Plug-in Electric Vehicle Charging Demand Estimation based on Queueing Network Analysis

Hao Liang, Student Member, IEEE, Isha Sharma, Student Member, IEEE, Weihua Zhuang, Fellow, IEEE, and Kankar Bhattacharya, Senior Member, IEEE

Abstract—Charging stations are critical infrastructure for the integration of plug-in electric vehicles (PEVs) in the future distribution systems. With a steadily increasing PEV penetration level, the PEV charging demands of charging stations are expected to constitute a significant portion of the total electric power demands. An accurate estimation of PEV charging demands is crucial for the planning and operation of future distribution systems. However, the estimation remains a challenging issue, as the charging demands of nearby charging stations are closely correlated to each other and depend on vehicle drivers’ response to charging prices. The evaluation of charging demands is further complicated by the highly dynamic vehicle mobility, which results in random PEV arrivals and departures. In order to address these challenges, a BCMP queueing network model is presented in this paper, in which each charging station is modeled as a service center with multiple servers (chargers) and PEVs are modeled as the customers in the service centers. Based on the stationary distribution of the number of PEVs in each charging station, the statistics of PEV charging demands can be obtained. The analytical model is validated by a case study based on realistic vehicle statistics extracted from 2009 National Household Travel Survey and New York State Transportation Federation Traffic Data Viewer.

Index Terms—Charging station, plug-in electric vehicle, queueing network.

I. INTRODUCTION

There is a clear trend of transportation electrification around the globe. For instance, the Government of Canada is committed to reducing Canada’s total greenhouse gas (GHG) emissions by 17% between 2005 and 2020, and the transportation sector is the single largest source of GHG emissions, accounting for 23% of Canada’s total GHG emissions [1]. Given the fact that approximately 75% of Canada’s electricity is generated without the use of fossil fuels, electrifying the transportation system is a promising solution to reduce Canada’s GHG emissions. It is estimated by Natural Resources Canada that there will be at least 500,000 highway-capable plug-in electric vehicles (PEVs) on Canadian roads by 2018, as well as a possibly larger number of hybrid-electric vehicles [2].

A critical infrastructure for transportation electrification is the charging stations, which can be deployed at not only residential/private parking lots, but also public parking lots of work places and shopping centers or even street parking decks [3]. To shorten the charging duration of plug-in electric vehicles (PEVs) at public charging stations, DC fast charge (DCFC) technology with off-board chargers is typically used. The DCFC can provide about 50% recharge of a PEV in 10 to 15 minutes with a charging power of at least 30 kW [4].

Due to the fast-growing PEV penetration level and high charging power of the DCFC, the PEV charging demands of charging stations are expected to constitute a significant portion of the total electric power demands in the future smart grid. However, the highly dynamic PEV mobility poses significant technical challenges on charging demand estimation for charging stations due to the random arrivals and departures of PEVs. Moreover, PEV drivers can select different charging stations in response to charging prices [5]. As a result, the interactions among multiple charging stations should be investigated in charging demand estimation.

A few recent research works use queueing analysis to characterize the charging demand of a single charging station, taking into account the randomness in PEV arrivals and departures [6]–[8]. When multiple charging stations coexist in a distribution system, a network model of charging stations needs to be developed [9]. With a centralized network coordinator to route PEVs to different charging stations, the charging stations can be modeled as independent single queues [9]. However, since PEV drivers may deviate from the assignments by the centralized network coordinator because of their own preferences, how to use charging price as an incentive for PEV routing needs further study.

In this paper, a BCMP queueing network model is presented to characterize relations among the charging demands of multiple charging stations, taking into account the factors such as single PEV charging demand, charging decision making processes of PEV drivers, PEV traffic flow, and road system. The PEV charging demands are calculated based on the stationary distribution of the number of PEVs in each charging station. A case study based on 2009 National Household Travel Survey (NHTS) and New York State Transportation Federation Traffic Data Viewer (TDV) is presented to validate the proposed analytical model. The analytical results can provide a useful reference for distribution system planning with PEV integration. More importantly, the analytical model can be incorporated in various distribution system operation schemes [10], such that certain operation objectives (e.g., minimizing feeder losses and minimizing PEV charging costs) can be achieved.

The authors are with the Department of Electrical and Computer Engineering, University of Waterloo, 200 University Avenue West, Waterloo, Ontario, Canada N2L 3G1 (e-mail: {hliang, i4sharma, wzhuang, kankar}@uwaterloo.ca).

This work was supported by research grants from the Natural Sciences and Engineering Research Council (NSERC) of Canada.
II. SYSTEM MODEL

In this paper, the integration of PEV charging stations in a distribution system is studied [6], [10]. Let $S$ denote the total number of charging stations served by the distribution system. The charging stations are interconnected by a road system such that the PEVs may travel from one charging station to another before the drivers make a charging decision. After the charging decision is made by a PEV driver, the PEV battery is charged at the corresponding charging station. Without loss of generality, a specific period (e.g., one hour [10]) of a day is considered, within which the PEV traffic statistics and electricity price do not change significantly. In the following subsections, detailed models are presented for charging stations, charging demand of a single PEV, charging decision making, PEV traffic flow, and road system.

A. Charging Station Model

Each charging station $s (s \in \{1, 2, \cdots, S\})$ is characterized by a limited number ($c_s$) of chargers and sufficient waiting spaces [6], [7]. Denote the charging power of each charger in charging station $s$ as $P_c^s$, which depends on the charger manufacturer and is specified at the charging station planning stage [4]. The charging price of charging station $s$ is denoted by $r_s$. The PEV charging demand ($P_{chg}^s$) of charging station $s$ equals the aggregated charging demand of all PEVs in the charging station. Due to the random arrivals and departures of PEVs in a period, $P_{chg}^s$ is a random variable.

B. Single PEV Charging Demand

Consider the single PEV charging demand model developed in [11], which is based on the authoritative source of travel patterns of light-duty vehicles in the United States, i.e., the 2009 NHTS database [12]. The daily energy consumption ($\epsilon$) of a PEV in an urban (or rural) area on a weekday (or weekend) depends on the overall wall-to-wheels efficiency\(^1\), and two random variables in terms of the vehicle miles traveled (VMT) and charge-depleting range (CDR). Specifically, the light-duty vehicle (LDV) fleet (including cars, vans, sport utility vehicles, and pickup trucks with gross vehicle weight less than 8500 pounds) is considered, which is the main source of energy consumption in transportation sector. Based on the statistics obtained from the 2009 NHTS, $\epsilon$ can be modeled as a random variable with lognormal distribution. However, since the lognormal distribution is not analytically tractable in our subsequent queueing analysis, the exponential distribution is used in this work to approximately model the probabilistic behavior of $\epsilon$ [6]–[9], with mean value $\bar{\epsilon}$ equal to that of the lognormal distribution. At 40-mile average CDR, the values of $\bar{\epsilon}$ are given by 4.16 kWh and 4.88 kWh, respectively, for weekdays in urban and rural areas [11].

Consider a daily charging scenario where each PEV is charged once a day. The main reason is that most daily travel requirements of PEVs in LDV fleet can be satisfied by their battery capacities [11]. Given the PEV daily energy consumption $\epsilon$ (which equals the energy demand of each PEV at a charging station) and the charging power $P_c^s$ of the chargers in charging station $s$, the charging time ($\epsilon/P_c^s$) of each PEV in charging station $s$ is exponentially distributed with an average value of $\bar{\epsilon}/P_c^s$. Let $\mu_s$ denote the service rate (i.e., the reciprocal of average charging time) of each PEV in charging station $s$, given by $\mu_s = P_c^s/\bar{\epsilon}$.

C. Charging Decision Making

The charging decision is made by each PEV driver spontaneously based on the charging prices. The selection probability $\beta(r_s)$ of a charging station by a PEV driver can be modeled by a decreasing and differentiable price sensitivity function with respect to the charging price ($r_s$) [5], given by

$$\beta(r_s) = \max \left\{ 1 - \left( \frac{r_s - r_{\min}}{r_{\max} - r_{\min}} \right)^2, 0 \right\}, r_s > r_{\min} \quad (1)$$

where $r_{\min}$ represents the minimum charging price which equals the real-time electricity price (i.e., the price for the charging station operator to purchase electricity). In (1), $r_{\max} > r_{\min}$ is the maximum acceptable charging price by a PEV driver. The typical value of $r_{\max}$ is 2-3 times the value of $r_{\min}$ [5]. Moreover, $r_s > r_{\min}$ ensures that the charging station operator can make a profit.

Due to the limited size of a distribution system, the charging stations are typically separated by a few miles such that the energy consumption for a PEV to travel among different charging stations is negligible [9]. As a result, the charging decision making processes are mainly determined by electricity prices. A possible extension to investigate the traveling energy consumption is to incorporate the solutions of a traditional tour gas station problem [13], which is left for our future research.

Upon arriving at a charging station, a driver decides whether or not to charge his/her PEV based on the charging price offered by the charging station. With a negligible decision delay, a PEV is charged (or served) immediately if the charging price is satisfactory to the driver. Otherwise, the PEV routes towards the next charging station on its trajectory without delay. The PEV leaves the distribution system if there is no more charging station on its trajectory within the distribution system.

D. PEV Traffic Flow and Road System

The arrival of PEVs at a specific charging station follows a Poisson process [6]. This observation conforms with the existing vehicle mobility models verified by experiments [14]. Denote by $\lambda_s$ the arrival rate of PEVs at charging station $s$. Note that only newly arrival PEVs are considered in the calculation of $\lambda_s$. In other words, charging station $s$ is the first charging station in the distribution system to be visited by the PEVs with respect to $\lambda_s$. As a result, the inter-arrival time of PEVs at a charging station is exponentially distributed.

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\(^1\)The wall-to-wheels efficiency is a product of the efficiencies of charger, battery (round-trip), power electronics, traction motor, and mechanical transmission plus accessory loads.
with mean $1/\lambda_s$. The value of $\lambda_s$ can be obtained based on traditional vehicle traffic monitoring techniques [15] and assuming a certain penetration level of PEVs. The distance from charging station $s$ to charging station $s'$ is denoted by $d_{s,s'}$. The speed of PEVs traveling from charging station $s$ to charging station $s'$ is modeled by a random variable with arbitrary probability distribution and an average value of $v_{s,s'}$. For specific road system (e.g., one-way road), it is possible that $d_{s,s'} ≠ d_{s',s}$ and $v_{s,s'} ≠ v_{s',s}$.

When a PEV travels in the area under consideration, it may encounter multiple charging stations on its trajectory. The driver can select one (or none) of the encountered charging stations for PEV charging based on the charging prices. Let $\alpha_{s'|s}$ be the probability for a PEV to visit charging station $s'$ after visiting charging station $s$ ($s ≠ s'$). The probability depends on the PEV mobility and can be estimated based on vehicle traffic monitoring techniques [15]. A similar model is used in vehicular communication network research [16]. It is possible that $\sum_{s'=1,2,\ldots,s\neq s} \alpha_{s'|s} < 1$ since a PEV is considered to leave the distribution system if there is no more charging station on its mobility trajectory.

III. QUEUING NETWORK ANALYSIS

Queueing network analysis is performed to calculate the PEV charging demand at each charging station. Different from traditional single queue analysis, the queueing network analysis can capture the interactions among multiple charging stations as a result of PEV drivers’ responses to charging prices. By definition, a queueing network model is concerned with a collection of interconnected single service center queueing systems which provide services to a set of customers [17]. A queueing network can be defined with respect to a group of service centers, customers, and network topology. Each service center is characterized by its service time, buffer space, queueing scheduling discipline, and number of servers. In an open queueing network (with exogenous arrivals), customers are described by the service demand and arrival process to each service center. Network topology models the interconnections of service centers and how customers move among them.

In order to model the charging stations in the distribution system, three kinds of service centers are defined:

- **Charging service center** to model each individual charging station;
- **Routing service center** to model the road system from one charging station to another before charging decisions are made by the drivers;
- **Decision service center**, defined for each charging station, to model the charging decision making processes of PEV drivers. It is a “virtual” service center for analysis without any actual system element, since a charging decision is made by a driver at the instance when he/she arrives at a charging station (i.e., without delay).

The customers in the queueing network correspond to the PEVs in the distribution system. The network topology of the queueing network is determined by the PEV traffic flow, road system, and charging decision making processes by PEV drivers. An illustration of the queueing network model for two charging stations is shown in Fig. 1. In the following subsections, a detailed queueing network analysis is presented to evaluate the PEV charging demands of charging stations.

A. Queueing Network Model

Denote the decision service center with respect to charging station $s$ as $DS_s$, as shown in Fig. 1. The exogenous arrival rate of PEVs to $DS_s$ equals $\lambda_s$, which corresponds to the newly arrive PEVs to the distribution system. Since a decision service center is used to model the charging decision making process by PEV drivers, the time that each PEV spent in a decision service center should be negligible in accordance with the negligible decision making delay. Therefore, both the number of servers and the service rate of each server in $DS_s$ are set to infinite. After the service of a decision service center, each PEV can make a decision on choosing one of the following two options:

1) Get charged at the corresponding charging station;
2) Go to the next charging station or leave the distribution system according to its mobility trajectory.

Upon the choice of option 1), the PEV enters the charging service center. Let $CS_s$ denote the charging service center corresponding to charging station $s$, as shown in Fig. 1, based on a first-come-first-serve (FCFS) discipline. The service rate of the charging service center depends on the number of chargers and the number of PEVs. Let $x_s(n)$ denote the service rate of $CS_s$ when there are $n$ PEVs in it (either charging or waiting), relative to the service rate when $n = 1$, given by

$$x_s(n) = \begin{cases} n_s, & \text{if } 0 \leq n < c_s \\ c_s, & \text{otherwise.} \end{cases}$$  \hspace{1cm} (2)

The probability for a PEV to enter $CS_s$ and get charged after leaving $DS_s$ is a function of charging price, denoted by $p_s,s(r_s) = \beta(r_s)$. 

![Fig. 1: An illustration of the queueing network model for two charging stations.](image-url)
On the other hand, upon the choice of option 2), a PEV routes to the next charging station \( s' \) (\( s' \in \{1, 2, \ldots, S\} \), \( s' \neq s \)) on its trajectory with a probability given by \( p_{s,s'}(r_s) = (1 - \beta(r_s))(1 - \sum_{s' \neq s} \alpha_{s'|s}) \), where \( \alpha_{s'|s} \) depends on the PEV mobility. The PEV may also leave the area (without being charged) with a probability given by \( p_{s,L}(r_s) = (1 - \beta(r_s))(1 - \sum_{s' \neq s} \alpha_{s'|s}) \). To model the routing process of PEVs moving from one charging station to another, let \( RS_{ss'} \) denote the routing service center from charging station \( s \) to charging station \( s' \). The model of each routing service center can be established based on an existing model of road system [16]. The routing service center between two charging stations is modeled as an infinite server (IS) service center. The number of servers is considered to be infinite for a road system without congestions. The model can be extended to a congested road system by investigating maneuver behaviors of vehicle drivers [16]. The average service rate (\( \mu_{ss'} \)) of \( RS_{ss'} \) depends on the distance \( (d_{s,s'}) \) from charging station \( s \) to charging station \( s' \) and the average PEV speed \( (v_{s,s'}) \), given by \( \mu_{ss'} = v_{s,s'}/d_{s,s'} \). After the service at routing service center \( RS_{ss'} \), a PEV enters decision service center \( DS_{s'} \) with probability 1.

### B. Average Charging Demands of Charging Stations

Let \( n = \{n^d_s, n^c_s, n^r_{s,s'}|s,s' \in \{1, 2, \ldots, S\}, s \neq s'\} \) be a set which represents the number of PEVs in each service center, where \( n^d_s, n^c_s, \) and \( n^r_{s,s'} \) represent the number of PEVs in \( DS_s, CS_s, \) and \( RS_{ss'} \), respectively. The queueing network model belongs to a class of BCMP open queueing network model, which has a product-form stationary distribution of \( n \) [18]. Define \( \xi^d_s, \xi^c_s, \) and \( \xi^r_{s,s'} \) as the absolute arrival rate of PEVs to \( DS_s, CS_s, \) and \( RS_{ss'} \), respectively. The traffic balance equations are given by [18]:

\[
\xi^d_s = \lambda_s + \sum_{s' \neq s} \xi^r_{s,s'}, \quad s \in \{1, 2, \ldots, S\} \tag{3}
\]

\[
\xi^c_s = p_{s,s}(r_s) \cdot \xi^d_s, \quad s \in \{1, 2, \ldots, S\} \tag{4}
\]

\[
\xi^r_{s,s'} = p_{s,s'}(r_s) \cdot \xi^d_s, \quad s, s' \in \{1, 2, \ldots, S\}, s \neq s' \tag{5}
\]

where (3) holds since both newly arrival PEVs and routing PEVs (from routing service centers) may arrive at a decision service center, as shown in Fig. 1, (4) holds since a PEV enters the charging service center only after a charging decision is made, and (5) holds since a PEV enters a routing service center only after the decision making at a decision service center. By solving these equations, the absolute arrival rate (in terms of \( \xi^d_s, \xi^c_s, \) and \( \xi^r_{s,s'} \)) at each service center can be obtained. Let \( \pi(n) \) be the stationary distribution of the number of PEVs in service centers, given by [18]:

\[
\pi(n) = \phi \prod_{s=1}^{S} G^d_s(n^d_s) \prod_{s=1}^{S} G^c_s(n^c_s) \prod_{s=1}^{S} \prod_{s' \neq s} G^r_{s,s'}(n^r_{s,s'}) \tag{6}
\]

where \( \phi \) is a normalization factor. Since both decision and routing service centers have an infinite number of servers (i.e., type-3 service centers [18]), \( G^d_s(n^d_s) \) and \( G^r_{s,s'}(n^r_{s,s'}) \) can be calculated as

\[
G^d_s(n^d_s) = (\xi^d_s/\mu_\infty)^{n^d_s} / n^d_s! \tag{7}
\]

\[
G^r_{s,s'}(n^r_{s,s'}) = (\xi^r_{s,s'}/\mu_{s,s'})^{n^r_{s,s'}} / n^r_{s,s'}! \tag{8}
\]

where \( \mu_\infty = \infty \) represents an infinitely large service rate. On the other hand, since each charging service center has a finite number of servers and is based an FCFS discipline (i.e., a type-1 service center [18]), the value of \( G^c_s(n^c_s) \) is given by

\[
G^c_s(n^c_s) = (\xi^c_s)^{n^c_s} / \left[ (\mu^c_s)^{n^c_s} \prod_{a=1}^{n^c_s} x(a) \right] \tag{9}
\]

Given \( n \), the PEV charging demand of charging station \( s \) is given by \( P_{s,ch}(n) = n^c_s P^c_s \). Then, the average charging demand \( P_{s,cho} \) of charging station \( s \) can be calculated based on (6). When \( c_s^r \mu^c_s < \xi^c_s \), charging station \( s \) is considered to be congested with \( P_{s,cho} = c_s P^c_s \). In other words, all chargers are occupied in a congested charging station.

### IV. CASE STUDY

The performance of the proposed charging station operation scheme is evaluated in a case study. Consider two charging stations with a distance of 10 km between them (i.e., \( d_{s,s'} = 10 \)). The charging power \( (P^c_s) \) of each charger in each charging station is 30 kW [4]. Without loss of generality, hour 10 of a day is considered. The minimum charging price \( r_{min} \) is set to 28.39 $/MWh which is based on the Hourly Ontario Energy Price (HOEP)² of July 8, 2013 [19]. The maximum charging price \( r_{max} = 2 r_{min} \) which is consistent with the range of price variation of Ontario electricity Time-of-Use price [20]. The PEV penetration level is set to 30%. The PEV traffic statistics \( v_{s,s'} \) and \( \lambda_s \) are obtained based on TDV [21]. The statistics of two typical weekdays for Delaware avenue (collected in 2008) and Hunt road (collected in 2009) in the New York State are used to represent urban and rural scenarios with average PEV speeds \( (v_{s,s'}) \) equal to 30 mph and 45 mph, respectively. Moreover, the PEV arrival rates are given by \( \lambda_1 = 191.4 \) vehicle/h and \( \lambda_2 = 223.2 \) vehicle/h for urban scenario, and \( \lambda_1 = 48.6 \) vehicle/h and \( \lambda_2 = 32.4 \) vehicle/h for rural scenario. Due to the lack of detailed PEV traces in TDV, it is assumed that all new arrival PEVs at each charging station (with respect to \( \lambda_s \)) need to be charged. The corresponding results can be considered as the worst case scenario (or upperbound) of PEV charging demands. The visiting probability \( (\alpha_{s'|s}) \) is set to 0.5.

The charging demand versus charging price and number of chargers for an urban scenario is shown in Fig. 2(a) and Fig. 2(b), where the charging prices of charging station 2 and charging station 1 are fixed to \( r_2 = 1.5 r_{min} \) and \( r_1 = 1.5 r_{min} \),

²Note that Ontario electricity price is considered since the daily price is online accessible while Ontario is in close vicinity of the New York State.
respectively. As shown in Fig. 2(a), the charging demand ($P_{1}^{chg}$) of charging station 1 decreases as $r_1$ increases and finally reaches 0 when $r_1 = 2r_{min}$. The main reason is that, less PEV drivers select charging station 1 for higher charging prices so that their charging demands are shifted to charging station 2. Moreover, when $r_1 = 2r_{min}$, the probability for a PEV driver to select charging station 1 equals 0. For the same reasons, the charging demand ($P_{2}^{chg}$) of charging station 2 increases as $r_1$ increases. However, the increment diminishes due to the congestion of charging station 2 with a limited number of chargers. Similarly, $P_{1}^{chg}$ increases as $r_1$ decreases, and the decrement diminishes when $r_1$ is small. For both $P_{1}^{chg}$ and $P_{2}^{chg}$, the congestion of charging station is more obvious for 25 chargers in comparison with that for 30 chargers. The charging demand versus $r_2$ is shown in Fig. 2(b). Although the trends of curves are similar to that in Fig. 2(a), the value of $P_{1}^{chg}$ when $r_2 = r_{min}$ in Fig. 2(b) is smaller than the value of $P_{2}^{chg}$ when $r_1 = r_{min}$ in Fig. 2(a). Specifically, the PEV arrival rate at charging station 2 (223.2 vehicle/h) is higher than that at charging station 1 (191.4 vehicle/h). Without any PEV been routed from the other charging station as $r_2 = r_{min}$ and $r_1 = r_{min}$, higher PEV arrival rate at charging station 2 leads to higher charging demand. Fig. 2(c) shows the charging demand versus charging price and number of chargers for a rural scenario with $r_1 = 1.5r_{min}$. Due to the low PEV arrival rate at the charging stations, none of the charging stations is congested when charging price varies within $[r_{min}, 2r_{min}]$. As a result, the curves for $P_{1}^{chg}$ (or $P_{2}^{chg}$) with respect to 30 and 25 chargers are overlapped.

V. CONCLUSIONS

In this paper, a BCMP queueing network model is developed to analyze the charging demands of multiple charging stations by taking account single PEV charging demand, charging decision making processes of PEV drivers, PEV traffic flow, and road system. The analytical model is validated by a case study based on real vehicle traffic statistics. Future research directions include the utilization of the analytical results to facilitate distribution system planning and the incorporation of the analytical model in distribution load flow analysis for optimal distribution system operation.