On Packet-Level Non-Altruistic Node Cooperation in Wireless Networks

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Abstract—In this paper, we investigate packet-level non-altruistic node cooperation in wireless networks with regenerative nodes. Since each node has its own data to transmit, pure relays do not exist. We prove that the split of transmit power has no impact on the diversity performance of non-altruistic cooperative transmissions. Despite the beneficial diversity gain, our results show that non-altruistic cooperative transmissions are not always superior to ordinary direct transmissions. We also evaluate the performance gains due to beneficial packet-level node cooperations in a multi-node wireless network. Simulation results show that favorable packet-level cooperative transmissions provide a substantial gain over ordinary direct transmissions.

I. INTRODUCTION

Spatial diversity has been demonstrated promising in mitigating channel fading impairments and enhancing system performance, facilitating wireless multimedia and interactive internet services. Although traditional co-located multi-antenna techniques are quite attractive for future broadband wireless access, setting up a large antenna array at wireless terminals (e.g., cellphones) is impractical due to size and power limitations. In contrast, cooperative diversity or node cooperation can provide a comparable spatial diversity gain without imposing extra hardware complexity.

In the literature, there exists a rich body of research work on node cooperation [1]–[5]. However, most of the work focuses on altruistic node cooperation (i.e., pure relays). In practice, most of the wireless terminals have their own data to transmit. Non-altruistic node cooperation is, therefore, more reasonable and realistic, where there is no pure relay in a wireless network. Due to the necessity of the split of transmit power (i.e., part of the transmit power of a node is dedicated to transmitting its own data, while the rest is dedicated to assisting other nodes), cooperative transmissions are not always advantageous over ordinary direct transmissions. Therefore, the results obtained based on the notion of altruistic node cooperation are not applicable to non-altruistic node cooperative wireless networks. On the other hand, whether a cooperative transmission can be carried out is contingent on the availability of a potential relay and how favorable that cooperative transmission is, giving rise to the notion of packet-level non-altruistic node cooperation.

In this paper, we study the impact of packet-level non-altruistic node cooperation on the system performance of a wireless network. In particular, we consider regenerative wireless nodes. Our results show that, although the diversity performance can be maintained in a high signal-to-noise ratio (SNR) regime, non-altruistic cooperative transmissions are not always beneficial to system performance enhancement, particularly in a low SNR regime. We also observe that in a multi-node wireless network, the performance gains due to beneficial packet-level node cooperations increase with the number of nodes, providing a substantial gain over ordinary direct transmissions. For presentation clarity, all the proofs are given in Appendix.

II. NON-ALTRUISTIC NODE COOPERATION

Consider a wireless network consisting of three nodes, namely Node $S$, Node $R$, and Node $D$. All three nodes are equipped with a single antenna. Node $S$ is to transmit data to Node $D$, while Node $R$ is viewed as a relay to help Node $S$ forward the data to Node $D$. We employ the Cooperation Protocol I suggested in [2] as our node cooperation strategy throughout this paper. In the first symbol interval, Node $S$ transmits a symbol to both Node $R$ and Node $D$. In the second symbol interval, Node $R$ forwards the symbol received from Node $S$ to Node $D$, while Node $S$ transmits another symbol to Node $D$. In this work, we assume that Node $D$ has complete knowledge of channel state information (CSI). After receiving the symbols in the two intervals, Node $D$ then employs the maximum likelihood (ML) detection to decode the symbols originally transmitted from Node $S$ [6]. Notice that Node $R$ can choose the decode-and-forward (DF) mode or amplify-and-forward (AF) mode of cooperation in the second symbol interval. In this work, although we consider the DF mode of cooperation (i.e., regenerative nodes), we examine the outage performances of both the AF and DF modes for the sake of completeness.

In the following, to comply with the conventional analysis, we first examine the benefits of node cooperation on the symbol level, and then we will extend the concepts to packet-level node cooperation. Let $x_1$ and $x_2$ be the first symbol and the second symbol transmitted by Node $S$, respectively. We assume that the energy of a symbol is one and the mean of the value of a symbol is zero, i.e., $E[|x_i|^2] = 1$ and $E[x_i] = 0$, where $i = 1, 2$. Denote $y_{D,1}$ as the signal received at Node $D$ during the first symbol interval, $y_{R,1}$ the signal received at Node $R$ during the first symbol interval, and $y_{D,2}$ the signal received at Node $D$ during the second symbol interval.

A. AF Mode of Cooperation

Consider a frequency-flat slow Rayleigh fading environment where the carrier phase distortion due to the fading channel can be estimated at the receiver and removed. For the AF mode of cooperation, the outputs of the matched filter receivers at the end of the symbol intervals are

$$y_{D,1} = \sqrt{E_{SD}h_{SD}}x_1 + \eta_{D,1}$$  \hspace{1cm} (1)
\[ y_{R,1} = \sqrt{E_{SR}h_{SR}x_1} + \eta_{R,1} \]  
\[ y_{D,2} = \sqrt{E_{SD}h_{SD}x_2 + E_{RD}h_{RD}} + \eta_{D,2} \]  

where \( E_{XY} \geq 0 \) is the average energy of a symbol received at Node \( Y \) from Node \( X \), i.e., \( X, Y \in \{ S, R, D \} \), \( h_{XY} \) is the Rayleigh fading coefficient for the \( X \rightarrow Y \) link modeled as an independent zero-mean complex Gaussian random variable with unit variance, and \( \eta_{D,1}, \eta_{R,1}, \) and \( \eta_{D,2} \) are independent zero-mean complex Gaussian random variables with variance \( \sigma^2/2 \) per dimension. Here, we make a reasonable assumption that the fading channels are quasi-static over a period of two symbol intervals. Notice that the introduction of a factor of \( \sqrt{E} |y_{R,1}|^2 \) in (3) to normalize the received signal at Node \( R \) during the first symbol interval so that the average energy of a symbol is one, where \( \sqrt{E} |y_{R,1}|^2 = E_{SR} + \sigma^2 \). Thus, the received signals can be represented in a matrix form given by

\[ y_{AF} = H_{AF}x + Q_{AF} \]

where \( x = [x_1 \ x_2] \), \( y_{AF} = [y_{D,1} \ y_{D,2}/w] \), \( Q_{AF} = [\eta_{D,1} \ \eta/w] \), and \( H_{AF} \) is the channel matrix given by

\[
H_{AF} = \begin{bmatrix}
\sqrt{E_{SD}h_{SD}} & 0 \\
\frac{\sqrt{E_{SR}h_{RD}}}{\sqrt{E_{RD}h_{RD}}} & \frac{\sqrt{E_{SD}h_{SD}}}{\sqrt{E_{RD}h_{RD}}}
\end{bmatrix}
\]

with \( \gamma = 1/\sigma^2 \), \( \eta = \eta_{D,2} + \sqrt{\frac{E_{RD}}{E_{SR}+\sigma^2}}h_{RD}\eta_{R,1} \), and \( w = \sqrt{\frac{(E_{SR}+E_{RD}+\sigma^2)/(E_{SR}+\sigma^2)}{w}} \). Notice that a factor of \( 1/w \) is multiplied to (3) to ensure the variance of the noise to be the same as that of \( \eta_{D,1} \) [2].

**B. DF Mode of Cooperation**

For the DF mode of cooperation, we have

\[ y_{D,1} = \sqrt{E_{SD}h_{SD}x_1} + \eta_{D,1} \]
\[ y_{R,1} = \sqrt{E_{SR}h_{SR}x_1} + \eta_{R,1} \]
\[ y_{D,2} = \sqrt{E_{SD}h_{SD}x_2 + E_{RD}h_{RD}} + \eta_{D,2} \]

In (8), we assume that Node \( R \) can decode the received symbol \( x_1 \) successfully, yet this assumption will be lifted when we perform the analysis of outage probability, to be discussed in Section II-C. The received signals can be represented in a matrix form given by

\[ y_{DF} = H_{DF}x + Q_{DF} \]

where \( y_{DF} = [y_{D,1} \ y_{D,2}] \), \( Q_{DF} = [\eta_{D,1} \ \eta_{D,2}] \), and \( H_{DF} \) is the channel matrix given by

\[
H_{DF} = \begin{bmatrix}
\sqrt{E_{SD}h_{SD}} & 0 \\
\sqrt{E_{RD}h_{RD}} & \sqrt{E_{SD}h_{SD}}
\end{bmatrix}
\]

**C. Outage Performance**

Here, we study the outage probabilities of both the AF and DF modes of cooperation. Let \( C_{AF} \), \( C_{DF} \), and \( R_{tar} \) be the channel capacity achieved by the AF mode, the channel capacity achieved by the DF mode, and a target transmission rate, respectively. An outage event occurs when the channel capacity is smaller than the target transmission rate, e.g., \( C_{AF} < R_{tar} \).

The channel capacity achieved by the AF mode is given by

\[ C_{AF} = \frac{1}{2} \log_2 \det (I_2 + \gamma H_{AF}^* H_{AF}) \]

\[ = \frac{1}{2} \log_2 \left( \left(1 + \gamma \alpha_{SD} \right) \left(1 + \frac{1}{w} \gamma \alpha_{SD} \right) + \frac{1}{w^2} \left( \frac{\gamma^2}{\gamma_{ESR} + 1} \right) \alpha_{SR} \alpha_{RD} \right) \]

where \( \alpha_{SR} = E_{SR} |h_{SR}|^2 \), \( \alpha_{SD} = E_{SD} |h_{SD}|^2 \), and \( \alpha_{RD} = E_{RD} |h_{RD}|^2 \). Notice that \( \alpha_{SR}, \alpha_{SD}, \) and \( \alpha_{RD} \) are exponential random variables. On the other hand, assuming that Node \( R \) can perfectly decode the symbols transmitted from Node \( S \), the channel capacity achieved by the DF mode, denoted by \( C_{DF} \), is given by

\[ C_{DF} = \frac{1}{2} \log_2 \det (I_2 + \gamma H_{DF}^* H_{DF}) \]

\[ = \frac{1}{2} \log_2 \left( \left(1 + \gamma \alpha_{SD} \right)^2 + \gamma \alpha_{RD} \right) \]

**Proposition 1:** In a three-node wireless network employing the cooperation protocol of interest, the AF mode of cooperation achieves the diversity order of two; however, the DF mode of cooperation can achieve the diversity order of two only if the decoding at the relay is perfect.

**D. Relay Selection**

In a large-scale wireless network, it is likely that a source node can exploit the notion of selection diversity. With a number of potential relays, the source node can choose the best node as its relay to assist its data transmissions.

**Corollary 1:** In a wireless network with \( m \) potential relays employing the cooperation protocol of interest with relay selection, both the AF cooperation mode and DF cooperation mode with perfect decoding achieves the full diversity order.

**E. Cooperation versus Non-Cooperation**

Consider a non-altruistic cooperative wireless network consisting of a number of nodes, including Node \( S \), Node \( R \), and Node \( D \). Suppose Node \( D \) is a receiving node at the time period of interest, and both Node \( S \) and Node \( R \) have their own data to transmit. Here, we adopt the DF cooperation mode to illustrate the idea of whether and when node cooperation is beneficial. According to the discussion given in Section II-C, the DF mode of cooperation should not be initiated unless the decoding at Node \( R \) is reliable. Therefore, the DF mode of cooperation is employed only if Node \( R \) can reliably decode the symbols sent from Node \( S \) and vice versa. Assuming perfect decoding at Node \( R \), we have

\[ y_{D,1} = \sqrt{a_S E_{SD} h_{SD} x_1} + \eta_{D,1} + I \]
\[ y_{R,1} = \sqrt{a_S E_{SR} h_{SR} x_1} + \eta_{R,1} + I \]
\[ y_{D,2} = \sqrt{a_S E_{SD} h_{SD} x_2 + \sqrt{a_S E_{SR} h_{SR} h_{RD}} x_1} + \eta_{D,2} + I \]

where \( a_X \) is the scaling factor for the transmit power of Node \( X \), i.e., \( 0 \leq a_X \leq 1 \) and \( I \) is the co-channel interference (e.g., generated by the transmissions from other clusters). With effective node clustering, the co-channel interference level can be strictly bounded [7], and we denote \( \sigma^2 \) as the aggregate
noise-plus-co-channel interference power. Thus, (13) and (15) can be represented in the following matrix form

$$y_{DF,p} = H_{DF,p}x + Q_{DF,p}$$

where $y_{DF,p} = [y_{D,1}, y_{D,2}]$, $Q_{DF,p} = [I_{D,1} + I_{D,2} + \eta_{D,2} + \eta_{D,2}]$, and $H_{DF,p}$ is the channel matrix given by

$$H_{DF,p} = \begin{bmatrix} \sqrt{\eta_{SD}E_{SD}}h_{SD} & 0 \\ \sqrt{\eta_{RD}E_{RD}}h_{RD} & \sqrt{\eta_{SD}E_{SD}}h_{SD} \end{bmatrix}. \quad (17)$$

Denote $C^c$ as the channel capacity achieved by means of node cooperation from Node $S$ to Node $D$ (with the help of Node $R$), which is given by

$$C^c = \frac{1}{2} \log_2 \det \left( I_2 + \frac{1}{\sigma^2} H_{DF,p}^* H_{DF,p} \right) = \frac{1}{2} \log_2 \left( \left( 1 + a_S \gamma_{SD} \right)^2 + a_R \gamma_{RD} \right). \quad (18)$$

where $\gamma_{SD} = E_{SD}[h_{SD}]^2/\sigma^2$ and $\gamma_{RD} = E_{RD}[h_{RD}]^2/\sigma^2$.

**Corollary 2:** Given perfect decoding at Node $R$, arbitrarily positive power allocation has no impact on the diversity performance in the DF mode of cooperation.

**Corollary 3:** Given perfect decoding at Node $R$, arbitrarily positive power allocation has no impact on the diversity performance in the DF mode of cooperation with relay selection.

Denote $C^{nc}$ as the channel capacity achieved from Node $S$ to Node $D$ without node cooperation (i.e., ordinary direct transmissions), which is given by

$$C^{nc} = \log_2 (1 + \gamma_{SD}). \quad (19)$$

In the case of non-altruistic node cooperation, power allocation is imperative as part of the transmit power of a node is dedicated to transmitting its own data (i.e., direct transmissions) and the rest of the transmit power is dedicated to relaying the data from other nodes (i.e., assisted transmissions). The variables $a_S$ and $a_R$ in (18) capture the power allocation. Here, we derive a sufficient condition for a cooperative transmission being advantageous over an ordinary direct transmission, which is given by

$$C^c \geq C^{nc} \Rightarrow (1 + a_S \gamma_{SD})^2 + a_R \gamma_{RD} \geq (1 + \gamma_{SD})^2 \Rightarrow a_S \geq \frac{-1 + \sqrt{1 + \gamma_{SD}}^2 - a_R \gamma_{RD}}{\gamma_{SD}}. \quad (20)$$

Notice that in the case where relay selection is considered, $R = \arg \max_{R} \{a_R \gamma_{RD} \}$. Consider $\gamma_{SD} = \gamma_{RD} = \gamma$ and $a_R = \rho a_S$, where $\rho \geq 0$. The sufficient condition for a cooperative transmission being advantageous over an ordinary direct transmission is given by

$$a_S \geq \frac{-2 + \rho + \sqrt{(2 + \rho)^2 + 4 \gamma (\gamma + 2)}}{2 \gamma}. \quad (21)$$

First, we consider the boundary case where $a_R = 0$. Since $a_S$ has to be positive (i.e., $a_S > 0$) for a feasible cooperative transmission, $\rho = 0$. When $\rho = 0$, from (21), $a_S \geq 1$; however, since $0 \leq a_S \leq 1$, $a_S = 1$. Thus, for the case of $a_S = 1$ and $a_R > 0$, the sufficient condition is (strictly) satisfied. In fact, this corresponds to the scenario of altruistic node cooperation (e.g., with pure relays in IEEE 802.16j). On the other hand, given the value of $\rho$, the value of $a_S$ is lower-bounded by (21) in order for node cooperative transmissions to be beneficial. This scenario of interest corresponds to non-altruistic node cooperation in wireless networks where every node has its own data to transmit and only a portion of its transmit power can be dedicated to relaying data from other neighboring nodes. In fact, the condition (20) refers to the cooperative transmissions being beneficial to an individual node only. For the sake of overall system performance, cooperation among nodes should be considered in a holistic manner, to be discussed in Section III.

### III. Packet-Level Node Cooperation

Concerning packet-level node cooperation, after overhearing the packet transmission from Node $S$, Node $R$ can verify the medium access control (MAC)-layer error correction code(s) of the received packet (e.g., forward error correction (FEC) and cyclic redundancy check (CRC)) to determine whether that packet is corrupted or not. If a packet is received successfully, Node $R$ can become one of Node $S$’s potential relays, and cooperative transmissions can be carried out hereafter whenever feasible and favorable.

Here, we investigate the performance gain due to beneficial packet-level node cooperations in a synchronized wireless network consisting of a number of nodes (or links). We assume that perfect CSI is available. Since the nodes that cannot reliably decode other neighboring nodes transmissions will not be considered as relays, with perfect CSI, the nodes in a system can have complete knowledge on which other nodes can be their potential relays. As mentioned in Section II-E, since non-altruistic cooperation is not always advantageous (to be further validated in Section IV-A), non-altruistic node cooperation can only be triggered as long as it is feasible and beneficial for system performance mitigation. Let $\bar{R}_m$ and $R^*_m$ be the achievable data rate obtained of the $m^{th}$ link without node cooperation and that with node cooperation, respectively. Suppose the $u^{th}$ link’s transmitter is a potential relay for the $m^{th}$ link’s transmitter. Considering that the $m^{th}$ link does not receive any assistance from any other nodes and the $u^{th}$ link’s transmitter does not offer any assistance to any other links, we have

$$\bar{R}_m = \log_2 \left( 1 + a_{mm} \gamma_{mm} \right) \quad (22)$$

$$R^*_m = \frac{1}{2} \log_2 \left( 1 + a_{mm} \gamma_{mm} \right)^2 + a_{mu} \gamma_{mu} \right) > \bar{R}_m \quad (23)$$

$$\bar{R}_u = \log_2 \left( 1 + a_{uu} \gamma_{uu} \right) \quad (24)$$

$$R^*_u = \frac{1}{2} \log_2 \left( 1 + a_{uu} \gamma_{uu} \right)^2 + a_{uj} \gamma_{uj} \right) < \bar{R}_u \quad (25)$$

where $\gamma_{mm}$ is the received signal-to-noise-plus-interference ratio (SNIR) at the $m^{th}$ link’s receiver from the $u^{th}$ link’s transmitter and $a_{mu}$ is the normalized portion of the total transmit power of the $u^{th}$ link’s transmitter allocated for the $m^{th}$ link’s transmission. Notice that, for link $u$, $a_{mu} + a_{uu} = 1, \forall m$. In the case of non-altruistic node cooperation, $a_{mu} = \rho a_{uu}$, whereas in the case of node non-cooperation, $a_{uu} = 1$ and
$a_{m,u} = 0, \forall m \neq u$. Equation (22) refers to the achievable rate of the $m^{th}$ link obtained by an ordinary direct transmission. Note that the $m^{th}$ link’s transmitter can be offering assistance to some other link in the system. Equation (23) refers to the achievable rate of the $m^{th}$ link obtained by a node cooperative transmission with the help of the $u^{th}$ link’s transmitter. Equation (24) refers to the achievable rate of the $u^{th}$ link obtained before offering assistance to the $m^{th}$ link. Note that the $u^{th}$ link can be receiving assistance from some other node in the system. Equation (25) refers to the achievable rate of the $u^{th}$ link obtained after offering assistance to the $m^{th}$ link. As such, a node cooperative transmission is considered beneficial and can be carried out if the following condition is valid:

$$R_m + R_u > \tilde{R}_m + \tilde{R}_u.$$  

(26)

If condition (26) is not valid, the node cooperative transmission is not beneficial, and the direct transmission is used instead. In the presence of multiple potential relays, relay selection can be incorporated in (22)-(25), where the best relay is selected for cooperative transmissions. For the sake of simplicity, we only consider a two-node cooperation model in this work; addressing the issue of multi-node cooperation, however, is left for further work.

IV. Performance Evaluation

A. Outage Performance

In simulations, we compare the outage probabilities of non-altruistic cooperative transmissions and ordinary direct transmissions. Regarding non-altruistic node cooperation with regenerative wireless nodes, the nodes that can successfully decode some neighboring node’s transmissions become its potential relays. Here, if the rate achieved by a source-relay link is larger than that by a cooperative transmission given in (18), we assume that the relay of interest can reliably decode the source node’s data and become one of its potential relays [1]. Figs. 1 and 2 depict the outage probabilities of cooperative transmissions using the DF cooperation mode and ordinary direct transmissions in a high SNR regime and a low SNR regime, respectively. As observed in Fig. 1, without considering perfect decoding, no diversity gain can be obtained even in the presence of multiple relays (i.e., Proposition 1). On the other hand, given perfect decoding, node cooperative transmissions with one relay achieve the diversity order of two (i.e., Corollary 2), while the ones with three relays achieve the diversity order of four (i.e., Corollary 3). It is also worth mentioning that, unlike altruistic cooperation, ordinary direct transmissions can perform better than non-altruistic cooperative transmissions in a low SNR regime (see Fig. 2). This phenomenon is ascribed to the split of transmit power and/or severe error propagation in non-altruistic node cooperation, as discussed in Section II-E. The results confirm that, in the context of non-altruistic node cooperation, cooperative transmissions are not always superior to ordinary direct transmissions, especially in a low SNR regime.

B. Throughput Performance

Here, we evaluate performance gains due to beneficial node cooperations on the packet level. We consider a synchronized wireless network consisting of $M$ wireless nodes randomly located in a 1km x 1km coverage area. We adopt the channel model employed in [8]. Time is partitioned into DATA slots. The duration of each DATA slot is 5ms. The maximum power constraint of a node is 1mW, and the noise-plus-interference power is assumed to follow a Gaussian distribution with zero mean and standard deviation $10^{-3}$W. In the simulations, we consider a saturated traffic model, where each node always has packets to transmit. Since the duration of a DATA slot is fixed, the size of a packet can be varied, depending upon the transmission rate. In order to merely focus on the merits of beneficial non-altruistic node cooperation, we employ a simple round-robin MAC protocol, where only one node (without
node cooperation) or two cooperating nodes can transmit at one time and, therefore, no packet collisions occur. We further consider that packet transmissions are from a node to a destination located at the center of the coverage area. Simulations are performed for 10,000 runs and the results are averaged, where each simulation run sustains 5,000 DATA slots.

Define node cooperation gain (NCG) as the normalized throughput gain due to node cooperation and use it as a performance measure. For $\rho = 1/3$, the NCG versus the number of nodes is given in Table I. As expected, the more the nodes, the more the potential relays and hence the cooperation opportunities. With increased cooperation opportunities, a source node is likely to attain a higher throughput by means of beneficial node cooperation. Thus, as $M$ increases, a higher NCG can be obtained. However, the rate of increment in the NCG decreases as $M$ increases. In particular, the NCG is leveled off at about 16% from $M = 36$ onward. This phenomenon stems from the fact that the additional performance benefits due to node cooperation are saturated when $M$ increases. Similar to the case of multi-user diversity [6], since the channel variations among relays become less significant at a large $M$, the NCG becomes flattened. Nonetheless, with beneficial non-altruistic node cooperation, we can achieve a significant performance gain over ordinary direct transmissions. Further work includes performance analysis for multi-node cooperation and study of cross-layer node cooperative resource allocation.

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### APPENDIX

#### A. Proof of Proposition 1

Denote $P_{\text{out}}(R_{\text{tar}})$ as the outage probability of the AF cooperation mode at a target rate $R_{\text{tar}}$, which is given by

$$P_{\text{out}}(R_{\text{tar}}) = P(C_{\text{AF}} < R_{\text{tar}}) \leq P\left(\frac{1}{2} \log_2 \left(1 + \varepsilon_{\min} (\alpha_{SD} + \alpha_{SR} \alpha_{RD})\right) < R_{\text{tar}}\right)$$

$$= P\left(\alpha_{SD} + \alpha_{SR} \alpha_{RD} < \frac{2^{R_{\text{tar}} - 1}}{\varepsilon_{\min}}\right)$$

(A-1)

where $\varepsilon_{\min} = \min \left\{ \left(1 + \frac{1}{\lambda_{SD}}\right) \gamma \frac{1}{\lambda_{SR} \left(\frac{2^{R_{\text{tar}} - 1}}{\varepsilon_{\min}}\right)} \right\}$. Let $\delta = \frac{2^{R_{\text{tar}} - 1}}{\varepsilon_{\min}}$. Let $\lambda_{SR}$, $\lambda_{SD}$, and $\lambda_{RD}$ be the parameters of the exponential random variables $\alpha_{SR}$, $\alpha_{SD}$, and $\alpha_{RD}$, respectively. Equation (A-1) becomes

$$\int_{0}^{\delta} P\left(\alpha_{SR} \alpha_{RD} < \delta - x\right) \lambda_{SD} e^{-\lambda_{SD} x} dx$$

$$\leq \int_{0}^{\delta} P\left(\frac{\alpha_{SR} \alpha_{RD}}{\alpha_{SR} + \alpha_{RD} + 1} < \delta - x\right) \lambda_{SD} e^{-\lambda_{SD} x} dx$$

$$= \delta^2 \int_{0}^{1} P\left(\frac{\alpha_{SR} \alpha_{RD}}{\alpha_{SR} + \alpha_{RD} + 1} < \delta x'\right) x' \lambda_{SD} e^{-\lambda_{SD} (1-x')} dx'$$

(A-2)

where $x' = 1 - x/\delta$. Let $h(\delta) = \delta x'$. As $\gamma \to \infty$, $\delta \to 0$ and $h(\delta) \to 0$. Applying the results obtained in [1], we have

$$\lim_{\gamma \to \infty} \frac{1}{h(\delta)} P\left(\frac{\alpha_{SR} \alpha_{RD}}{\alpha_{SR} + \alpha_{RD} + 1} < h(\delta)\right) = \lambda_{SR} + \lambda_{RD}$$

Therefore, in a high SNR regime, (A-2) can be re-written as

$$\delta^2 \int_{0}^{1} (\lambda_{SR} + \lambda_{RD}) x' \lambda_{SD} dx' = \frac{\lambda_{SD} (\lambda_{SR} + \lambda_{RD})}{2} \delta^2$$

Then, the outage probability of the AF cooperation mode at a target rate $R_{\text{tar}}$ is upper-bounded according to

$$P_{\text{out}}(R_{\text{tar}}) \leq \frac{\lambda_{SD} (\lambda_{SR} + \lambda_{RD})}{2} \delta^2$$

(A-3)

From (A-3), it is clear that the diversity order of two can be attained in the AF cooperation mode [6].

For the case of the DF cooperation mode, denote $P_{\text{out}}(R_{\text{tar}})$ as the outage probability at a target rate $R_{\text{tar}}$, given by

$$P_{\text{out}}(R_{\text{tar}}) = P(C_{\text{DF}} < R_{\text{tar}})$$

### REFERENCES


where [1] 
\[ C_{DF} = \min \left\{ \frac{1}{2} \log_2 \left( 1 + \gamma \alpha_{SR}, C_{\text{DF}} \right) \right\}. \tag{A-4} \]

In (A-4), the first term refers to the maximum transmission rate at which Node \( R \) can decode the symbols sent from Node \( S \) successfully, whereas the second term is given by (12). Thus, the outage probability of the DF mode is given by

\[
P_{\text{DF}}^{\text{out}}(R_{\text{tar}}) = P(C_{DF} < R_{\text{tar}})
= P\left( \min \{ \alpha_{SR}, 2\alpha_{SD} + \alpha_{RD} + \gamma \alpha_{SD}^2 \} < \frac{2^{2R_{\text{tar}} - 1}}{\gamma} \right)
\leq P\left( \min \{ \alpha_{SR}, 2\alpha_{SD} + \alpha_{RD} \} < \frac{2^{2R_{\text{tar}} - 1}}{\gamma} \right). \tag{A-5} \]

Let \( \delta = \frac{2^{2R_{\text{tar}} - 1}}{\gamma} \), we have

\[
P_{\text{DF}}^{\text{out}}(R_{\text{tar}}) \leq P(\alpha_{SR} < \delta) + P(\alpha_{SR} \geq \delta) P(2\alpha_{SD} + \alpha_{RD} < \delta). \]

As \( \gamma \to \infty \), \( \delta \to 0 \). Applying the results obtained in [1], we have

\[
\lim_{\gamma \to \infty} \frac{1}{\gamma} P(\alpha_{SR} < \delta) = \lambda_{SR}
\lim_{\gamma \to \infty} \frac{1}{\gamma} P(\alpha_{SR} \geq \delta) = 1
\lim_{\gamma \to \infty} \frac{1}{\gamma} P(2\alpha_{SD} + \alpha_{RD} < \delta) = \frac{\lambda_{SD}\lambda_{RD}}{4}. \tag{A-6} \]

Thus, the outage probability of the DF cooperation mode at a target rate \( R_{\text{tar}} \) is upper-bounded according to

\[
P_{\text{DF}}^{\text{out}}(R_{\text{tar}}) \leq \lambda_{SR}\delta + \frac{\lambda_{SD}\lambda_{RD}}{4}\delta^2. \tag{A-7} \]

From (A-7), the DF cooperation mode provides no diversity gain even in a high SNR regime. The rationale is that the diversity benefit vanishes due to the decoding capability of Node \( R \). However, given that the decoding is perfect at Node \( R \), the diversity order of two can be achieved in the DF cooperation mode. Denote \( P_{\text{DF}}^{\text{out}}(R_{\text{tar}}) \) as the outage probability of the DF cooperation mode with perfect decoding at a target rate \( R_{\text{tar}} \). In a high SNR regime, we have

\[
P_{\text{DF}}^{\text{out}}(R_{\text{tar}}) = P(C_{DF} < R_{\text{tar}}) \leq \frac{\lambda_{SD}\lambda_{RD}}{4}\delta^2. \]

\[ \]

**B. Proof of Corollary 1**

Following the similar steps shown in the proof of Proposition 1, we consider the outage probability of the AF cooperation mode with relay selection at a target rate \( R_{\text{tar}} \), denoted by \( P_{\text{AF,sel}}^{\text{out}}(R_{\text{tar}}) \). Denote \( E_{\text{XR}} \), as the average energy of a symbol received at Node \( R \), from Node \( X \), where \( i \in \{1, 2, \ldots, m\} \), \( \alpha_{SR} = E_{\text{SR}} |h_{\text{SR}}|^2 \), and \( \alpha_{RD} = E_{\text{RD}} |h_{\text{RD}}|^2 \). We have

\[
P_{\text{AF,sel}}^{\text{out}}(R_{\text{tar}}) = P(1 + \gamma \alpha_{SD}) \left\{ \frac{1}{\gamma \alpha_{SR} \alpha_{RD}} \right\} \leq P(\alpha_{SD} + \max_{i} \{ \alpha_{SR}, \alpha_{RD} \} < 2^{2R_{\text{tar}} - 1} \epsilon_{\text{min}}^{-1}) \tag{B-1} \]

\[ \]

where \( \epsilon_{\text{min}} = \min \left\{ \left( 1 + \frac{1}{\epsilon_{\text{min}}} \right) \gamma, \min \left\{ \frac{1}{\epsilon_{\text{min}}} \left( \frac{\gamma^2}{\gamma E_{\text{SR}} + 1} \right) \alpha_{SR}, \alpha_{RD} \right\} \right\}. \]

Let \( \lambda_{SR} \) and \( \lambda_{RD} \) be the parameters of the exponential random variables \( \alpha_{SR} \) and \( \alpha_{RD} \), respectively. Inequality (B-1) becomes

\[
P_{\text{AF,sel}}^{\text{out}}(R_{\text{tar}}) \leq \int_{0}^{\delta} P(\alpha_{SR} \max_{i} \{ \alpha_{RD} \} < \delta - x) \lambda_{SD} e^{-\lambda_{SD} x} dx
= \int_{0}^{\delta} \prod_{i=1}^{m} P(\alpha_{SR} \alpha_{RD} < \delta - x) \lambda_{SD} e^{-\lambda_{SD} x} dx. \]

By Proposition 1, in a high SNR regime, we have

\[
P_{\text{AF,sel}}^{\text{out}}(R_{\text{tar}}) \leq \frac{\lambda_{SD} \prod_{i=1}^{m} (\lambda_{SR} + \lambda_{RD})}{m+1}\delta^{m+1}. \]

Therefore, the full diversity order can be obtained with relay selection in the AF cooperation mode.

The outage probability of the DF cooperation mode with perfect decoding and relay selection at a target rate \( R_{\text{tar}} \), denoted by \( P_{\text{DF,sel}}^{\text{out}}(R_{\text{tar}}) \), is given by

\[
P_{\text{DF,sel}}^{\text{out}}(R_{\text{tar}}) = P\left( 2\gamma \alpha_{SD} + \gamma \alpha_{SD}^2 + \max_{i} \{ \alpha_{RD} \} < 2^{2R_{\text{tar}} - 1} \right)
\leq P\left( \alpha_{SD} + \max_{i} \{ \alpha_{RD} \} < \delta \right)
= \int_{0}^{\delta} \prod_{i=1}^{m} P(\alpha_{RD} < \delta - x) \lambda_{SD} e^{-\lambda_{SD} x} dx
= \int_{0}^{\delta} \prod_{i=1}^{m} \lambda_{RD} \delta^{m+1}. \]

where \( \delta = \frac{2^{2R_{\text{tar}} - 1}}{\gamma} \). Thus, by Proposition 1, in a high SNR regime, we have

\[
P_{\text{DF,sel}}^{\text{out}}(R_{\text{tar}}) \leq \frac{\lambda_{SD} \prod_{i=1}^{m} \lambda_{RD}}{m+1}\delta^{m+1}. \]

With relay selection, the DF cooperation mode with perfect decoding achieves the full diversity order.

**C. Proof of Corollary 2**

Similar to the proof of Proposition 1, we consider the outage probability of the DF cooperation mode with arbitrarily positive power allocation, denoted by \( P_{\text{DF,c}}^{\text{out}}(R_{\text{tar}}) \), where \( \alpha_{SR}, \alpha_{RD} > 0 \). We have

\[
P_{\text{c}}^{\text{out}}(R_{\text{tar}}) = P(C_{\text{c}} < R_{\text{tar}})
= P\left( 2a_{\text{SD}} \gamma_{SD} + a_{\text{SR}} \gamma_{SD} + a_{\text{RD}} \gamma_{RD} < 2^{2R_{\text{tar}} - 1} \right)
\leq P\left( 2a_{\text{SD}} \gamma_{SD} + a_{\text{RD}} \gamma_{RD} < 2^{2R_{\text{tar}} - 1} \right)
\leq P(2a_{\text{SD}} + a_{\text{RD}} < \delta)
where \( a_{\text{SD}} = E_{\text{SD}} |h_{\text{SD}}|^2, a_{\text{RD}} = E_{\text{RD}} |h_{\text{RD}}|^2, \delta = \frac{2^{2R_{\text{tar}} - 1}}{\gamma} \), and \( \epsilon_{\text{min}} = \min\{ \frac{a_{\text{SR}}}{\sigma_{\text{SR}}^2}, \frac{a_{\text{RD}}}{\sigma_{\text{RD}}^2} \} \). As \( \sigma^2 \to 0 \), \( \delta \to 0 \). Therefore, by (A-6), we have

\[
\lim_{\delta \to 0} \frac{1}{\delta} P_{\text{c}}^{\text{out}}(R_{\text{tar}}) \leq \frac{\lambda_{SD}\lambda_{RD}}{4}. \tag{B-1} \]

**D. Proof of Corollary 3**

It can be proved by following the same arguments given in the proof of Corollaries 1 and 2.