An introduction on game theory for wireless networking [1]

Ning Zhang
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Roadmap

1 Introduction
2 Static games
3 Extensive-form games
4 Summary
Introduction to Game Theory

• Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players.

• Components
  - A set of players
  - For each player, a set of actions
  - Payoff function or utility function
## Classification of games

<table>
<thead>
<tr>
<th>Non-cooperative</th>
<th>Cooperative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static</td>
<td>Dynamic (repeated)</td>
</tr>
<tr>
<td>Strategic-form</td>
<td>Extensive-form</td>
</tr>
<tr>
<td>Perfect information</td>
<td>Imperfect information</td>
</tr>
<tr>
<td>Complete information</td>
<td>Incomplete information</td>
</tr>
</tbody>
</table>

Non-cooperative game theory is concerned with the analysis of strategic choices. By contrast, the cooperative describes only the outcomes that result when the players come together in different combinations.

**Strategic-form:** simultaneous moves, matrix  
**Extensive-form:** sequential moves, tree

**Complete info:** each player knows the identity of other players and, for each of them, the payoff resulting of each strategy.

**Perfect info:** each player can observe the action of each other player.
Complete information vs Perfect information

• A game with complete information is a game in which each player knows the game $G = (N; S; U)$, notably the set of players $N$, the set of strategies $S$ and the set of payoff functions $U$.

• The players have a perfect information in the game, meaning that each player always knows the previous moves of all players when he has to make his move.
Cooperation in self-organized wireless networks

Usually, the devices are assumed to be cooperative. But what if they are selfish and rational?
4 Examples

upper layers

networking layer

medium access layer

physical layer

Forwarder’s Dilemma

Joint Packet Forwarding Game

Multiple Access Game

Jamming Game
Roadmap

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Ex 1: The Forwarder’s Dilemma

- Reward for packet reaching the destination: 1
- Cost of packet forwarding: \( c \) (\( 0 < c << 1 \))
From a problem to a game

- users controlling the devices are *rational* = try to maximize their benefit
- game formulation: \( G = (P, S, U) \)
  - \( P \): set of players
  - \( S \): set of strategy
  - \( U \): set of payoff functions
- strategic-form representation

<table>
<thead>
<tr>
<th></th>
<th>Forward</th>
<th>Drop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward</td>
<td>((1-c, 1-c))</td>
<td>((-c, 1))</td>
</tr>
<tr>
<td>Drop</td>
<td>((1, -c))</td>
<td>((0, 0))</td>
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- Reward for packet reaching the destination: 1
- Cost of packet forwarding: \( c \) (\(0 < c << 1\))
Solving the Forwarder’s Dilemma (1/2)

**Strict dominance:** strictly best strategy, for any strategy of the other player(s)

Strategy $s_i$ strictly dominates if

$$u_i(s'_i, s_{-i}) < u_i(s_i, s_{-i}), \forall s_{-i} \in S_{-i}, \forall s'_i \in S_i$$

where:

- $u_i \in U$ payoff function of player $i$
- $s_{-i} \in S_{-i}$ strategies of all players except player $i$

In Example 1, strategy Drop *strictly dominates* strategy Forward
Solving the Forwarder’s Dilemma (2/2)

Solution by iterative strict dominance (ie., by iteratively eliminating strictly dominated strategies):

\[
\begin{array}{c|c|c}
\text{Green} & \text{Forward} & \text{Drop} \\
\hline
\text{Blue} & (1-c, 1-c) & (c, 1) \\
\hline
\text{Forward} & (1, -c) & (0, 0) \\
\end{array}
\]

Drop \textit{strictly dominates} Forward

Forward would result in a \textit{better outcome}
Ex2: The Joint Packet Forwarding Game

- Reward for packet reaching the destination: 1
- Cost of packet forwarding: $c$ ($0 < c << 1$)

No strictly dominated strategies!
Weak dominance

**Weak dominance:** strictly better strategy for at least one opponent strategy

Strategy $s'_i$ is weakly dominated by strategy $s_i$ if

$$u_i(s'_i, s_{-i}) \leq u_i(s_i, s_{-i}), \forall s_{-i} \in S_{-i}$$

with strict inequality for at least one $s_{-i}$

![Diagram showing weak dominance and iterative weak dominance with a payoff matrix and arrows indicating strategy dominance.](image-url)
Nash equilibrium (1/2)

The best response of player $i$ to the profile of strategies $s_{-i}$ is a strategy $s_i$ such that:

$$b_i(s_{-i}) = \arg \max_{s_i \in S_i} u_i(s_i, s_{-i})$$

Strategy profile $s^*$ constitutes a Nash equilibrium if, for each player $i$,

$$u_i(s^*_i, s^*_{-i}) \geq u_i(s_i, s^*_{-i}), \forall s_i \in S_i$$

where: $u_i \in U$ payoff function of player $i$

$s_i \in S_i$ strategy of player $i$

Nash Equilibrium = Mutual best responses
**Nash equilibrium (2/2)**

**Nash Equilibrium:** no player can increase its payoff by deviating unilaterally

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<td><strong>Blue</strong></td>
<td><strong>Forward</strong></td>
<td><strong>Drop</strong></td>
<td><strong>Forward</strong></td>
<td><strong>Drop</strong></td>
</tr>
<tr>
<td><strong>Forward</strong></td>
<td>$(1-c, 1-c)$</td>
<td>$(0, 0^*)$</td>
<td>$(1-c^<em>, 1-c^</em>)$</td>
<td>$(0^<em>, 0^</em>)$</td>
</tr>
<tr>
<td><strong>Drop</strong></td>
<td>$(1^*, -c)$</td>
<td>$(0^<em>, 0^</em>)$</td>
<td>$(0^<em>, 0^</em>)$</td>
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**E1: The Forwarder’s Dilemma**

**E2: The Joint Packet Forwarding game**

**Caution!** Many games have more than one Nash equilibrium
### Efficiency of Nash equilibria

#### E2: The Joint Packet Forwarding game

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#### How to choose between several Nash equilibria?

**Pareto-optimality:** A strategy profile is Pareto-optimal if it is not possible to increase the payoff of any player without decreasing the payoff of another player.
Ex 3: The Multiple Access game

Reward for successful transmission: 1
Cost of transmission: c (0 < c << 1)

There is no strictly dominating strategy
There are two Nash equilibria
### Mixed strategy Nash equilibrium

The mixed strategy of player $i$ is a probability distribution over his pure strategies.

- **$p$:** probability of transmit for Blue
- **$q$:** probability of transmit for Green

The payoff matrices are given by:

- **Blue**
  - Quiet: $(0, 0)$
  - Transmit: $(1-c, 0)$

- **Green**
  - Quiet: $(0, 1-c)$
  - Transmit: $(-c, -c)$

The utility functions are:

- $u_{blue} = p(1-q)(1-c) - pqc = p(1-c-q)$
- $u_{green} = q(1-c-p)$
Mixed strategy Nash equilibrium

\[ u_{\text{green}} = q(1 - c - p) \]

\[ u_{\text{blue}} = p(1 - q)(1 - c) - pqc = p(1 - c - q) \]

Objectives
- Blue: choose \( p \) to maximize \( u_{\text{blue}} \)
- Green: choose \( q \) to maximize \( u_{\text{green}} \)

Green:
- If \( p < 1-c \), setting \( q = 1 \)
- If \( p > 1-c \), setting \( q = 0 \)
- If \( p = 1-c \), any \( q \) is best response

Blue:
- If \( q < 1-c \), setting \( p = 1 \)
- If \( q > 1-c \), setting \( p = 0 \)
- If \( q = 1-c \), any \( p \) is best response

\[ p = 1 - c, \quad q = 1 - c \]

is a Nash equilibrium
Ex 4: The Jamming game

Two channels: $C_1$ and $C_2$

Transmitter:
• Reward for successful transmission: 1
• Loss for jammed transmission: -1

Jammer:
• Reward for successful jamming: 1
• Loss for missed jamming: -1

There is no pure-strategy Nash equilibrium

$p = \frac{1}{2}, \quad q = \frac{1}{2}$ is a Nash equilibrium
Theorem by Nash, 1950

Theorem:
Every finite strategic-form game has a mixed-strategy Nash equilibrium.
Roadmap

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Extensive-form games

- usually to model sequential decisions
- game represented by a tree
- Example 3 modified: the **Sequential** Multiple Access game: blue plays first, then green plays.

Reward for successful transmission: 1

Cost of transmission: c

\(0 < c \ll 1\)
Strategies in Extensive-form games

• The strategy defines the moves for a player for every node in the game, even for those nodes that are not reached if the strategy is played.

strategies for blue: T, Q

strategies for green: TT, TQ, QT and QQ

TQ means that player p2 transmits if p1 transmits and remains quiet if p1 remains quiet.
Backward induction

• Solve the game by reducing from the final stage

Backward induction solution: \( h=\{T, Q\} \)
Summary

• Game theory can help modeling rational behaviors in wireless networks
• Iterated Dominance, best response function
• Pure strategies vs Mixed Strategies
• More advanced games dealing with imperfect information or incomplete information
Thank you for your attentions 😊😊