed. Each DA has 15 sub-channels and a coverage area of $\Lambda_a = 40m$. In Figure 3, we show the impact of varying the DA density on the average transmission power per device as well as the number of successfully served devices. Each result is an average of 100 Monte Carlo runs.

As can be seen from Figure 3, compared to all the baseline algorithms, the proposed algorithm significantly improves both the number of successfully served devices and the transmission energy per device. For instance, at 32 deployed DAs, the proposed algorithm serves 108 more devices than the random approach, and 53 more compared to optimized max-RSS. At the same time, it achieves 40% reduction in energy consumption compared to the random approach and performs similarly to optimized max-RSS. These results demonstrate that the proposed algorithm achieves the maximum number of successfully served devices at the minimum energy consumption.

2) Performance comparison at different numbers of sub-channels per DA: We simulate the same network with 280 devices, the two BSs, and 30 randomly deployed DAs. The number of sub-channels per DA is varied from 10 to 25 in a step size of 5. Figure 4 shows the impact of varying the number of sub-channels per DA on the average transmission power per device as well as the number of successfully served devices. Each result is an average of 100 Monte Carlo runs.

As can be seen from Figure 4, increasing the number of sub-channels,
increases the number of supported devices. However, as we are operating in the plateau region, due to the high number of DAs deployed, the increase in the number of supported devices is not as significant as shown in Figure 3. On the other hand, increasing the number of sub-channels significantly improves the energy consumption per device. This is expected, as devices have more sub-channels to choose from, resulting in a lower number of interfering devices per sub-channel. Figure 4 also shows that the proposed algorithm outperforms all other examined baseline schemes in both power consumption and number of successfully served devices. At 30 DAs and 25 sub-channel per DA, the proposed algorithm can successfully serve all the 280 devices in the network.

Fig. 4: Impact of changing the number of sub-channels per DA on the average energy consumption per device (left) and on the number of satisfied served-devices (right)

VII. CONCLUSION

In this paper, we study a two-hop DA infused cellular network to support a large number of battery power IoT devices. The objective is to maximize the number of satisfied devices while minimizing their energy consumption and achieving their desired minimum data rate. Complementing the cellular network with a layer of low-cost yet powerful aggregator nodes to relay data from the devices in a two-hop manner is an effective approach. However, the aggregator nodes are cellular devices themselves and thus use radio resources at the BSs, leading to a dependency that requires the optimization of the number of aggregators to be deployed given the available resources and the number of devices to be supported. We formulate the joint user association, power consumption minimization, and resource utility maximization problem as a MINLP multi-objective optimization problem from the view point of both devices and the network. We propose a novel algorithm, PAUSE, based on coalition formation games and D.C programming to simplify the problem into continuous convex non-linear sub-problems that can be solved using traditional optimization methods in polynomial time. Numerical results are presented to study the impact of summation weighted factor $\epsilon$, DA density $\lambda_d$, and number of DA sub-channels $N$ on the algorithm performance and demonstrate the benefits of the proposed two-hop DA infused architecture in supporting future massive cellular IoT applications. The performance of the proposed algorithm is compared to three baseline AP association schemes and is shown to outperform all of them in terms of the number of successfully served devices and energy consumption.

APPENDIX: EQUIVALENCY OF P1 AND P2

Let $(R^*, S^*, X^*, Y^*, P^*)$ be the optimal solution to $P_1$. Further, let $(\tilde{R}, \tilde{S}, \tilde{X}, \tilde{Y}, \tilde{P})$ be an optimal solution to $P_2$. Since $P_1$ and $P_2$ have the same constraints, $(R^*, S^*, X^*, Y^*, P^*)$ is a feasible solution of $P_2$. Next, we prove that $(R^*, S^*, X^*, Y^*, P^*)$ is the optimal solution of $P_2$. For simplicity of notations, let

\[
a^* = \left( \sum_{b_j \in \Psi_d} \sum_{d_i \in \Psi_d} x_{ij}^* + \sum_{a_i \in \Psi_d} \sum_{d_i \in \Psi_d} y_{ih}^* \right)
\]

\[
b^* = \sum_{d_i \in \Psi_d} (x_{ij}^* + y_{ih}^*) q_{di}
\]

\[
\tilde{a} = \left( \sum_{b_j \in \Psi_d} \sum_{d_i \in \Psi_d} \tilde{x}_{ij} + \sum_{a_i \in \Psi_d} \sum_{d_i \in \Psi_d} \tilde{y}_{ih} \right)
\]

\[
\tilde{b} = \sum_{d_i \in \Psi_d} (\tilde{x}_{ij} + \tilde{y}_{ih}) q_{di}
\]

Since $x_{ij}^*, y_{ih}^*, \tilde{x}_{ij}$ and $\tilde{y}_{ih}$ are positive integers that assumes a maximum value of 1, define $\Delta A$ as

\[
\Delta A = (a^* - \tilde{a}) \geq 1.
\]

Accordingly, using the preceding definitions and the formulation of problem $P_1$ and problem $P_2$, we have the following

\[
e\Delta B - (1 - e)\Delta A \leq \frac{\Delta B}{1 + \sum_{q_{di} \in \Psi_d} Q_{di,max}} - \frac{\Delta A \sum_{q_{di} \in \Psi_d} Q_{di,max}}{1 + \sum_{q_{di} \in \Psi_d} Q_{di,max}}
\]

\[
\leq \frac{\Delta B}{1 + \sum_{q_{di} \in \Psi_d} Q_{di,max}} - \frac{\sum_{q_{di} \in \Psi_d} Q_{di,max}}{1 + \sum_{q_{di} \in \Psi_d} Q_{di,max}} = \sigma
\]

where $\Delta B = b^* - \tilde{b}$, (14) follows from the definition $0 \leq e \leq 1$, and (15) follows from the fact that the minimum value $\Delta A$ can assume is 1, based on (13). Furthermore, since $- \sum_{q_{di} \in \Psi_d} Q_{di,max} \leq \Delta B \leq \sum_{q_{di} \in \Psi_d} Q_{di,max}$, we have

\[
-2 \sum_{q_{di} \in \Psi_d} Q_{di,max} \leq \sigma \leq 0.
\]

As both $(R^*, S^*, X^*, Y^*, P^*)$ and $(\tilde{R}, \tilde{S}, \tilde{X}, \tilde{Y}, \tilde{P})$ are feasible solutions of $P_2$, they result in the optimal objective function. Thus,

\[
e\Delta B - (1 - e)\Delta A \leq 0
\]

\[
\nu(b^* - \tilde{b}) - (1 - \nu)(a^* - \tilde{a}) \leq 0
\]

\[
\nu b^* - (1 - \nu) a^* \leq \nu \tilde{b} - (1 - \nu) \tilde{a}.
\]

Accordingly, as $(R^*, S^*, X^*, Y^*, P^*)$ results in a smaller objective function in $P_2$ than $(\tilde{R}, \tilde{S}, \tilde{X}, \tilde{Y}, \tilde{P})$, we can conclude that $(R^*, S^*, X^*, Y^*, P^*)$ is in fact the optimal solution of both $P_1$ and $P_2$. 


REFERENCES


