Energy Efficient Sleep/Wake Scheduling for Multihop Sensor Networks: Non-convexity and Approximation Algorithm

Yan Wu, Sonia Fahmy, Ness B. Shroff
In Proc. INFOCOM 2007

Presented by David (Bong Jun) Choi
ECE, University of Waterloo (2012-01-11)
Presentation Outline

- Introduction
- System Model
- Optimal Sleep/Wake Scheduling Problem
  - PART 1: Single-hop Intra-Cluster
  - PART 2: Multi-hop
- Conclusion
Introduction

- Contribution of the Paper
  - When to wakeup/sleep?
    - Single-hop
    - Multi-hop
  - Uniqueness of the problem
    - Consider synchronization error
  - Technical merit of the paper
    - Non-convex Optimization
      - Formulate into Convex Optimization
      - Approximation Method (0.73-approximation algorithm)
Introduction

- **Motivation**
  - Idle listening consumptions a proportion of total energy
  - Impact of synchronization error is non-negligible

---

**Fig. 3.** Wake up interval to capture the message

- **Tradeoff**
  - Energy consumption vs. Message delivery performance
Introduction

- Optimal sleep/wake scheduling algorithm
  - Goal: min energy consumption (max lifetime)
  - Condition: Satisfies QoS
    - Single-hop: per-hop message capture probability threshold
    - Multi-hop: source (sensors) to destination (BS)

Fig. 1. A three-level cluster hierarchy
System Model

- **Cluster Hierarchy**
  - Existing clustering algorithms

- **Scheduling**
  - TDMA (Sync + M Tx)
  - CH - CMs

Fig. 1. A three-level cluster hierarchy

Fig. 2. Equispaced upstream transmissions
System Model

- **Assumptions**
  - Neighboring clusters do not interfere with each other
    - Orthogonal frequency channel (1000’s of channels available)
  - Data aggregation
    - Length of the aggregated message:
      \[
      \chi(L_0, \ldots L_M) = r \sum_{i=0}^{M} L_i + c. \quad (1)
      \]
      \[\text{CH} \quad \text{“compression ratio”} \quad \text{“length of message”} \quad \text{“overhead”}\]
  - Negligible collision probability
    - Large separation distance between transmissions
  - Negligible propagation delay
Problem Formulation

- **Synchronization**
  - Phase offset
  - Clock skew

\[ t_i(j, k) = a_i(j)C(j, k) + b_i(j) + e_i(j, k), \quad (2) \]

“node i time” “CH time”
Problem Formulation

- **Goal**
  - When to wake/sleep?

\[
E = \text{(if arrival outside } wp \text{ and } sp \rightarrow E_{\text{idle}} \text{ )} + \text{(if arrival inside } wp \text{ and } sp \rightarrow E_{\text{idle}} + E_{\text{receive}}) }
\]

**Fig. 3.** Wake up interval to capture the message

\[
(A) \ \text{Min } E = (s_p - w_p)\alpha_I \text{Prob}\{\tau'_p \notin (w_p, s_p)\} +\int_{w_p}^{s_p} \left\{ (x - w_p)\alpha_I + \frac{L_p}{R} \alpha_r \right\} f_{\tau'_p}(x)dx
\]

such that $\text{Prob}\{\tau'_p \in (w_p, s_p)\} \geq th$,

\[f_{\tau'_p}(\cdot) \text{ (PDF) of } \tau'_p\]
Single-hop

**Simplification**

- Normal distribution: $f_{\tau'_p}(\cdot)$ (PDF) of $\tau'_p$

\[
\begin{align*}
(A) \text{ Min } E &= (s_p - w_p)\alpha_I \text{Prob}\{\tau'_p \notin (w_p, s_p)\} + \\
& \int_{w_p}^{s_p} ((x - w_p)\alpha_I + \frac{L_p}{R} \alpha_R) f_{\tau'_p}(x) dx \\
& \text{such that } \text{Prob}\{\tau'_p \in (w_p, s_p)\} \geq th,
\end{align*}
\]

\[
E(\tau'_p) = \tau_p, \quad \text{(4)}
\]

\[
\text{VAR}(\tau'_p) \equiv \sigma^2_p = \frac{\sigma_0^2}{a^2_j(j)} \frac{1}{N_s} [1 + \frac{(\tau_p - \overline{C(j,k)})^2}{\overline{C^2(j,k)} - (\overline{C(j,k)})^2}],
\]

where \( \overline{C(j,k)} = \frac{\sum_{k=1}^{N_s} C(j,k)}{N_s}, \overline{C^2(j,k)} = \frac{\sum_{k=1}^{N_s} C^2(j,k)}{N_s} \).

\[
\hat{r} = \frac{\tau'_p - \tau_p}{\sigma_p}, \quad w = \frac{w_p - \tau_p}{\sigma_p}, \quad s = \frac{s_p - \tau_p}{\sigma_p}
\]

\[
(A1) \text{ Min } F(w, s) = (s - w)\sigma_p\alpha_I - [Q(w) - Q(s)] s\sigma_p\alpha_I \\
+ [g(w) - g(s)] \sigma_p\alpha_I + [Q(w) - Q(s)] \frac{L_p}{R} \alpha_R,
\]

such that $Q(w) - Q(s) \geq th$, $g(\cdot)$ is the PDF for the standard normal distribution, $Q(\cdot)$ is the complementary cumulative distribution function.
Part 1: Single-hop

- Non-Convexity of (A1)
  - Hessian Matrix

(A1) \( \text{Min } F(w, s) = (s - w)\sigma_p\alpha_I - [Q(w) - Q(s)]s\sigma_p\alpha_I \]
\[+ [g(w) - g(s)]\sigma_p\alpha_I + [Q(w) - Q(s)] \frac{L_p}{R}\alpha_r, \]
such that \( Q(w) - Q(s) \geq th, \)

Proposition 1: For Problem (A1), any optimal solution \((w^*, s^*)\) satisfies \(Q(w^*) - Q(s^*) = th.\)

(A2) \( \text{Min } G(w) = (1 - th)s(w) - w + g(w) - g(s(w)), \)
such that \( s(w) = Q^{-1}(Q(w) - th) \) and \( w < Q^{-1}(th). \)

(Convex Optimization) Second derivative of \( G(w) > 0 \)
Single-hop

- Finally!

![Diagram showing wake and sleep intervals](image)

**Fig. 3.** Wake up interval to capture the message

After we obtain $w^*$, $s^*$, we compute the optimal sleep/wake schedule as $(w_p^* = \tau_p + w^*\sigma_p, s_p^* = \tau_p + s^*\sigma_p)$. 
Part 2: Multi-hop

- Different from the single hop
  - Per-hop threshold $\rightarrow$ Src. to Dest. QoS

- System Model (Modified)
  - Cluster Head of node $n$: $H(n)$
  - $d(n)$: hop distance to node $n$ from $H$
  - $M(n)$: members of node $n$
  - $D(n)$: descendants of node $n$

Ex) node $A = n$
$H(A) = BS$
$d(n) = 1$
$M(n) = C, D$
$D(A) = C, D, E, F$

Fig. 1. A three-level cluster hierarchy
Multi-hop

- **Goal**
  
  \[
  \text{(B) Max } T_L \text{ such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in S, \]

  \[z(n): \text{capture probability threshold of } H(n)\]

- **Solution**

  **Satisfy “leaf node” → Satisfy “all ancestors” (on the path to the leaf node)**

  \[
  \text{(B) Max } T_L \text{ such that } \prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF, \]
Multi-hop

- **System Model**
  - **Message Aggregation**
    \[
    L^{avg}(n) = r(l + \sum_{i \in M(n)} z(i) L^{avg}(i)) + c,
    \]
    \[
    L^{avg}(n) = rl + c + \sum_{i \in D(n)} (rl + c) \prod_{k=0}^{d(i) - d(n) - 1} [r^z(H^{(k)}(i))].
    \]

- **Energy Consumption**
  - **Avg. Consumption by node n**
    \[
    \eta(n, \bar{z}) = \frac{\varepsilon_s(n) + \varepsilon_{syn}(n) + \varepsilon_t(n) + \varepsilon_r(n)}{T_e} + A(n) + \sum_{i \in M(n)} P(n, i)\gamma(z(i)) + \sum_{i \in D(n)} Q(n, i) \prod_{k=0}^{d(i) - d(n) - 1} z(H^{(k)}(i)),
    \]
    where
    \[
    A(n) = \frac{1}{T_e} [\varepsilon_s(n) + \varepsilon_{syn}(n) + N\alpha_t(n)\frac{rl + c}{R}],
    \]
    \[
    P(n, i) = \frac{1}{T_e} \sum_{h=1}^{N} \alpha_l \sqrt{\sigma_0^2 \frac{1}{N_s} \left[1 + \frac{(T_s + \theta(i)T_{[M(n)]}) + hT - \frac{1+N_s T_s}{2 N_s T_s}}{\sum_{k=1}^{N_s} (k \frac{T_s}{N_s} - \frac{1+N_s T_s}{2 N_s T_s})^2}\right]},
    \]
    \[
    Q(n, i) = \frac{1}{T_e} rN\alpha_t(n) + N\alpha_r (rl + c)r^{d(i) - d(n) - 1}.\]
Multi-hop

Formulation + Energy Conditions

(B) Max $T_L$
   such that $\prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,$

Max $T_L$
   such that $\prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,$
   $\eta(n, \vec{z}) \leq \xi(n)/T_L, \forall n \in S.$

“avg. energy consumption” “initial energy”

Introduce life-time penalty function: $\Psi(1/T_L) u = 1/T_L$

(B) Min $\Psi(u)$
   such that $\prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF,$
   $\eta(n, \vec{z}) \leq \xi(n)u, \forall n \in S.$
Multi-hop

Non-Convex Optimization Problem

(B) Min $\Psi(u)$ such that $\prod_{i=0}^{d(n)-1} z(H^{(i)}(n)) \geq \Lambda, \forall n \in LF$,
$\eta(n, z) \leq \xi(n) u, \forall n \in S.$

\[
\eta(n, z) = \frac{\varepsilon_s(n) + \varepsilon_{syn}(n) + \varepsilon_t(n) + \varepsilon_r(n)}{T_e} 
= A(n) + \sum_{i \in M(n)} P(n, i) \gamma(z(i)) + \\
\sum_{i \in D(n)} Q(n, i) \prod_{k=0}^{d(i)-d(n)-1} z(H^{(k)}(i)).
\]

Non-convex

Fig. 4. $\gamma(z)$

$\gamma(th) = \min\{G(w) : w < Q^{-1}(th)\}$ (7)
Multi-hop

Approximation

Proposition 3: (1) For $z \geq 0.86$, $\gamma(z)$ is strictly convex;
(2) For $z \in [0, 0.99]$, $1.86z < \gamma(z) < 2.52z$.

Approximation (Fig. 5):

$$\gamma_1(z) = \begin{cases} 
2z + 0.001z^2 & 0 \leq z \leq Z_0 \\
\gamma(z) & Z_0 \leq z < 1 
\end{cases}$$

$Z_0 \approx 0.95$. 

Fig. 4. $\gamma(z)$

Fig. 5. Approximating $\gamma(z)$
Multi-hop

- How close is the approximation to the optimal?
  - Achievable approximation ratio: **0.73**

**Proposition 4:**
(1) \(0.929 \leq \frac{\gamma(z)}{\gamma_1(z)} \leq 1.26\);
(2) \(\gamma_1(z)\) is strictly convex.

\[
T_L(\vec{z}^*) \leq \frac{1}{0.929 u_1^*}
\]

\[
T_L(\vec{z}_1^*) \geq 0.73 T_L(\vec{z}^*)
\]
Multi-hop

- Solve B1 ...

(B1') Min $\Psi(u)$ such that $\sum_{i=0}^{d(n)-1} v(H^{(i)}(n)) \geq \ln \Lambda, \forall n \in LF$,

$$
\eta'_1(n, \overline{v}) = A(n) + \sum_{i \in M(n)} P(n, i) \gamma_1(e^{v(i)}) +
\sum_{i \in D(n)} Q(n, i) e^{\sum_{k=0}^{d(i)-d(n)-1} v(H^{(k)}(i))} \leq \xi(n)u, \forall n \in S.
$$

$$
\max_{\lambda \geq 0, \mu \geq 0} \Phi(\overline{\lambda}, \overline{\mu}), \quad \overline{\lambda}, \overline{\mu} \text{ are Lagrange multipliers}
$$

$$
\Phi(\overline{\lambda}, \overline{\mu}) = \min_{u \geq 0, \overline{v} < 0} \Psi(u) + \sum_{n \in LF} \lambda_n (\ln \Lambda - (15)
$$

$$
\sum_{i=0}^{d(n)-1} v(H^{(i)}(n))) + \sum_{n \in S} \mu_n (\eta'_1(n, \overline{v}) - \xi(n)u).
$$

- Then, use “subgradient method” to solve dual problem.
Simulation Results

- Vs. scheme with “equal” thresholds

- Capture probability: 0.7

![Graph showing Performance Gain vs. compression ratio r]

**Fig. 7. Performance gain**

$r$: compression ratio

$r=1$, no compression
Simulation Results

- $r=1$ (no compression)
  - Node 1 becomes bottleneck, $z(2)$ and $z(3)$ should be small

- $r=0$ (high compression)
  - Node 2 and 3 becomes bottleneck (to receive from more member nodes)

Fig. 6. Simulation topology

- $z(1)=1$
- $z(2)=z(3)=0.71$
- $z(4)=\ldots=z(11)=0.99$

- $z(2)=z(3)=0.999$
- $z(4)=\ldots=z(11)=0.703$
Conclusion

- Sleep/wake scheduling for low duty-cycle sensor networks
- Consider synchronization error
- Achieve given capture probability threshold with min energy consumption
  - Non-convex optimization
  - 0.73-approximation algorithm
  - Correctly identifies the bottlenecks to extend the network lifetime