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Opportunistic cooperation in wireless ad hoc networks with interference correlation

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Abstract Compared with conventional direct transmissions, the cooperative transmissions are not always beneficial and redistribute the interference over the network due to relay transmissions. In this paper, we propose an opportunistic cooperation strategy for a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays, where each source-destination pair activates the cooperative transmission only when the number of potential relays is not smaller than a cooperation threshold. Such a threshold determines the proportion of concurrent cooperative transmissions and it can be adjusted to enhance the overall network performance. The correlation of nodes' locations induces spatial and temporal correlation of interference. Based on tools from stochastic geometry, we derive the correlation coefficient of interference at a destination during the transmission periods of the sources and relays. The outage probability of opportunistic cooperation is derived for selection combining, while taking into account the interference correlation, relay selection strategy, and spatial distributions of sources and relays. Extensive simulations are conducted to validate the performance analysis. The analytical results evaluate the effectiveness of opportunistic cooperation and provide useful insights on the protocol design and parameter setting for large-scale networks.

Keywords Opportunistic cooperation · Relay selection · Poisson point process · Interference correlation · Selection combining

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1 Introduction

Research in mobile ad hoc networks has been attracting more and more attentions in recent years because of their low costs and broad applications [1]. The wireless channel in such a mobile network suffers from path loss, multi-path fading due to node mobility, and interference from concurrent transmissions on the same radio channel. All these detrimental aspects should be addressed to use limited radio resources for an increasing service demand and to satisfy quality-of-service (QoS) requirements. As a result, cooperative communication, as an effective technique for realizing spatial diversity, has been proposed to enhance transmission reliability [2–4].

Due to the scarcity of the radio spectrum, supporting concurrent transmissions via space division on the same radio channel is necessary to enhance spectrum utilization. Interference due to concurrent transmissions from other unintended sources cannot be avoided, which undesirably affects the packet reception at a target destination. As the main performance-limiting factor, interference depends on the networking environment, including propagation channel, medium access scheme, and node spatial distribution. Via modeling the node locations by a homogeneous Poisson point process (PPP), the characterization of interference and the analysis of outage probability for direct transmissions are carried out in [5, 6] based on tools from stochastic geometry. Such a method is extended to analyze the performance of cooperative transmissions, e.g., [7,8]. Most of existing works assume that cooperation is always beneficial. However, the transmission reliability of a cooperative link may not always be higher than that of a direct link, as the effectiveness of cooperation depends on many factors, including the relay selection strategy, number of available relays, interference level, and link length. Hence, cooperation should be activated by each source-destination pair when necessary and beneficial. In particular, the overall network performance can be enhanced by allowing only a fraction of source-destination pairs to activate cooperative transmissions, which motivates this work.

The direct and cooperative links coexist in the network and generate interference to each other. The interference incurred by a cooperative link is different from the interference incurred by a direct link, as an additional relay or relays take part in the transmission of one packet. In other words, the cooperative links redistribute the interference over the network due to relay transmissions. Because of the common and adjacent locations of the interferers, the interference power is spatially correlated across adjacent locations and temporally correlated in consecutive time-slots. The correlation level of interference power depends on the proportion of concurrent cooperative transmissions as well as the distance between each interfering source and its selected relay for a single-relay case, which is determined by the relay selection strategy. The correlation of interference power results in the correlation of successful packet receptions. Such correlation poses significant challenges on characterizing interference power as well as on making network-wide beneficial cooperation decisions. Although there have been significant efforts in demonstrating the effectiveness of cooperative communication for a single isolated source-destination pair [9–11], the analysis for cooperative communication from a perspective of the overall network performance while taking into account the interference redistribution and correlation is still very limited. For broad applications of cooperative communication, it is desirable to fully understand its benefits and limitations in a wireless ad hoc network.

In this paper, we propose a single-relay opportunistic cooperation strategy for a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays, where the cooperative transmission is activated by each source-destination pair only when the number of potential relays is not smaller than a cooperation threshold. Such a threshold can be adjusted to vary the proportion of cooperative transmissions over the network as well as the level of interference correlation. We construct a constrained relay selection region, within which a relay with the best channel to the destination and having received a packet from the source is selected as the best relay for a cooperative link. Via modeling the spatial locations of sources and relays by homogeneous PPPs, we derive the correlation coefficient of interference power at a destination during the transmission periods of the sources and relays based on tools from stochastic geometry. Over such a network model, we derive the outage probability of opportunistic cooperation with selection combining, while taking into account the cooperation threshold, interference correlation, and spatial distributions of sources and relays. We demonstrate that the overall network performance can be enhanced by activating cooperative transmissions when necessary and beneficial.

The main contributions of this paper are three-fold:

i) We develop a theoretical performance analysis framework for opportunistic cooperation in a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays, while taking into account the interference redistribution and correlation. We show that cooperation is not always beneficial and should be activated when necessary and beneficial from a perspective of the overall network performance. Such an analytical framework provides a better understanding of the overall network performance of cooperative communication in wireless networks with interference correlation;

ii) We derive the correlation coefficient of interference power at the destination during the transmission periods of the sources and relays, as a function of the cooperation threshold and the distances between the interfering sources and their selected relays. It is observed that the interference redistribution incurred by cooperative transmissions reduces the level of interference correlation;

iii) The outage probability of opportunistic cooperation is derived for selection combining at the destination in terms of important network and protocol parameters. Extensive simulations are conducted to validate the performance analysis and demonstrate the effectiveness of opportunistic cooperation. The analytical results provide useful insights on the protocol design and parameter setting for large-scale networks while incorporating the effect of interference correlation.

The rest of this paper is organized as follows. The related work is reviewed in Section 2. In Section 3, the system model under consideration, proposed opportunistic cooperation strategy, and interference characterization are presented. We derive the correlation coefficient of interference power in Section 4. In Section 5, we analyze the outage probability of opportunistic cooperation by conditioning on the number of potential relays. Numerical results are given in Section 6. Finally, Section 7 concludes this work. The important symbols are summarized in Table 1.

2 Related work

Two categories of performance analysis for cooperative communication with random relay locations can be distinguished in the literature. The first category focuses on the performance analysis for a single isolated source-destination pair, while the secondary category takes into account the co-channel interference.

The scenario with a single isolated source-destination pair is studied in [9–11]. By treating node locations in a probabilistic manner, stochastic geometry as an effective mathematical tool is used to deal with random network topologies [12, 13]. Via modeling the relay locations by a homogeneous PPP, the outage probabilities of both opportunistic relaying (i.e., selecting the relay that maximizes the minimum signal-to-noise ratio (SNR) of the source-relay and relay-destination links) and selection cooperation (i.e., selecting the relay with best channel to the destination) are analyzed for Rayleigh fading channels in [9]. The performance analysis is extended to general fading channels in [10]. Zhai *et al.* propose an uncoordinated cooperation scheme, where each relay forwards a packet with a specific probability calculated based on its local channel state information (CSI) [11]. These studies focus on the performance analysis for a single isolated source-destination pair, which provide useful insights on the potential benefits of cooperation, but cannot characterize the performance of cooperation in the network with co-channel interference.

The scenario with co-channel interference is studied in [7, 8, 14–16]. Based on point process theory, the authors in [7] analyze the asymptotic outage probability and diversity gain for downlink relaying in a cellular network. The performance analysis is extended to an interference-limited hierarchical spectrum sharing network in [8], where the dominant interference is eliminated by forming a primary exclusive region around each receiver. A QoS region, within which any relay can be selected to satisfy a specified QoS constraint, is introduced in [14]. Altieri *et al.* propose a relay activation strategy to activate cooperation in a random manner and investigate the tradeoff between cooperation gain and additional interference [15]. In our previous work [16], we study the network throughput of cooperative transmissions to investigate the performance tradeoff achieved by spatial diversity and by spatial frequency reuse. These studies are carried out without taking into account the interference correlation.

The impact of interference correlation on the performance of direct transmissions is investigated recently in [17–21]. It is shown that the interference correlation significantly reduces the probability of successful packet transmissions. Schilcher *et al.* investigate three main sources of interference correlation, i.e., node locations, propagation channel, and traffic [21]. The packet delivery probability and diversity order of a cooperative link in a Poisson field of interferers are derived in [22] and [23] respectively, where the interference power at the fixed relay and destination is correlated. The performance analysis is extended by selecting the best relay from multiple potential relays in [24]. By assuming the same set of interferers during the transmission periods of the source and relay, these studies focus on the scenario where only the considered link is activating the cooperative transmission. Without activating cooperation opportunistically and taking into account the interference redistribution incurred by concurrent cooperative transmissions, the network-wide performance of cooperative communication cannot be characterized. A comprehensive study of the impact

Table 1. Summary of Important Symbols

Symbol	Definition
$g(s)$	Path loss over a link of length $\|s\|$
$H_{S_iD_i:n}$	Fading coefficient between nodes S_i and D_i in the nth sub-time-slot
$I_{CD_0:1}(\Phi_C)$	Aggregate interference power from the source nodes of cooperative links in the first sub-time-slot at destination D_0
$I_{D_0:1}(\Phi_D, \Phi_C)$	Aggregate interference power in the first sub-time-slot at destination D_0
$I_{D_0:2}(\Phi_D, \Phi_F)$	Aggregate interference power in the second sub-time-slot at destination D_0
$I_{DD_0:n}(\Phi_D)$	Aggregate interference power from the source nodes of direct links in the nth sub-time-slot at destination D_0
$I_{FD_0:2}(\Phi_F)$	Aggregate interference power from the relays of cooperative links in the second sub-time-slot at destination D_0
K_i	Number of potential relays for source-destination pair i
L	Distance between a source and its intended destination
q_e	Probability of an empty relay set
$q_{out}^{CT}, q_{out}^{DT}$	Outage probabilities of the cooperative and direct transmissions
q_{out}^{OC}	Outage probability of opportunistic cooperation
r_C	Radius of a constrained relay selection region
R_b^0	Best relay of the target link when activating cooperative transmission
s_i, r_i, d_i	Location coordinates of source S_i, relay R_i, and destination D_i
α	Path loss exponent
$\gamma_{S_iD_i:1}$	SIR at destination D_i in the first sub-time-slot
κL	Distance between the source and center of a relay selection region
λ_S	Spatial density of PPP Φ_S
Φ_C, Φ_D	PPPs formed by the locations of sources of cooperative and direct links
Ω_0	Set of qualified relays for the target source-destination pair
Φ_R, Φ_S	PPPs formed by the locations of relays and sources
ρ	Correlation coefficient
τ	Coordinate difference between a source and its selected relay
θ_C	Cooperation threshold in terms of the number of potential relays

of interference redistribution and correlation on the overall network performance is important for the design of efficient cooperative transmission schemes.

In this paper, we propose an opportunistic cooperation strategy for a wireless ad hoc network, in which the cooperative transmissions are activated when necessary and beneficial for the overall network performance. Taking into account the interference redistribution and correlation as well as the spatial distributions of the sources and relays, we evaluate the correlation coefficient of interference power and outage probability of opportunistic cooperation, in terms of the source density, relay density, cooperation threshold, and link length.

3 System model

3.1 Network topology

Consider a two-dimensional (2D) spatial network coverage area with nodes independently and randomly distributed. The locations of source nodes at any time instant can be specified by a homogeneous PPP $\Phi_S = \{s_1, s_2, \cdots\} \subset \mathbb{R}^2$. The spatial density of PPP Φ_S is denoted as λ_S (average number of nodes per unit area). Source node, S_i, has an associated destination node, D_i, located at L meters away with a random direction, i.e., $d_i \in \mathbb{R}^2$, $i = 1, 2, \cdots$ [8, 15, 16]. Extension to consider random link length is straightforward [25]. The destinations are not part of PPP Φ_S. All other nodes, without their own packets to transmit and receive, are denoted as relays (e.g., R_i) and they are always willing to forward packets from the sources [7–11, 14–16]. At any time instant, they form a homogeneous PPP $\Phi_R = \{r_1, r_2, \cdots\} \subset \mathbb{R}^2$ with density λ_R. Due to the stationary property[1] of the homogeneous PPP, we focus on the performance of a target source-destination pair, indexed as $i = 0$. As shown in Fig. 1, the target source and destination are located at $s_0 = (L, 0)$ and $d_0 = (0, 0)$, respectively. Note that the capital letters (i.e., S, D, and R) and lowercase letters (i.e., s, d, and r) denote the nodes and their locations, respectively.

3.2 Propagation channel

Consider an interference-limited wireless network, where the effect of noise is neglected. The channel between any pair of nodes is characterized by both Rayleigh fading and path loss. All the distance-independent fading coefficients are independent and identically distributed (i.i.d.) random variables with unit mean. A general path loss model is given by

$$g(s-d) = \frac{1}{\epsilon + \|s-d\|^{\alpha}}, \quad \epsilon \geq 0, \tag{1}$$

where ϵ is a parameter to model the singular (i.e., $\epsilon = 0$) and non-singular (i.e., $\epsilon > 0$) path loss models, $\|s-d\|$ is the Euclidean distance between two points in the 2D plane with coordinates s and d, and α denotes the path loss exponent.

3.3 Opportunistic cooperation

Consider a time-slotted packet transmission over a single frequency channel. The time-slot duration is a constant, and all nodes are synchronized in time. The coordination signaling among a source, neighboring relays, and its intended destination is required before a packet transmission [27]. All nodes transmit with the same power, which is normalized to one without loss of generality. Each source always has a packet for transmission and it transmits to its intended destination via either one-hop direct transmission or two-hop cooperative transmission with a single best relay node. All packets have equal length and each packet is transmitted in exactly one time-slot. Each node has a single omni-directional antenna and operates in half-duplex mode.

For fair comparison with the direct transmission, a higher transmission rate is required in both hops of the cooperative transmission due to the half-duplex constraint.

[1] Adding a node does not affect the statistics of the PPP. According to the Slivnyak's theorem [26], the average performance of the target source-destination pair represents that of any source-destination pair in the network.

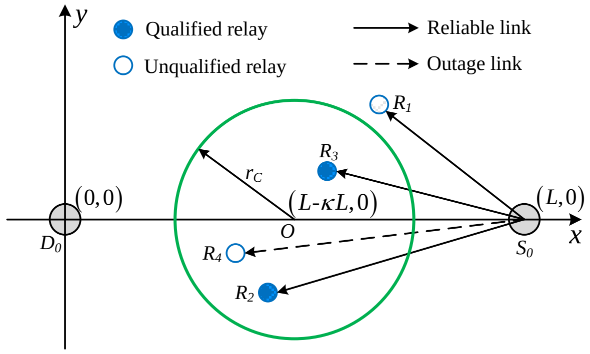


Fig. 1 An illustration of the constrained relay selection region centered at O with radius r_C for the target source-destination pair under the 2D Cartesian coordinate system. Only the relays within the constrained relay selection region (e.g., R_2, R_3, and R_4) that successfully receive the packet from the source (e.g., R_2 and R_3) are qualified relays (e.g., R_2 and R_3)

To support a higher transmission rate, a higher reception threshold in terms of the signal-to-interference ratio (SIR) is required for successful packet receptions. When no relays that have good channel quality to both the source and destination can be found to forward the packet, the cooperative transmission performs worse than the direct transmission because of the higher reception threshold. Due to the randomness of the channel fading and nodes' locations, the availability of reliable relays varies for different source-destination pairs. Hence, from a perspective of the overall network performance, an opportunistic cooperation strategy is required to activate cooperative transmissions when necessary and beneficial, leading to a mixture of direct and cooperative transmissions.

For each source-destination pair, a constrained region[2] is considered for relay selection. For instance, the relay selection region for the target source-destination pair is centered at $O = (L - \kappa L, 0)$, $0 \leq \kappa \leq 1$, with radius $r_C \leq L/2$, as shown in Fig. 1. Such a constrained relay selection region can be identified by coordination signaling and location estimation at each node via the localization technique [29]. The relays within the relay selection region are referred to as potential relays and only the potential relays contend to be the best relay. The existence of more potential relays implies a higher probability of selecting a reliable relay. Hence, the number of potential relays determines the achievable performance of cooperative transmissions and such local information can be obtained by each source via coordination signaling. Note that each source does not have two-hop instantaneous CSI (i.e., channel qualities of source-relay and relay-destination links) to help making the cooperation decision, which is always the case in decentralized wireless networks. As a result, each source makes an independent decision on whether or not to enable a coopera-

[2] It is desirable to select the best relay within a constrained region due to the following reasons: 1) the relays geographically far away from the source and/or destination are not beneficial [14]; 2) the protocol overhead and implementation complexity of a relay selection scheme increase with the number of relays that contending to be the best relay [27, 28]; and 3) the efficiency of spatial frequency reuse reduces with a larger relay selection region [16].

tive transmission based on limited available information (i.e., the number of potential relays).

Threshold-based opportunistic cooperation is considered here due to its efficiency and simplicity for implementation. To make a cooperation decision, source node, S_i, compares the number of potential relays, denoted as K_i, with a cooperation threshold, θ_C. For example, source S_0 schedules a cooperative transmission when $K_0 \geq \theta_C$, and enables a direct transmission otherwise. To guarantee an acceptable outage probability for each source-destination pair, the concurrent transmissions should keep a distance away with high probability, which leads to a low possibility of having overlapped constrained relay selection regions. Hence, we assume that the numbers of potential relays are independent for different source-destination pairs. Because of the independent cooperation decisions of all sources and by applying the independent thinning property of the PPP, the final cooperative transmission set (consisting of the spatial locations of source nodes of cooperative links)

$$\Phi_C = \{s_i \in \Phi_S : K_i \geq \theta_C\} \tag{2}$$

is a homogeneous PPP with density

$$\begin{aligned} \lambda_C &= \lambda_S \cdot \sum_{k=\theta_C}^{\infty} \mathbb{P}\left(K = k\right) \\ &\overset{(a)}{=} \lambda_S \cdot \sum_{k=\theta_C}^{\infty} \frac{\left(\lambda_R A_R\right)^k}{k!} \exp\left(-\lambda_R A_R\right), \end{aligned} \tag{3}$$

where $A_R = \pi r_C^2$ is the area of a relay selection region and (a) holds as the number of potential relays is a Poisson random variable with mean $\lambda_R A_R$. Similarly, the final direct transmission set $\Phi_D = \{s_i \in \Phi_S : K_i < \theta_C\}$ is a homogeneous PPP with density $\lambda_D = \lambda_S - \lambda_C$.

Based on the opportunistic cooperation strategy, the concurrent transmissions within the network are a mixture of direct and cooperative transmissions, as shown in Fig. 2, except two special cases (i.e., direct transmissions only when $\theta_C = \infty$ and cooperative transmissions only when $\theta_C = 0$). For a one-hop direct transmission, the source utilizes the whole time-slot to transmit a packet at rate ν (in bit/s). An outage occurs when the received SIR at the destination within the time-slot is smaller than the required reception threshold, β_ν. Define the required reception threshold $\beta_\nu \equiv 2^{\nu/B} - 1$ so that $\nu = B \cdot \log_2(1 + \beta_\nu)$, based on Shannon's formula, where B denotes the channel bandwidth in Hz.

On the other hand, a single relay is considered for a two-hop cooperative transmission; hence, each source and its best relay share one time-slot in transmitting the same packet. The decode-and-forward (DF) scheme at the best relay is considered. As in [30], a time-slot is partitioned equally to two sub-time-slots. The fading coefficients remain invariant during one sub-time-slot and vary independently in different sub-time-slots. Each source transmits a packet at rate 2ν in the first sub-time-slot. Due to the broadcast nature of wireless communications, each potential relay can successfully receive the packet if the received SIR is not smaller than $\beta_{2\nu}$, depending on both instantaneous signal and interference power. Note that for $\beta_{2\nu} > 1$, each potential relay can correctly decode at most one packet over each time-slot [7]. The potential relays that successfully receive the packet from source S_0 in the first sub-time-slot are referred to as qualified relays, which form a relay set, Ω_0. With a potential of more than one qualified relay for each cooperative link, a distributed relay selection scheme is required. Assuming that, via coordination signaling, each qualified relay has instantaneous CSI between itself and the intended destination. A back-off scheme can be employed to select the best relay in a distributed manner [31]. When the relay set is not empty, a qualified relay with the best channel to the destination is selected as the best relay, R_b^0. The location of the best relay is given by

$$r_{R_b^0} = \arg \max_{R_i \in \Omega_0} \{H_{R_i D_0:2} \cdot g\left(r_i\right)\}, \tag{4}$$

where $H_{R_i D_0:2}$ denotes the fading coefficient between relay R_i and destination D_0 in the second sub-time-slot.

In the second sub-time-slot, the best relay forwards the packet to the intended destination at rate 2ν and the source does not repeat the packet transmission. If the relay set is empty (i.e., no qualified relays), the packet is not forwarded to the intended destination. Finally, the destination decodes the packet using selection combining, by which the destination selects a better link from the direct and forward links for packet decoding. Hence, an outage occurs when both the direct and forward links cannot support the required transmission rate. As the main focus of this paper is to study the outage probability of opportunistic cooperation, the protocol overhead incurred by coordination signaling and relay selection is not considered.

3.4 Interference characterization

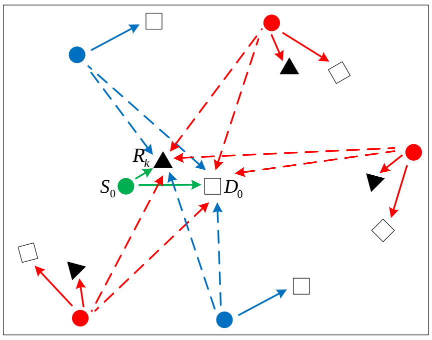
Under the physical interference model, power levels of the signals transmitted from all unintended transmitters are added and the sum is considered as interference power. Due to the mixture of direct and cooperative transmissions, as shown in Fig. 2(a), the aggregate interference power at a potential relay (e.g., R_k) of the target source-destination pair in the first sub-time-slot is given by

$$I_{R_k:1}\left(\Phi_D, \Phi_C\right) = I_{DR_k:1}\left(\Phi_D\right) + I_{CR_k:1}\left(\Phi_C\right), \tag{5}$$

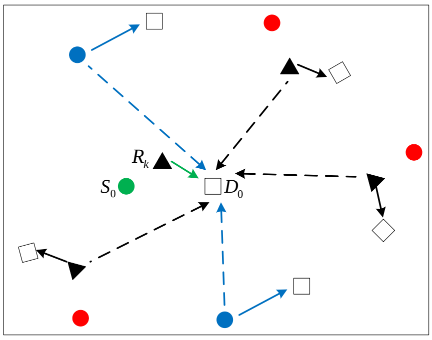
where $I_{DR_k:1}\left(\Phi_D\right) = \sum_{s_i \in \Phi_D} H_{S_i R_k:1} g(s_i - r_k)$ is the interference power at relay R_k from the source nodes of direct links, and $I_{CR_k:1}\left(\Phi_C\right) = \sum_{s_j \in \Phi_C} H_{S_j R_k:1} g(s_j - r_k)$ denotes the interference power at relay R_k from the source nodes of cooperative links.

Due to the interference redistribution incurred by concurrent cooperative transmissions, as shown in Fig. 2(b), the aggregate interference power observed by destination D_0 in the first and second sub-time-slots can be expressed respectively as

$$\begin{aligned} I_{D_0:1}\left(\Phi_D, \Phi_C\right) &= I_{DD_0:1}\left(\Phi_D\right) + I_{CD_0:1}\left(\Phi_C\right) \\ I_{D_0:2}\left(\Phi_D, \Phi_F\right) &= I_{DD_0:2}\left(\Phi_D\right) + I_{FD_0:2}\left(\Phi_F\right), \end{aligned} \tag{6}$$



(a) Transmissions in the first sub-time-slot



(b) Transmissions in the second sub-time-slot

Fig. 2 An illustration of the interference power, originating from both the direct and cooperative links over the network, observed by relay R_k and destination D_0 in the first and second sub-time-slots. Each circle, triangle, and square represent a source, selected relay, and destination, respectively. The solid and dashed lines represent the transmitted signal and interference, respectively

where $I_{DD_0:1}\left(\Phi_D\right)=\sum\limits_{s_i\in\Phi_D} H_{S_iD_0:1}\cdot g(s_i)$ and $I_{DD_0:2}\left(\Phi_D\right)=\sum\limits_{s_i\in\Phi_D} H_{S_iD_0:2}\cdot g(s_i)$ are the aggregate interference power from the source nodes of direct links in the first and second sub-time-slots respectively, $I_{CD_0:1}\left(\Phi_C\right) = \sum\limits_{s_j\in\Phi_C} H_{S_jD_0:1}\cdot g(s_j)$ and $I_{FD_0:2}\left(\Phi_F\right) = \sum\limits_{r_m\in\Phi_F} H_{R_mD_0:2}\cdot g(r_m)$ denote the aggregate interference power from the sources and selected relays of cooperative links in the first and second sub-time-slots respectively, and Φ_F denotes the PPP formed by the locations of the selected relays that forward the packets in the second sub-time-slot [32].

Due to the common locations of the source nodes of direct links in both sub-time-slots, interference power $I_{DD_0:1}\left(\Phi_D\right)$ and $I_{DD_0:2}\left(\Phi_D\right)$ are temporally correlated. Similarly, as the source node (e.g., S_j) locates close to its selected relay (e.g., R_b^j) for each cooperative link, interference power $I_{CD_0:1}\left(\Phi_C\right)$ and $I_{FD_0:2}\left(\Phi_F\right)$ are temporally correlated. As a result, interference power observed by destination D_0 in two sub-time-slots, i.e., $I_{D_0:1}\left(\Phi_D,\Phi_C\right)$ and $I_{D_0:2}\left(\Phi_D,\Phi_F\right)$ defined in (6), are temporally correlated. On the other hand, the potential relays (e.g., R_k) of the target source-destination pair and destination D_0 are geographically close and suffer from the interference originated from the same or adjacent interferers, which yields to the spatial correlation between interference power $I_{R_k:1}\left(\Phi_D,\Phi_C\right)$ and $I_{D_0:1}\left(\Phi_D,\Phi_C\right)$ or $I_{D_0:2}\left(\Phi_D,\Phi_F\right)$. The correlation of interference power is taken into account in the following analysis of correlation coefficient and outage probability.

4 Temporal correlation coefficient of interference

In this section, we analyze the temporal correlation coefficient of interference power observed by destination D_0 in the first and second sub-time-slots, i.e., $I_{D_0:1}\left(\Phi_D,\Phi_C\right)$ and $I_{D_0:2}\left(\Phi_D,\Phi_F\right)$. The spatial correlation coefficient of interference power can be derived similarly. As in [17], a non-singular path loss model (i.e., $\epsilon > 0$) is utilized to ensure the mean and variance of interference power to be finite when deriving the correlation coefficient, while a singular path loss model (i.e., $\epsilon = 0$) is used to determine whether or not a packet is successfully received[3]. The correlation coefficient can be represented in terms of important network and protocol parameters, as stated in the following proposition (with proof in Appendix).

Proposition 1 *For Rayleigh fading channels, the temporal correlation coefficient of interference power observed by destination D_0 in the first and second sub-time-slots (i.e., $I_{D_0:1}(\Phi_D, \Phi_C)$ and $I_{D_0:2}(\Phi_D, \Phi_F)$) is given by*

$$\rho = \frac{\lambda_D \int_{\mathbb{R}^2} g^2(s)\mathrm{d}s + \lambda_F \int_{\mathbb{R}^2} g(s)\,\mathbb{E}_\tau\left[g(s+\tau)\right]\mathrm{d}s}{2\sqrt{\lambda_S}\sqrt{\lambda_D + \lambda_F}\int_{\mathbb{R}^2} g^2(s)\mathrm{d}s}, \tag{7}$$

where $\int_{\mathbb{R}^2} g^2(s)\mathrm{d}s = \delta(1-\delta)\pi^2 \big/ \left[\epsilon^{2-\delta}\sin(\pi\delta)\right]$, $\delta = 2/\alpha$, τ denotes the coordinate difference between the source and the best relay, and $\lambda_F = \lambda_C(1-q_e)$ is the spatial density of PPP Φ_F with

$$q_e = \sum_{k=\theta_C}^{\infty} \frac{(\lambda_R A_R)^k}{k!}\exp(-\lambda_R A_R)\sum_{m=0}^{k}\binom{k}{m}(-1)^m \exp(-\lambda_S Q) \tag{8}$$

and

$$Q = -\pi\delta\beta_{2\nu}^{\delta}(\kappa L)^2 \Gamma(-\delta)\,\Gamma(\delta+m)\Big/\Gamma(m). \tag{9}$$

In (9), $\Gamma(x)$ is the Gamma function.

Due to random relay locations, the coordinate difference between the source and the best relay is a random variable and utilizing its probability density function to calculate the correlation coefficient in (7) is complex [9]. For simplicity of performance analysis, we obtain the lower bound of the temporal correlation coefficient, as stated in the following corollary.

Corollary 1 *For Rayleigh fading channels, the temporal correlation coefficient of interference power observed by destination D_0 in the first and second sub-time-slots (i.e., $I_{D_0:1}(\Phi_D, \Phi_C)$ and $I_{D_0:2}(\Phi_D, \Phi_F)$) is lower bounded by*

$$\rho \geq \frac{\lambda_D \int_{\mathbb{R}^2} g^2(s)\mathrm{d}s + \lambda_F \int_{\mathbb{R}^2} g(s)\,g(s+\mathbb{E}_\tau[\tau])\mathrm{d}s}{2\sqrt{\lambda_S}\sqrt{\lambda_D + \lambda_F}\int_{\mathbb{R}^2} g^2(s)\mathrm{d}s}. \tag{10}$$

Proof As $g(s)$ is a convex function and according to Jensen's inequality, we have

$$\mathbb{E}_\tau\left[g(s+\tau)\right] \geq g(s+\mathbb{E}_\tau[\tau]). \tag{11}$$

As a result, (10) follows directly from (7).

[3] The singularity has a negligible effect on determining whether or not a packet is successfully received [5]. For instance, if an interferer locates very close to a receiver, the singular path loss model would result in a very small SIR and hence an unsuccessful packet reception at the receiver. On the other hand, the receiver would also very likely fail to decode the packet even if the singularity is removed (i.e., non-singular path loss model).

As mentioned in Section 3.4, both common locations of the sources of direct links and adjacent locations of the sources and relays of cooperative links yield to the temporal correlation of interference power in the first and second sub-time-slots. To distinguish the interference correlation incurred by these two factors, we obtain the temporal correlation coefficient due to the common locations of the sources of direct links, as stated in the following corollary.

Corollary 2 *The temporal correlation coefficient of interference power observed by destination D_0 due to the common locations of the sources of direct links in the first and second sub-time-slots is given by*

$$\rho_D = \frac{\lambda_D}{2\sqrt{\lambda_S}\sqrt{\lambda_D + \lambda_F}}. \tag{12}$$

Similarly, the temporal correlation coefficient of interference power observed by destination D_0 due to the adjacent locations of the sources and relays of cooperative links in the first and second sub-time-slots is $\rho_C = \rho - \rho_D$. The temporal correlation coefficient incorporates the effect of interference redistribution incurred by concurrent cooperative transmissions and reflects the level of interference correlation at the destination in two sub-time-slots. The impact of interference redistribution on the temporal correlation coefficient is illustrated in Section 6.1.

5 Outage probability of opportunistic cooperation

In this section, we derive the outage probability of opportunistic cooperation based on tools from stochastic geometry, while taking into account the spatial distributions of sources and relays, cooperation threshold, and interference correlation. By conditioning on whether or not cooperation is activated by the target source-destination pair, the outage probability of opportunistic cooperation can be expressed as

$$q_{out}^{OC} = \sum_{k=0}^{\theta_C - 1} \mathbb{P}\left(K_0 = k\right) q_{out}^{DT} + \sum_{k=\theta_C}^{\infty} \mathbb{P}\left(K_0 = k\right) q_{out}^{CT}\left(k\right), \tag{13}$$

where q_{out}^{DT} and $q_{out}^{CT}\left(k\right)$ denote the outage probabilities of the direct transmission and cooperative transmission given k potential relays, derived in the following two subsections, respectively.

5.1 Direct transmission

For a direct transmission of the target source-destination pair, an outage occurs when the received SIR at destination D_0 in either the first or second sub-time-slot is smaller than β_ν. The outage probability, denoted as q_{out}^{DT}, is given by

$$q_{out}^{DT} = 1 - \mathbb{P}\left(\gamma_{S_0 D_0 :1} \geq \beta_\nu, \gamma_{S_0 D_0 :2} \geq \beta_\nu\right), \tag{14}$$

where $\gamma_{S_0D_0:1} = \frac{H_{S_0D_0:1}L^{-\alpha}}{I_{D_0:1}(\Phi_D,\Phi_C)}$ and $\gamma_{S_0D_0:2} = \frac{H_{S_0D_0:2}L^{-\alpha}}{I_{D_0:2}(\Phi_D,\Phi_F)}$ denote the received SIR at destination D_0 in the first and second sub-time-slots, respectively.

Over a Rayleigh fading channel, the received signal power at the destination follows an exponential distribution. As PPP Φ_D is independent of PPP Φ_C and PPP Φ_F, we have

$$\begin{aligned} q_{out}^{DT} = 1 - &\underbrace{\mathbb{E}\left[\exp\left(-\beta_\nu L^\alpha \left[I_{DD_0:1}\left(\Phi_D\right) + I_{DD_0:2}\left(\Phi_D\right)\right]\right)\right]}_{\mathcal{B}_1} \\ &\times \underbrace{\mathbb{E}\left[\exp\left(-\beta_\nu L^\alpha \left[I_{CD_0:1}\left(\Phi_C\right) + I_{FD_0:2}\left(\Phi_F\right)\right]\right)\right]}_{\mathcal{B}_2}. \end{aligned} \tag{15}$$

Taking the Laplace transforms of the fading coefficients between the interferers and target destination, we have

$$\begin{aligned} \mathcal{B}_1 &= \mathbb{E}\left[\prod_{s_i \in \Phi_D} \frac{1}{\left(1+\beta_\nu L^\alpha g\left(s_i\right)\right)^2}\right] \\ &\overset{(a)}{=} \exp\left(-\lambda_D 2\pi \int_0^\infty \left[1 - \frac{1}{\left(1+\beta_\nu L^\alpha l^{-\alpha}\right)^2}\right] l \mathrm{d}l\right) \\ &\overset{(b)}{=} \exp\left(-\lambda_D \pi \beta_\nu^\delta \Gamma\left(1+\delta\right)\Gamma\left(1-\delta\right)\left(1+\delta\right)L^2\right), \end{aligned} \tag{16}$$

where (a) follows from the probability generating functional (PGFL) of the PPP [13] and (b) follows from the diversity polynomial [33].

As PPP Φ_C is not independent of PPP Φ_F, we have

$$\begin{aligned} \mathcal{B}_2 &= \mathbb{E}\left[\left(\prod_{s_i \in \Phi_C} \frac{1}{1+\beta_\nu L^\alpha g\left(s_i\right)}\right)\left(\prod_{r_m \in \Phi_F} \frac{1}{1+\beta_\nu L^\alpha g\left(r_m\right)}\right)\right] \\ &\overset{(a)}{=} \mathbb{E}\left[\left(\prod_{s_i \in \Phi_C} \frac{1}{1+\beta_\nu L^\alpha g\left(s_i\right)}\right)\left(\prod_{s_i \in \Phi_C}\left[\frac{1-q_e}{1+\beta_\nu L^\alpha g\left(s_i+\tau\right)} + q_e\right]\right)\right] \\ &\overset{(b)}{=} \exp\left(-\lambda_C \int_{\mathbb{R}^2}\left[1 - \frac{1}{1+\beta_\nu L^\alpha g\left(s\right)}\left(\frac{1-q_e}{1+\beta_\nu L^\alpha g\left(s+\tau\right)} + q_e\right)\right]\mathrm{d}s\right), \end{aligned} \tag{17}$$

where (a) follows from the transformation between PPP Φ_C and PPP Φ_F and expectation with respect to the event of an empty relay set, and (b) follows from the PGFL of the PPP.

By substituting (16) and (17) into (15), we can derive the outage probability of the direct transmission, which can be efficiently calculated such as in Mathematica.

5.2 Cooperative transmission

For a cooperative transmission of the target source-destination pair with selection combining at the destination, an outage occurs when both the direct and forward links cannot support the required transmission rate. Specifically, the direct link fails when the SIR at the destination is smaller than $\beta_{2\nu}$, as the source transmits a packet at rate 2ν in the first sub-time-slot. On the other hand, the forward link fails when one of the following events occurs: 1) Event ξ_1: relay set Ω_0 is empty; 2) Event ξ_2: the received SIR at destination D_0 in the second sub-time-slot is smaller than $\beta_{2\nu}$ when relay set Ω_0 is not empty. Hence, the conditional outage probability given that there exist k potential relays, denoted as $q_{out}^{CT}(k)$, is given by

$$q_{out}^{CT}(k) = \mathbb{P}\left(\gamma_{S_0 D_0:1} < \beta_{2\nu}, \xi_1 \cup \xi_2 \left| K_0 = k \right.\right), \tag{18}$$

where $\gamma_{S_0 D_0:1} = H_{S_0 D_0:1} L^{-\alpha} / I_{D_0:1}\left(\Phi_D, \Phi_C\right)$, and outage events ξ_1 and ξ_2 can be expressed as

$$\begin{aligned} &\xi_1 = \{\Omega_0 = \emptyset\} \\ &\xi_2 = \{\Omega_0 \neq \emptyset, \gamma_{R_b^0 D_0:2} < \beta_{2\nu}\}. \end{aligned} \tag{19}$$

Outage event ξ_1 means that no potential relays have a reliable link to source S_0, while outage event ξ_2 means that no qualified relays have a reliable link to destination D_0 given that relay set Ω_0 is not empty. Hence, outage event $(\xi_1 \cup \xi_2)$ is equivalent to the event that no potential relays have reliable links to both the source and destination. Given that k potential relays locate within a relay selection region, we have

$$q_{out}^{CT}(k) = \mathbb{E}\left[\underbrace{\mathbb{P}\left(\gamma_{S_0 D_0:1} < \beta_{2\nu}\right)}_{\mathcal{C}_1} \prod_{n=1}^{k}\left(1 - \underbrace{\mathbb{P}\left(\gamma_{S_0 R_n:1} \geq \beta_{2\nu}\right)}_{\mathcal{C}_2} \underbrace{\mathbb{P}\left(\gamma_{R_n D_0:2} \geq \beta_{2\nu}\right)}_{\mathcal{C}_3}\right)\right]. \tag{20}$$

The success probability of the relay-destination link can be expressed as

$$\begin{aligned} \mathcal{C}_3 &\overset{(a)}{=} \mathbb{E}\left[\exp\left(-\beta_{2\nu} d_{R_n D_0}^{\alpha} I_{D D_0:2}\left(\Phi_D\right)\right) \cdot \exp\left(-\beta_{2\nu} d_{R_n D_0}^{\alpha} I_{F D_0:2}\left(\Phi_F\right)\right)\right] \\ &\overset{(b)}{=} \prod_{s_i \in \Phi_D} \frac{1}{1 + \beta_{2\nu} d_{R_n D_0}^{\alpha} g\left(s_i\right)} \cdot \prod_{r_m \in \Phi_F} \frac{1}{1 + \beta_{2\nu} d_{R_n D_0}^{\alpha} g\left(r_m\right)} \\ &\overset{(c)}{=} \prod_{s_i \in \Phi_D} \frac{1}{1 + \beta_{2\nu} d_{R_n D_0}^{\alpha} g\left(s_i\right)} \cdot \prod_{s_j \in \Phi_C} \left[\frac{1 - q_e}{1 + \beta_{2\nu} d_{R_n D_0}^{\alpha} g\left(s_j + \tau\right)} + q_e\right], \end{aligned} \tag{21}$$

where $d_{R_n D_0} = \|r_n\|$ denotes the distance between potential relay R_n and destination D_0, (a) follows from the expectation with respect to the channel fading between the relay and destination, (b) follows from the Laplace transforms of the channel fading between the interferers and destination, and (c) follows from the transformation between PPP Φ_C and PPP Φ_F.

Similarly, the outage and success probabilities of the source-destination and source-relay links can be expressed respectively as

$$\begin{aligned} &\mathcal{C}_1 = 1 - \prod_{s_i \in \Phi_D} \left[1 + \beta_{2\nu} L^{\alpha} g\left(s_i\right)\right]^{-1} \prod_{s_j \in \Phi_C} \left[1 + \beta_{2\nu} L^{\alpha} g\left(s_j\right)\right]^{-1} \\ &\mathcal{C}_2 = \prod_{s_i \in \Phi_D} \left[1 + \beta_{2\nu} d_{S_0 R_n}^{\alpha} g\left(s_i - r_n\right)\right]^{-1} \prod_{s_j \in \Phi_C} \left[1 + \beta_{2\nu} d_{S_0 R_n}^{\alpha} g\left(s_j - r_n\right)\right]^{-1}, \end{aligned} \tag{22}$$

where $d_{S_0 R_n} = \|s_0 - r_n\|$ denotes the distance between source S_0 and potential relay R_n.

The conditional outage probability, given that there exist k potential relays, is given by

$$\begin{aligned} q_{out}^{CT}(k) &= \mathbb{E}\left[\mathcal{C}_1 \cdot \prod_{n=1}^{k} (1 - \mathcal{C}_2 \cdot \mathcal{C}_3)\right] \\ &\overset{(a)}{=} \mathbb{E}\left[\mathcal{C}_1 \cdot (1 - \mathcal{C}_2 \cdot \mathcal{C}_3)^k\right] \\ &\overset{(b)}{=} \sum_{m=0}^{k} \binom{k}{m} (-1)^m \underbrace{\mathbb{E}\left[\mathcal{C}_1 \cdot \mathcal{C}_2^m \cdot \mathcal{C}_3^m\right]}_{\mathcal{C}}, \end{aligned} \tag{23}$$

where (a) follows as k potential relays are uniformly distributed within the relay selection region, and (b) follows from the binomial expansion [33, 34]. Note that the spatial and temporal correlation of interference power observed by the destination and potential relays is considered by taking a joint expectation over the spatial locations of the same set of interferers.

Instead of averaging over all possible relay locations within the constrained relay selection region, we divide it evenly into ζ equal sub-regions and average over the centers of all sub-regions, so as to reduce the computation complexity. As shown in the simulations, a small value of ζ provides an accurate approximation. As PPP Φ_D and PPP Φ_C are independent, applying the PGFL of the PPP, we have

$$\begin{aligned} \mathcal{C} \approx \frac{1}{\zeta} \sum_{i=1}^{\zeta} \Bigg[&\exp\left(-\lambda_D \int_{\mathbb{R}^2} \left[1 - \mathcal{L}^m\left(\|s_0 - r_i\|, -r_i\right) \mathcal{L}^m\left(\|r_i\|, d_0\right)\right] \mathrm{d}s\right) \\ &\times \exp\left(-\lambda_C \int_{\mathbb{R}^2} \left[1 - \mathcal{L}^m\left(\|s_0 - r_i\|, -r_i\right) \left((1 - q_e)\, \mathcal{L}^m\left(\|r_i\|, r_i\right) + q_e\right)\right] \mathrm{d}s\right) \\ &- \exp\left(-\lambda_D \int_{\mathbb{R}^2} \left[1 - \mathcal{L}\left(L, d_0\right) \mathcal{L}^m\left(\|s_0 - r_i\|, -r_i\right) \mathcal{L}^m\left(\|r_i\|, d_0\right)\right] \mathrm{d}s\right) \\ &\times \exp\left(-\lambda_C \int_{\mathbb{R}^2} \left[1 - \mathcal{L}\left(L, d_0\right) \mathcal{L}^m\left(\|s_0 - r_i\|, -r_i\right) \left((1 - q_e) \mathcal{L}^m\left(\|r_i\|, r_i\right) + q_e\right)\right] \mathrm{d}s\right) \Bigg], \end{aligned} \tag{24}$$

where

$$\mathcal{L}(u, v) = \frac{1}{1 + \beta_{2\nu} u^{\alpha} g(s + v)}. \tag{25}$$

The conditional outage probability of opportunistic cooperation, given k potential relays, can be derived by substituting (24) and (25) into (23). Finally, the outage probability of opportunistic cooperation can be obtained by substituting (15) and (23) into (13).

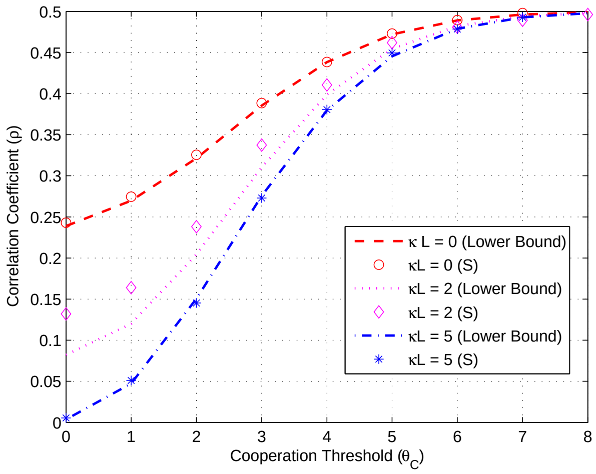


Fig. 3 Correlation coefficient of interference power $I_{D_0:1}(\Phi_D, \Phi_C)$ and $I_{D_0:2}(\Phi_D, \Phi_F)$ versus cooperation threshold θ_C and distance between a source and the center of a relay selection region κL when $\lambda_S = 0.001$ nodes/m^2, $\lambda_R = 0.2$ nodes/m^2, $r_C = 2$ m, and $L = 12$ m

6 Numerical results

This section presents both analytical (A) and simulation (S) results for the direct transmission and opportunistic cooperation. In the simulations, a circular network coverage area with radius 1000 m is considered. Based on Shannon's formula, the required reception thresholds for direct and cooperative transmissions (i.e., β_ν and $\beta_{2\nu}$) are set to be 2 and 8, respectively. In addition, we set both α and ζ equal to 4. The simulation results are obtained by averaging 10^6 realizations of the random network topology.

6.1 Correlation coefficient of interference power

Fig. 3 shows the temporal correlation coefficient of interference power $I_{D_0:1}(\Phi_D, \Phi_C)$ and $I_{D_0:2}(\Phi_D, \Phi_F)$ versus cooperation threshold θ_C and distance between a source and the center of a relay selection region κL with parameters $\lambda_S = 0.001$ nodes/m^2, $\lambda_R = 0.2$ nodes/m^2, $r_C = 2$ m, and $L = 12$ m. The analytical lower bound of temporal correlation coefficient is obtained based on (10). To guarantee that each source-destination pair has enough potential relays and to show the impact of the number of potential relays, source density λ_S is set to be much smaller than relay density λ_R. In addition, we set $\epsilon = 1$ for the non-singular path loss model. It is observed that the correlation coefficient increases with the cooperation threshold. This is due to the fact that, with an increase of the cooperation threshold, the probability of each source-destination pair activating cooperative transmissions becomes smaller, which results in a higher level of interference correlation. In other words, the interference

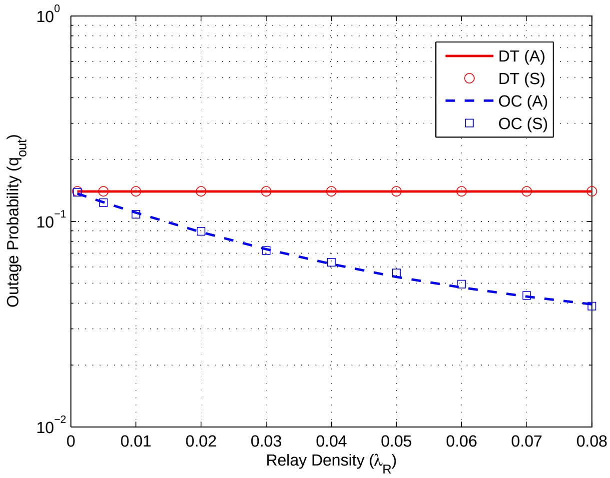


Fig. 4 Outage probability versus relay density (in nodes/m^2) when $\lambda_S = 0.0001$ nodes/m^2, $r_C = 4$ m, $\kappa = 0.5$, and $L = 12$ m

redistribution incurred by cooperative transmissions reduces the level of interference correlation. Over a quasi-static Rayleigh fading channel with $\mathbb{E}[H^2] = 2$, the correlation coefficient increases up to 0.5. On the other hand, with an increase of κL, the average distance between the source and the best relay becomes larger, which reduces the level of interference correlation as well as the correlation coefficient.

6.2 Outage probability and transmission capacity

In this subsection, we study the impact of the relay density, cooperation threshold, distance between a source and the center of a relay selection region, size of the relay selection region, and link length on the outage probability and transmission capacity[4] [25]. The transmission capacity measures the maximum spatial density of concurrent successful transmissions that can be supported in a network without violating the outage probability constraint, which is set to 5% in the simulation.

Fig. 4 shows the outage probabilities of direct transmission (DT) where $\theta_C = \infty$ and opportunistic cooperation (OC) with $\theta_C = 1$ versus the relay density with parameters $\lambda_S = 0.0001$ nodes/m^2, $r_C = 4$ m, $\kappa = 0.5$, and $L = 12$ m, where the analytical results are obtained based on (13) and (15). It is observed that the outage probability of opportunistic cooperation decreases with the relay density, while that of the direct transmission does not change. This is due to the fact that, with an increase of the relay density, more potential relays are available for each source-destination pair, which results in a higher probability of selecting a reliable relay. The outage probability of opportunistic cooperation is always lower than that of the direct trans-

[4] In terms of the average number of source-destination pairs per square kilometer.

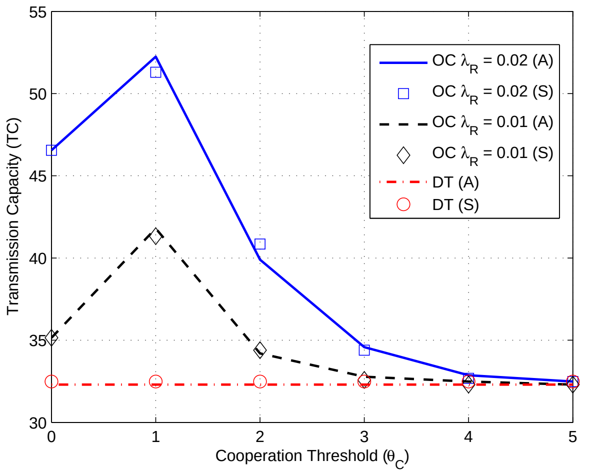


Fig. 5 Transmission capacity in links/km^2 versus cooperation threshold for $\lambda_R = 0.01$ nodes/m^2 and $\lambda_R = 0.02$ nodes/m^2 when $r_C = 4$ m, $\kappa = 0.5$, and $L = 12$ m

mission, as cooperation is activated only when there exists at least one potential relay within the best relay locations.

Fig. 5 illustrates the transmission capacity of DT and OC versus the cooperation threshold for $\lambda_R = 0.01$ nodes/m^2 and $\lambda_R = 0.02$ nodes/m^2 when $r_C = 4$ m, $\kappa = 0.5$, and $L = 12$ m. When all source-destination pairs activate cooperative transmissions regardless of the number of potential relays (i.e., $\theta_C = 0$), the achievable transmission capacity is smaller than that of opportunistic cooperation with $\theta_C = 1$. When there are no potential relays, the cooperative transmission performs worse than the direct transmission as the cooperative transmission requires a higher reception threshold for the SIR. It is observed that the transmission capacity of opportunistic cooperation reaches the maximum value when cooperation threshold $\theta_C = 1$. With an increase of the cooperation threshold, the transmission capacity of opportunistic cooperation approaches to that of the direct transmission as the probability of enabling cooperative transmissions decreases. Moreover, the transmission capacity increases with the relay density, as the probability of selecting a reliable relay becomes higher.

Fig. 6 shows the outage probabilities of DT and OC with $\theta_C = 1$ versus the distance between a source and the center of a relay selection region for $r_C = 3$ m and $r_C = 4$ m with parameters $\lambda_S = 0.0001$ nodes/m^2, $\lambda_R = 0.03$ nodes/m^2, and $L = 12$ m. It can be observed that the opportunistic cooperation achieves the best performance when the center of the relay selection region is located at the link center. This is because the performance of the two-hop cooperative transmission is determined by the qualities of both the source-relay and relay-destination links. Furthermore, with an increase of the size of the relay selection region, better performance is achieved as more relays are available for each source-destination pair.

Fig. 7 shows the transmission capacity of DT, cooperative transmission (CT, i.e., OC with $\theta_C = 0$), and OC with $\theta_C = 1$ versus the link length for $\lambda_R = 0.005$

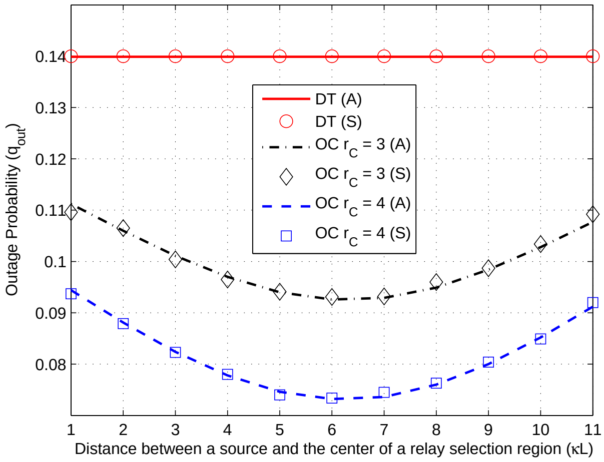


Fig. 6 Outage probabilities versus distance between a source and the center of a relay selection region for $r_C = 3$ m and $r_C = 4$ m when $\lambda_S = 0.0001$ nodes/m^2, $\lambda_R = 0.03$ nodes/m^2, and $L = 12$ m

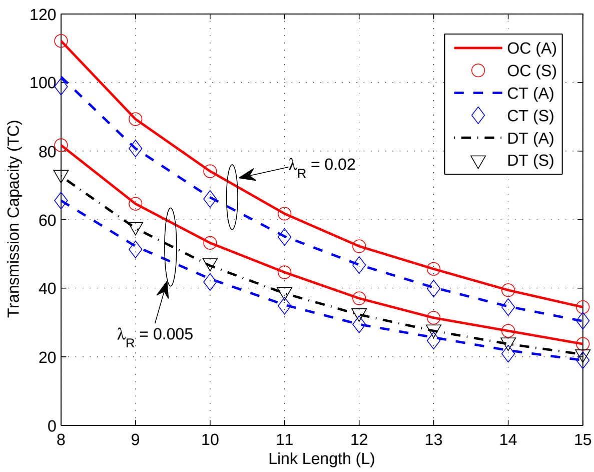


Fig. 7 Transmission capacity in links/km^2 versus link length for $\lambda_R = 0.005$ nodes/m^2 and $\lambda_R = 0.02$ nodes/m^2 when $r_C = 4$ m and $\kappa = 0.5$

nodes/m^2 and $\lambda_R = 0.02$ nodes/m^2 when $r_C = 4$ m and $\kappa = 0.5$. With an increase of the link length, the outage probabilities of all transmission schemes increase due to a larger path loss, which leads to a lower transmission capacity. When the relay density is low (e.g., $\lambda_R = 0.005$ nodes/m^2), the cooperative transmission performs worse than the direct transmission, because the probability of having a reliable relay is low and the cooperative transmission requires a higher reception threshold for the SIR. On the other hand, by activating cooperation when there exists at least one potential relay, the transmission capacity of opportunistic cooperation achieves better performance. Furthermore, with an increase of the relay density (e.g., $\lambda_R = 0.02$ nodes/m^2), higher performance gains are achieved by both cooperative transmission schemes.

7 Conclusions

In this paper, we propose an opportunistic cooperation strategy for a wireless ad hoc network with randomly positioned single-hop source-destination pairs and relays, which leads to a mixture of direct and cooperative transmissions as well as the spatial and temporal correlation of interference power. We derive the correlation coefficient of interference power and outage probability of opportunistic cooperation as a function of important network and protocol parameters. We demonstrate that cooperation is not always beneficial and its effectiveness depends on the number of available potential relays. Extensive simulations are conducted to validate the performance analysis. The analytical framework can be extended to analyze more advanced cooperation strategies. For further work, we aim to take into account the multi-hop transmission and protocol overhead, in order to fully understand the benefits and limitations of cooperative communication.

Appendix. Proof of Proposition 1

According to the definition of the correlation coefficient between two random variables, we have

$$\rho = \frac{\mathbb{E}\left[I_{D_0:1}\left(\Phi_D, \Phi_C\right) I_{D_0:2}\left(\Phi_D, \Phi_F\right)\right] - \mathbb{E}\left[I_{D_0:1}\left(\Phi_D, \Phi_C\right)\right]\mathbb{E}\left[I_{D_0:2}\left(\Phi_D, \Phi_F\right)\right]}{\sqrt{\text{Var}\left(I_{D_0:1}\left(\Phi_D, \Phi_C\right)\right)}\sqrt{\text{Var}\left(I_{D_0:2}\left(\Phi_D, \Phi_F\right)\right)}}, \quad (26)$$

where $\mathbb{E}[X]$ and $\text{Var}(X)$ represent the mean and variance of random variable X, respectively.

Due to the unit mean of fading coefficients, the mean of interference power $I_{D_0:1}\left(\Phi_D, \Phi_C\right)$ is given by

$$\begin{aligned} \mathbb{E}\left[I_{D_0:1}\left(\Phi_D, \Phi_C\right)\right] &= \mathbb{E}\left[\sum_{s_i \in \Phi_D} H_{S_i D_0:1} g\left(s_i\right) + \sum_{s_j \in \Phi_C} H_{S_j D_0:1} g\left(s_j\right)\right] \\ &\overset{(a)}{=} \lambda_S \int_{\mathbb{R}^2} g\left(s\right) \mathrm{d}s, \end{aligned} \quad (27)$$

where (a) follows from the Campbell's Theorem [12].

Similarly, we have $\mathbb{E}\left[I_{D_0:2}\left(\Phi_D, \Phi_F\right)\right] = \left(\lambda_D + \lambda_F\right) \int_{\mathbb{R}^2} g\left(s\right) \mathrm{d}s$, where $\lambda_F = \lambda_C \cdot \mathbb{P}\left(\Omega_0 \neq \emptyset\right)$ denotes the spatial density of PPP Φ_F. As the cooperative transmission is activated only when there exist at least θ_C potential relays, the probability of an empty relay set (i.e., no qualified relays), denoted as $q_e = \mathbb{P}\left(\Omega_0 = \emptyset\right)$, is given by

$$q_e = \sum_{k=\theta_C}^{\infty} \mathbb{P}\left(K_0 = k\right) \cdot \underbrace{\mathbb{P}\left(\gamma_{S_0 R_1:1} < \beta_{2\nu}, \cdots, \gamma_{S_0 R_k:1} < \beta_{2\nu}\right)}_{\mathcal{A}}, \quad (28)$$

where $\gamma_{S_0R_k:1} = H_{S_0R_k:1}g(s_0 - r_k)/I_{R_k:1}(\Phi_D, \Phi_C)$ denotes the received SIR at relay R_k.

The probability that k potential relays fail to decode the packet from source S_0 is given by

$$\begin{aligned}
\mathcal{A} &\overset{(a)}{=} \mathbb{E}\left[\prod_{i=1}^{k}\left(1-\exp\left[-\beta_{2\nu}(\kappa L)^{\alpha}\left(I_{DR_i:1}(\Phi_D)+I_{CR_i:1}(\Phi_C)\right)\right]\right)\right] \\
&\overset{(b)}{=} \mathbb{E}\left[\left(1-\prod_{s_i\in\Phi_D}\frac{1}{1+\beta_{2\nu}(\kappa L)^{\alpha}g(s_i-r_k)}\prod_{s_j\in\Phi_C}\frac{1}{1+\beta_{2\nu}(\kappa L)^{\alpha}g(s_j-r_k)}\right)^{k}\right] \\
&\overset{(c)}{=} \sum_{m=0}^{k}\binom{k}{m}(-1)^m\underbrace{\mathbb{E}\left[\prod_{s_i\in\Phi_D}\frac{1}{\left(1+\beta_{2\nu}(\kappa L)^{\alpha}g(s_i-r_k)\right)^m}\right]}_{\mathcal{A}_1} \\
&\quad \times\underbrace{\mathbb{E}\left[\prod_{s_j\in\Phi_C}\frac{1}{\left(1+\beta_{2\nu}(\kappa L)^{\alpha}g(s_j-r_k)\right)^m}\right]}_{\mathcal{A}_2},
\end{aligned} \tag{29}$$

where (a) follows by taking expectations over independent exponential channel fading between source S_0 and potential relays, (b) follows by taking Laplace transforms of independent channel fading between the interferers and potential relays, and (c) follows from the binomial expansion and the independence between PPP Φ_D and PPP Φ_C. Note that the spatial correlation of interference power at potential relays is considered by taking a joint expectation over the spatial locations of the same set of interferers.

Via applying the PGFL of the PPP [13] and performing a coordinate transformation, we have

$$\begin{aligned}
\mathcal{A}_1 &= \exp\left(-2\pi\lambda_D\int_0^{\infty}\left[1-\left(1+\beta_{2\nu}(\kappa L)^{\alpha}l^{-\alpha}\right)^{-m}\right]l\mathrm{d}l\right) \\
&= \exp\left(-\lambda_D Q\right) \\
\mathcal{A}_2 &= \exp\left(-\lambda_C Q\right),
\end{aligned} \tag{30}$$

where Q is defined in (9).

By substituting (29) and (30) into (28), λ_F and $\mathbb{E}\left[I_{D_0:2}(\Phi_D, \Phi_F)\right]$ can be derived.

The mean product of $I_{D_0:1}(\Phi_D, \Phi_C)$ and $I_{D_0:2}(\Phi_D, \Phi_F)$ is given by

$$\begin{aligned}
&\mathbb{E}\left[I_{D_0:1}(\Phi_D,\Phi_C)\,I_{D_0:2}(\Phi_D,\Phi_F)\right] \\
&= \mathbb{E}\left[I_{DD_0:1}(\Phi_D)\,I_{DD_0:2}(\Phi_D)\right]+\mathbb{E}\left[I_{DD_0:1}(\Phi_D)\,I_{FD_0:2}(\Phi_F)\right] \\
&+ \mathbb{E}\left[I_{CD_0:1}(\Phi_C)\,I_{DD_0:2}(\Phi_D)\right]+\mathbb{E}\left[I_{CD_0:1}(\Phi_C)\,I_{FD_0:2}(\Phi_F)\right].
\end{aligned} \tag{31}$$

As PPP Φ_D is independent of PPP Φ_C and PPP Φ_F, we have

$$\begin{aligned} \mathbb{E}\left[I_{DD_0:1}\left(\Phi_D\right)I_{FD_0:2}\left(\Phi_F\right)\right] &= \lambda_D\lambda_F\left(\int_{\mathbb{R}^2} g\left(s\right)\mathrm{d}s\right)^2 \\ \mathbb{E}\left[I_{CD_0:1}\left(\Phi_C\right)I_{DD_0:2}\left(\Phi_D\right)\right] &= \lambda_C\lambda_D\left(\int_{\mathbb{R}^2} g\left(s\right)\mathrm{d}s\right)^2. \end{aligned} \tag{32}$$

As PPP Φ_C and PPP Φ_F are not independent of each other, the mean product of $I_{CD_0:1}\left(\Phi_C\right)$ and $I_{FD_0:2}\left(\Phi_F\right)$ is given by

$$\begin{aligned} &\mathbb{E}\left[I_{CD_0:1}\left(\Phi_C\right)I_{FD_0:2}\left(\Phi_F\right)\right] \\ &\overset{(a)}{=}\mathbb{E}\left[\left(\sum_{s_j\in\Phi_C} g\left(s_j\right)\right)\left(\sum_{r_m\in\Phi_F} g\left(r_m\right)\right)\right] \\ &\overset{(b)}{=}\mathbb{E}\left[\left(\sum_{s_j\in\Phi_C} g\left(s_j\right)\right)\left(\sum_{s_j\in\Phi_C}\left(1-q_e\right)\cdot g\left(s_j+\tau\right)\right)\right] \\ &=\mathbb{E}\left[\sum_{s_j\in\Phi_C}\left(1-q_e\right)\cdot g\left(s_j\right)g\left(s_j+\tau\right)\right]+\mathbb{E}\left[\sum_{s_i,s_j\in\Phi_C}^{s_i\neq s_j}\left(1-q_e\right)\cdot g\left(s_j\right)g\left(s_j+\tau\right)\right] \\ &\overset{(c)}{=}\lambda_F\int_{\mathbb{R}^2} g\left(s\right)\mathbb{E}_\tau\left[g\left(s+\tau\right)\right]\mathrm{d}s+\lambda_C\lambda_F\left(\int_{\mathbb{R}^2} g\left(s\right)\mathrm{d}s\right)^2, \end{aligned} \tag{33}$$

where (a) holds as the fading coefficients of different links are independent random variables with unit mean, (b) follows from the transformation between PPP Φ_C and PPP Φ_F and τ is the coordinate difference between a source and its selected relay (i.e., $\tau = r_m - s_j$), and (c) follows from the Campbell's Theorem and second-order product density formula of the PPP [12].

Similarly, by replacing τ with $(0,0)$, we have

$$\mathbb{E}\left[I_{DD_0:1}\left(\Phi_D\right)I_{DD_0:2}\left(\Phi_D\right)\right] = \lambda_D\int_{\mathbb{R}^2} g^2\left(s\right)\mathrm{d}s+\lambda_D^2\left(\int_{\mathbb{R}^2} g\left(s\right)\mathrm{d}s\right)^2. \tag{34}$$

By substituting (32-34) into (31), the numerator of (26), denoted as N, is given by

$$N = \lambda_D\int_{\mathbb{R}^2} g^2\left(s\right)\mathrm{d}s+\lambda_F\int_{\mathbb{R}^2} g\left(s\right)\mathbb{E}_\tau\left[g\left(s+\tau\right)\right]\mathrm{d}s. \tag{35}$$

The second moment of $I_{DD_0:1}\left(\Phi_D\right)$ is given by

$$\mathbb{E}\left[I_{DD_0:1}^2\left(\Phi_D\right)\right] \overset{(a)}{=} 2\lambda_D\int_{\mathbb{R}^2} g^2\left(s\right)\mathrm{d}s+\lambda_D^2\left(\int_{\mathbb{R}^2} g\left(s\right)\mathrm{d}s\right)^2, \tag{36}$$

where (a) follows from the similar arguments in (33) and $\mathbb{E}\left[H^2\right] = 2$ for Rayleigh fading channels.

Using (36), the variance of $I_{D_0:1}(\Phi_D, \Phi_C)$ and $I_{D_0:2}(\Phi_D, \Phi_F)$ can be expressed respectively as

$$\begin{aligned}\text{Var}\left(I_{D_0:1}(\Phi_D, \Phi_C)\right) &= \mathbb{E}\left[I^2_{D_0:1}(\Phi_D, \Phi_C)\right] - \mathbb{E}[I_{D_0:1}(\Phi_D, \Phi_C)]^2 \\ &= 2\lambda_S \int_{\mathbb{R}^2} g^2(s)\,\mathrm{d}s \\ \text{Var}\left(I_{D_0:2}(\Phi_D, \Phi_F)\right) &= 2(\lambda_D + \lambda_F) \int_{\mathbb{R}^2} g^2(s)\,\mathrm{d}s.\end{aligned} \tag{37}$$

For the non-singular path loss model defined in (1), we have

$$\int_{\mathbb{R}^2} g^2(s)\,\mathrm{d}s = \frac{\delta(1-\delta)\pi^2}{\epsilon^{2-\delta}\sin(\pi\delta)}. \tag{38}$$

By substituting (35), (37), and (38) into (26), we obtain (7).

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