Joint Relay, Subcarrier and Power Allocation for OFDMA-based Femtocell Networks

Amila Tharaperiya Gamage, Md Shamsul Alam, Xuemin (Sherman) Shen, Jon W. Mark
Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON N2L 3G1, Canada
Email: {amila.gamage, ms3alam, sshen, jwmark}@uwaterloo.ca

Abstract—Relaying in femtocell networks is a promising and economically viable option to reduce the co-channel interference while improving indoor coverage and the network capacity in the next generation wireless networks. However, efficient relay selection as well as subcarrier and power allocation are critical in such networks when multiple users and multiple relays are considered. In this paper, an optimal resource (relay, subcarrier and power) allocation algorithm for co-channel deployed orthogonal frequency division multiple access (OFDMA) based femtocell systems is proposed. The resource allocation problem is formulated as a joint relay, subcarrier and power allocation problem with the objective of maximizing the sum of the weighted rates of the femtocell system subject to protecting the macrocell network’s communications. Due to the non-convex nature of the original resource allocation problem, we obtain an optimal solution for the original problem by solving a relaxed problem via dual decomposition. Simulation results demonstrate that our proposed resource allocation algorithm outperforms the resource allocation algorithms proposed in literature by achieving higher throughput at the expense of a slight increment of the system complexity.

I. INTRODUCTION

Femtocells have emerged as a promising technology to provide ubiquitous indoor connectivity in future broadband wireless networks [1], [2]. Femtocell systems use simple low power and low cost femto access points (FAPs) which connect to the operators via broadband communication links to provide the indoor coverage [2]. Orthogonal frequency division multiple access (OFDMA) is accepted as the most appropriate air interface for the next generation wireless technologies, such as 3GPP LTE-advanced and WiMAX, due to its inherent ability to combat frequency-selective fading while providing higher spectral efficiency. Therefore, the low cost OFDMA based femtocell systems provide one of the best ways to cater for the ever increasing demand for data at places, such as enterprise offices, apartment buildings and airports, from both technology and economic perspective [1], [2]. Co-channel deployed femtocell systems which share the same set of OFDM subcarriers with the macrocell network provide superior performance compared to the dedicated channel assignments between femtocell and macrocell networks [2], [5], [6]. However, in these co-channel deployments, femtocell and macrocell networks cause severe co-channel interference (CCI) among themselves [2], [4], [5]. Therefore, the co-channel deployed femtocell systems have to take extra measures to reduce CCI they introduce to macrocell network in order to protect macrocell network’s communications.

Relaying in femtocell networks is a cost effective solution to substantially reduce CCI as it improves the signal-to-noise ratio (SNR) of end-to-end links requiring smaller transmit power levels and there are many wireless devices, such as smart phones, PDAs, tablets etc., readily available at such indoor environments to act as relays. Moreover, employing relays improves the service coverage and the system throughput at in-building environments where building walls significantly attenuate the transmitting signals [3]. It also increases the reliability of transmission which is crucial specially for some indoor emergency applications such as home health care systems. However, the introduction of relays to femtocell networks further increases the complexity of the overall system including resource allocation. Therefore, efficient resource allocation schemes are crucial to fully exploit the benefit of relaying in such relay enhanced femtocell networks.

Resource allocation for OFDMA based femtocell networks has been studied in [4]–[8]. A subcarrier and power allocation scheme for femtocell systems which employ relays to reduce CCI is presented in [4]. However, this scheme is not optimal as relay selection has not been considered in the algorithm design, and subcarriers and power are allocated separately in two different stages of the algorithm. Open access scheme which allows users to connect to either femtocell or macrocell network in order to mitigate CCI is proposed in [5]. However, this scheme heavily depends on the availability of the backhaul capacities as well as FAP’s ability to synchronize itself with the macrocell base station. An algorithm for reducing inter-tier and intra-tier interference by controlling the number of subcarriers shared among the macrocell and the femtocell networks, and allocating subcarriers and power by a distributed auction based algorithm is proposed in [6]. Authors of [7] have proposed a distributed subchannel and power allocation algorithm for maximizing the sum of the weighted rates (SWR) of the femtocell network subjected to the CCI constraints, while the authors of [8] have proposed a power allocation scheme to minimize the required amount of transmit power while satisfying rate and CCI constraints. Furthermore, relay, subcarrier and power allocation for OFDMA systems have been studied in [9]–[11].

In this paper, we investigate the resource (relay, subcarrier and power) allocation for the downlink of a co-channel deployed OFDMA based femtocell network coexisting with a macrocell network. The objective is to maximize the SWR of the femtocell system while protecting the macrocell net-
work’s communications without causing excessive CCI. As the resource allocation problem formulated for this system is non-convex, we obtain the optimal solution for this problem by solving a relaxed version of the original problem using dual decomposition method. Simulation results have shown that the proposed resource allocation scheme outperforms the scheme proposed in [4] by 9% of SWR improvement at the expense of a slight increment of the complexity. The remainder of this paper is organized as follows. Section II introduces the system model and the problem formulation is presented in Section III. The analytical framework and the solution approach of the proposed resource allocation scheme are discussed in Section IV. Simulation results are given in Section V, followed by the conclusions in Section VI.

II. SYSTEM MODEL

![Co-channel deployed femtocell network coexisting with a macrocell network.](image)

Fig. 1: Co-channel deployed femtocell network coexisting with a macrocell network.

Downlink of a co-channel deployed OFDMA based femtocell system coexisting with a macrocell network as shown in Fig.1 is considered. There is K number of OFDM subcarriers in the system and which are shared among both femtocell and macrocell networks. Resources for the femtocell and the macrocell networks are allocated separately as it reduces high signaling overheads and complexity of the resource allocation protocol. There are M users in the macrocell network while N indoor users in the femtocell network. FAP can use multiple direct and/or relay links to communicate with these femtocell users. Each relay link can use at most one indoor user to act as an amplify-and-forward relay. A direct link from FAP to the $n^{th}$ femto user ($U_n$) completes transmission of a data symbol (i.e., QAM symbol) within one OFDM symbol period over the $i^{th}$ subcarrier while a relay link completes it within two OFDM symbol periods. In a relay link, as shown in Fig.2, FAP transmits a data symbol to both $U_n$ and the $m^{th}$ femto user who act as the relay node ($U_m$) over the $i^{th}$ subcarrier in the first OFDM symbol period, and $U_n$ forward the data symbol to $U_m$ over the $j^{th}$ subcarrier in the second OFDM symbol period. Then, $U_n$ combines the received data over the two paths using maximal ratio combining (MRC). Furthermore, during the second OFDM symbol period, FAP transmits another data symbol over the $i^{th}$ subcarrier while $U_m$ transmits over the $j^{th}$ subcarrier. Therefore, relay links also complete transmission of one data symbol per OFDM symbol period on average. Therefore, this scheme results in a high efficiency [4]. Furthermore, each subcarrier can be used by only one link in this system.

III. PROBLEM FORMULATION

The maximum achievable error free data rate over a relay link from FAP to $U_n$ using $(i, j)$ subcarrier pair and $U_m$ as the relay node is

$$ R_{i,j}^{n,m} = \Delta f \log_2 \left( 1 + \frac{P_{s,i,j}^{n,m} H_{a,b}^{0,i}}{P_{s,i,j}^{n,m} H_{b,k}^{0,i} + P_{r,i,j}^{n,m} H_{m,j}^{n,m} + 1} \right) $$

where

$$ H_{a,k}^{b} = \frac{|h_{a,k}^{b}|^2}{\sqrt{N_0 + Q_{b,k}}} $$

$P_{s,i,j}^{n,m}$ is the transmit power level of FAP over the $i^{th}$ subcarrier, $P_{r,i,j}^{n,m}$ is the transmit power level at $U_m$ over the $j^{th}$ subcarrier, $h_{a,k}^{b}$ is the channel gain of the channel between $U_n$ and $U_m$ over the $k^{th}$ subcarrier and FAP is represented by $U_0$. $Q_{b,k}$ is the CCI received by $U_b$ over the $k^{th}$ subcarrier, $N_0$ is the single sided power spectral density of additive white Gaussian noise and $\Delta f$ is the bandwidth of a OFDM subcarrier. Furthermore, it should be noted that if $P_{s,i,j}^{n,m} = 0$, then the data rate given by (1) reduces to the maximum achievable data rate over the direct link using only the $i^{th}$ subcarrier.

The transmit power levels in the femtocell network over the subcarriers which are being used by the macrocell network have to be selected such that these femtocell transmissions do not introduce excessive CCI for the macrocell network. Therefore, the CCI constraints on the transmit power levels in femtocell network are given by

$$ b_i P_{s,i,j}^{n,m} |h_{0,i}^U|^2 \leq I_i \quad \forall i, j, m, n $$

and

$$ b_j P_{r,i,j}^{n,m} |h_{m,j}^U|^2 \leq I_j \quad \forall i, j, m, n $$

where $I_k$ is the CCI threshold for the $k^{th}$ subcarrier, $b_k = 1$ if the $k^{th}$ subcarrier is being used by the macrocell network, or $b_k = 0$ otherwise. $h_{a,k}^{b}$ is the channel gain of the channel
between $U_a$ and the macrocell user who uses the $k^{th}$ subcarrier for its communications.

Now, we derive the constraints on subcarrier allocations in order to restrict each subcarrier to be allocated only for one link. For that purpose, we define the variable $\rho_{i,j}^{n,m}$ such that $\rho_{i,j}^{n,m} = 1$ if $(i, j)$ subcarrier pair and $U_m$ as the relay node are allocated for the downlink communications of the $n^{th}$ user, or $\rho_{i,j}^{n,m} = 0$ otherwise. Then, the constraint which restricts the $k^{th}$ subcarrier to be used only by one link can be written as

$$\sum_{n=1}^{N} \sum_{m=1}^{N} \left( \sum_{j=1}^{K} \rho_{k,j}^{n,m} + \sum_{i=1}^{K} \rho_{i,k}^{n,m} \right) \leq 1, \forall k \in \{1, ..., K\}. \quad (4)$$

Furthermore, this constraint also prevents the allocation of same subcarrier for both links: link between FAP and the relay node, and the link between relay node and the destination node.

### A. Resource Allocation Problem

The objective of the resource allocation is to allocate relays, subcarriers and power such that the SWR of the femtocell network, and the link between relay node and the destination node satisfies all the KKT conditions [14]. To derive KKT conditions, we first state the Lagrangian of the primal problem stated in (8) after some simplifications as follows.

$$\mathcal{L}(\mathbf{P}_s, \mathbf{P}_r, \rho, \alpha, \beta, \lambda, \xi, \varsigma, \gamma, \mu) =$$

$$\sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \left( \rho_{s,i,j}^{n,m} \right) - \mu P_T - \sum_{k=1}^{K} \lambda_k \quad (9)$$

where

$$\mathcal{L}_{i,j}^{n,m} = -w_n P_{i,j}^{n,m} + \left( \mu + \alpha_{i,j}^{n,m} \left| h_{0,i} \right|^2 - \xi_{i,j}^{n,m} \right) \rho_{s,i,j}^{n,m}$$

$$+ \left( \beta_{i,j}^{n,m} \left| h_{m,j} \right|^2 - \varsigma_{i,j}^{n,m} \right) \rho_{r,i,j}^{n,m}$$

$$+ \left( \gamma_{i,j} \right) \rho_{i,j}^{n,m}$$

and $\mu$ is a scalar while $\lambda$ is a $K \times 1$ vector consisting of the dual variable $\lambda_i$ as its elements. $\alpha, \beta, \xi, \varsigma$ and $\gamma$ are $K \times K \times N \times N$ dimensional matrices which consist of the dual variables, $\alpha_{i,j}^{n,m}, \beta_{i,j}^{n,m}, \xi_{i,j}^{n,m}, \varsigma_{i,j}^{n,m}$ and $\gamma_{i,j}$, as their elements.

### IV. Relay, Subcarrier and Power Allocation

Since the optimization problem stated in (8) is a convex optimization problem, Karush-Kuhn-Tucker (KKT) conditions guarantee the optimality of the solution when the solution satisfies all the KKT conditions [14]. To derive KKT conditions, we can relax the resource allocation problem as the objective function is a non-convex optimization problem, Karush-Kuhn-Tucker (KKT) conditions guarantee the optimality of the solution when the solution satisfies all the KKT conditions [14]. To derive KKT conditions, we can relax the resource allocation problem as the objective function is a non-convex optimization function [4], [12], and the feasible set is not a convex set as a fraction of a subcarrier cannot be allocated to a user. Therefore, finding the optimal solution for this non-convex optimization problem requires searching through all the possible relay, subcarrier and power allocations. However, these exhaustive search methods are prohibitively complex to employ in large systems. Therefore, we relax the problem in (5) to make it a convex optimization problem and then find the optimal solution as explained in the next sections [13].

### B. Relaxed Resource Allocation Problem

Throughput, $R_{i,j}^{n,m}$, given by (1) can be approximated at high SNR region as follows [4], [12].

$$R_{i,j}^{n,m} \approx \Delta f \log_2 \left( 1 + \frac{P_{s,i,j}^{n,m} H_{0,i}^{n,m}}{P_{s,i,j}^{n,m} H_{0,i}^{n,m} + \sum_{i,j}^{K} \rho_{i,j}^{n,m} H_{m,j}^{n,m}} \right) \quad (6)$$

Then, $R_{i,j}^{n,m}$ is a convex function. Furthermore, we relax the condition of each subcarrier can be allocated for only one link by allowing fractional subcarrier allocations (i.e., $0 \leq \rho_{i,j}^{n,m} \leq 1, \forall i, j, m, n$) [12], [13]. Define $\bar{P}_{s,i,j}^{n,m}$ and $\bar{P}_{r,i,j}^{n,m}$ as

$$\bar{P}_{s,i,j}^{n,m} = P_{s,i,j}^{n,m} + \rho_{i,j}^{n,m} H_{0,i}^{n,m}, \bar{P}_{r,i,j}^{n,m} = P_{r,i,j}^{n,m} + \rho_{i,j}^{n,m} H_{m,j}^{n,m} \quad (7)$$

and it is also a convex function [12], [13], [14]. Then, the relaxed resource allocation problem can be stated as follows.

$$\max \mathbf{P}_s, \mathbf{P}_r, \rho \quad \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{i,j}^{K} w_n R_{i,j}^{n,m}$$

s.t.

$$C_1: b_i P_{s,i,j}^{n,m} U_{i,j}^{n,m} \leq b_i P_{i,j}^{n,m} U_{i,j}^{n,m}, \forall i, j, m, n$$

$$C_2: b_j P_{r,i,j}^{n,m} U_{i,j}^{n,m} \leq b_j P_{i,j}^{n,m} U_{i,j}^{n,m}, \forall i, j, m, n$$

$$C_3: \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{i,j}^{K} P_{s,i,j}^{n,m} + P_{r,i,j}^{n,m} \leq P_T$$

$$C_4: \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{i=1}^{K} \sum_{j=1}^{K} \sum_{i,j}^{K} \sum_{i,j}^{K} \left( \bar{P}_{s,i,j}^{n,m} + \bar{P}_{r,i,j}^{n,m} \right) \leq P_T$$

$$C_5: 0 \leq \rho_{i,j}^{n,m} \leq 1, \forall i, j, m, n$$

$$C_6: \bar{P}_{s,i,j}^{n,m} \geq 0, \bar{P}_{r,i,j}^{n,m} \geq 0, \forall i, j, m, n \quad (8)$$

where $\mathbf{P}_s = \rho \odot \mathbf{P}_s, \mathbf{P}_r = \mathbf{P}_r \odot \rho$, and $\odot$ is the Hadamard product (i.e., element wise product) operator.
respectively. Furthermore, these sub-Lagrangians, \( \mathcal{L}_{i,j}^{n,m} \), are independent of each other for a set of given values of \( \mu \) and \( \lambda_k \), \( \forall k \in \{1, ..., K\} \).

Next, we find the optimal relay, subcarrier and power allocation for the problem stated in (8) for a set of given values of \( \mu \) and \( \lambda_k \), \( \forall k \in \{1, ..., K\} \) by using KKT conditions. This solution satisfies all the constraints except C3 and C4. Then, for a given value of \( \mu \), we find \( \lambda^*_k, \forall k \in \{1, ..., K\} \) and the optimal relay, subcarrier and power allocation such that C3 is also satisfied. In the final step, we change the value of \( \mu \) until the optimal relay, subcarrier and power allocation found by the previous two steps satisfies C4.

### A. Power Allocation

The optimal power levels for a relay link which uses \((i, j)\) subcarrier pair and node \( U_m \) as the relay can be calculated based on the following KKT conditions derived using \( \mathcal{L}_{i,j}^{n,m} \).

\[
\begin{align*}
\frac{\partial R_{n,m}^{i,j}}{\partial p_{n,m}^{i,j}} |_{\hat{P}_{n,m}^{i,j}} & = \mu^* + b_i \alpha_{n,m} |U_{0,i}|^2 - e_{n,m}, \tag{10} \\
\frac{\partial R_{n,m}^{i,j}}{\partial r_{n,m}^{i,j}} |_{\hat{P}_{n,m}^{i,j}} & = \beta_{n,m} |U_{0,j}|^2 - e_{n,m}, \tag{11} \\
\end{align*}
\]

where the symbol \( x^* \) denotes the optimal value of the variable denoted by \( x \). The optimal power levels for a given value of \( \mu \) can be calculated by using (10)–(11) as follows. First, let,

\[
\hat{P}_{n,m}^{i,j} = \min \left\{ \frac{I_{p}^{n,m}}{b_i |U_{0,i}|^2}, \left[ \frac{w_n p_{s,i,j}^{n,m}}{\mu} - \frac{\rho_{n,m}^{i,j}}{h_{0,i}} \right]^+ \right\} \tag{15}
\]

where \( |x|^+ = \max\{0, x\} \). Now, the optimal power levels of \( i^{th} \) and \( j^{th} \) subcarriers are calculated based on the following three cases.

- **Case-1:** \( \mu(\hat{P}_{n,m}^{i,j} H_{0,i} + \hat{p}_{s,i,j}^{n,m}) \geq w_n \rho_{n,m}^{i,j} H_{m,j} \)

  Power is not allocated for the \( j^{th} \) subcarrier assigned to the link between the relay and the destination for the given value of \( \mu \). Therefore, \( P_{n,m}^{s,i,j} = \hat{P}_{n,m}^{s,i,j} \) and \( \hat{P}_{r,i,j}^{n,m} = 0 \).

However, if the condition of Case 1 is not satisfied, then power is allocated for both \( i^{th} \) and \( j^{th} \) subcarriers. In this scenario, \( \hat{P}_{n,m}^{s,i,j} \) and \( \hat{P}_{r,i,j}^{n,m} \) have to be jointly calculated according to the following two cases.

- **Case-2:** \( \hat{P}_{n,m}^{s,i,j} = \frac{I_{p}^{n,m}}{b_i |U_{0,i}|^2}, \hat{P}_{r,i,j}^{n,m} = \frac{p_{s,i,j}^{n,m}}{H_{m,j}} \)

  The optimal power levels for a given value of \( \mu \) in independent of each other for a set of given values of \( \lambda \) and \( \lambda_k \). Then, the optimal value of \( \hat{P}_{r,i,j}^{n,m} \) is given by

\[
\hat{P}_{r,i,j}^{n,m} = \min \left\{ \frac{I_{p}^{n,m}}{b_i |U_{0,i}|^2}, \hat{P}_{r,i,j}^{n,m} \right\}. \tag{17}
\]

- **Case-3:** \( \hat{P}_{n,m}^{s,i,j} = \frac{w_n \rho_{n,m}^{i,j}}{\mu} - \frac{\rho_{n,m}^{i,j}}{H_{0,i}} \)

  \( P_{n,m}^{s,i,j} = \hat{P}_{n,m}^{s,i,j} \) and \( \hat{P}_{r,i,j}^{n,m} = \mu \)

  using Newton’s method. Then, \( \hat{P}_{r,i,j}^{n,m} \) is given by (17).

### B. Relay and Subcarrier Allocation

Optimal relay and subcarrier assignments can be determined based on the following KKT conditions,

\[
\begin{align*}
\frac{\partial R_{n,m}^{i,j}}{\partial l_{s,i,j}^{n,m}} |_{\hat{l}_{s,i,j}^{n,m}} & = \mu^* + b_i \alpha_{n,m} |U_{0,i}|^2 - e_{n,m}, \tag{12} \\
\frac{\partial R_{n,m}^{i,j}}{\partial l_{r,i,j}^{n,m}} |_{\hat{l}_{r,i,j}^{n,m}} & = \beta_{n,m} |U_{0,j}|^2 - e_{n,m}, \tag{13} \\
\end{align*}
\]

and the metric \( \Phi_{i,j}^{n,m} \) which is derived from (20) as

\[
\Phi_{i,j}^{n,m} = \frac{w_n \log(1 + \Gamma_{i,j}^{n,m}) - \frac{w_n}{\ln(2)} (1 + \Gamma_{i,j}^{n,m})}{b_i \alpha_{n,m} I_i + b_j \beta_{n,m} I_j} \tag{22}
\]

where

\[
\Gamma_{i,j}^{n,m} = \frac{P_{n,m}^{s,i,j} H_{0,i} + \hat{P}_{n,m}^{s,i,j} H_{m,j}}{P_{n,m}^{s,i,j} H_{0,i} + \hat{P}_{n,m}^{s,i,j} H_{m,j}} \tag{20}
\]

The values of \( \rho_{n,m}^{i,j} \) and \( \hat{P}_{n,m}^{i,j} \) can be calculated by using (10)–(11) once the optimal power levels are calculated. Therefore, \( \Phi_{i,j}^{n,m} \) can be calculated for every relay and subcarrier assignment (i.e., for every \( \rho_{n,m}^{i,j} \)). Therefore, based on (20)–(21), [12], [13],

\[
\rho_{n,m}^{i,j} = \begin{cases} 
1, & \text{if } \Phi_{i,j}^{n,m} > \lambda_i^* + \lambda_j^* \\
0, & \text{otherwise}
\end{cases} \tag{23}
\]
The optimal relay and subcarrier assignments can be determined by (23) if \( \lambda_0^* \), \( \forall k \in \{1, \ldots, K\} \) are known. \( \lambda_k^* \), \( \forall k \in \{1, \ldots, K\} \) can be found by decreasing each \( \lambda_k \) starting from a large value until the relay and subcarrier assignments determined by (23) satisfy C3. It should be noted that the relay and subcarrier assignments correspond to larger \( \Phi_{n,m}^* \) are allocated first compared to those corresponding to smaller \( \Phi_{n,m} \). Therefore, instead of finding the optimal value for \( K \) dual variables, we use the following simple method for finding the optimal relay and subcarrier assignments. First, we find the largest \( \Phi_{n,m}^* \) and set the corresponding \( \rho_{i,j} = 1 \). Then, we find the second largest \( \Phi_{n,m}^* \) searching over the subcarriers which have not been already allocated and set corresponding \( \rho_{i,j} = 1 \). This process continues until all the subcarriers are allocated.

Next, we change the value of \( \mu \) from a large value to a small value until the optimal power levels satisfy C4 (i.e., until \( \mu^* \) is found). (23) provides the optimal subcarrier and relay allocation for the problem stated in (8) when all \( \Phi_{n,m} \) have distinct values. This condition is a reasonable assumption due to the fact that the channel gain of each subcarrier over that subcarrier is 25dB. The remaining parameters that the SNR level at the macrocell user who receives the downlink communications. The transmit power levels of the macrocell users are assumed to be uniformly distributed between 0 and 1, and they are normalized to have a unit sum. Each macrocell user occupies one subcarrier for its downlink communications. The transmit power levels of the macrocell base station over each subcarrier are selected such that the SNR level at the macrocell user who receives the signal over that subcarrier is 25dB. The remaining parameters of the simulation setup are given in Table I.

The performance of the proposed resource allocation algorithm (RSPA) is compared with the performance of the scheme which combines subcarrier and power allocation algorithm proposed in [4] with the relay selection scheme proposed in [16]. The latter resource allocation scheme is termed as SPA. The subcarrier and power allocation algorithm proposed in [4] achieves results close to optimal in networks where the relays have already been allocated, and the relay selection scheme proposed in [16] provides the optimal relay selection assuming equal power allocations at the source and the relay stations.

Fig. 3 shows the comparison of the SWR achieved by RSPA and SPA for different number of femtocell and macrocell users. According to the simulation results, RSPA provides at least 9% of SWR improvement over SPA. The reason for such an improvement is that the joint allocation of relays, subcarriers and power is optimal than allocating those resources separately by different algorithms at different steps. SWR of the femtocell

### Algorithm 1: Relay, Subcarrier and Power Allocation

1. Initialize \( \mu \) with a large value
2. \( \mathcal{N} \leftarrow \{1, \ldots, N\} \) and \( \mathcal{K} \leftarrow \{1, \ldots, K\} \)
3. for Iterations = 1 to 100 do
4. for all \( i,j \in \mathcal{K} \) and \( n,m \in \mathcal{N} \) do
5. if \( (i = j)(n \neq m) \) or \( (i \neq j)(n = m) \) then
6. \( \bar{P}_{s,i,j} \leftarrow 0 \) and \( \bar{P}_{r,i,j} \leftarrow 0 \)
7. else
8. \( \rho_{i,j} \leftarrow 1 \)
9. Calculate \( \bar{P}_{s,i,j} \) and \( \bar{P}_{r,i,j} \) based on Case 1 \~ 3
10. end if
11. end for
12. Calculate \( \alpha_{i,j}^{n,m} \) and \( \beta_{i,j}^{n,m} \), \( \forall i,j,m,n \) by (10) \~ (11)
13. \( \mathcal{I} \leftarrow \mathcal{K}, \mathcal{J} \leftarrow \mathcal{K} \) and \( \rho_{i,j} \leftarrow 0 \), \( \forall i,j,m,n \)
14. for \( l = 1 \) to \( K \) do
15. \( \{i_l,j_l,n_l,m_l\} \leftarrow \argmax_{i \in \mathcal{I},j \in \mathcal{J},n,m \in \mathcal{N},m \in \mathcal{N}} \{ \Phi_{i,j,n,m} \} \)
16. \( \rho_{i_l,j_l} \leftarrow 1, \mathcal{I} \leftarrow \mathcal{I}\setminus i_l \) and \( \mathcal{J} \leftarrow \mathcal{J}\setminus j_l \)
17. end for
18. \( P_{\text{allocate}} \leftarrow \sum_{\forall i,j,n,m} \rho_{i,j} (\bar{P}_{s,i,j} + \bar{P}_{r,i,j}) \)
19. if \( |P_{\text{allocate}} - P_T| < \varepsilon \) then
20. Break
21. else if \( P_{\text{allocate}} > P_T \) then
22. Increase \( \mu \)
23. else
24. Decrease \( \mu \)
25. end if
26. end for
27. \( P_s^* \leftarrow \rho \odot \bar{P}_s^* \) and \( P_r^* \leftarrow \rho \odot \bar{P}_r^* \)

### Table I: Simulation Environment

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of OFDM subcarriers, ( K )</td>
<td>128</td>
</tr>
<tr>
<td>Bandwidth of OFDMA system ((K\Delta f))</td>
<td>5MHz</td>
</tr>
<tr>
<td>Total power for downlink communications, ( P_T )</td>
<td>30dBm</td>
</tr>
<tr>
<td>Single sided power spectra density of noise, ( N_0 )</td>
<td>-174dBm/Hz</td>
</tr>
<tr>
<td>Path-loss exponent, ( \eta )</td>
<td>4</td>
</tr>
<tr>
<td>Radius of the macro cell</td>
<td>1000m</td>
</tr>
<tr>
<td>Minimum distance to users from macro BS</td>
<td>50m</td>
</tr>
<tr>
<td>Radius of the femtocell</td>
<td>50m</td>
</tr>
<tr>
<td>Minimum distance to users from FAP</td>
<td>1m</td>
</tr>
<tr>
<td>Error tolerance, ( \varepsilon )</td>
<td>0.5%</td>
</tr>
</tbody>
</table>
network is slightly decreased as the number of femtocell users increases due to the fact that the values of weights which have been normalized to have a unit sum are decreased as the number of users increases. Furthermore, SWR of the system increases with the interference threshold as the constraints on the power levels of femtocell network are loosened as the interference threshold increases. Moreover, SWR decreases when the number of macrocell users increases due to the increment of the number of subcarriers with CCI constraints with the number of macrocell users.

The complexity of each algorithm is measured in terms of the number of iterations required for the algorithm to converge. Fig. 4 shows the average number of iterations required for each algorithm to converge at different scenarios. RSPA requires more iterations to converge compared to SPA. SPA converges faster as the number of power allocation variables is less in SPA due to relays and subcarriers have already been allocated prior to power allocation whereas RSPA requires more iterations than SPA as there are more power allocation variables in RSPA due to joint allocation of relays, subcarriers and power in RSPA to achieve optimal SWR of the femtocell network. Although RSPA requires more iterations than SPA, it still converges within average of 10.5 iterations. Moreover, the number of iterations required for RSPA is further reduced as the number of femtocell users decreases due to the fact that the number of power allocation variables is decreased when the number of femtocell users decreases.

**VI. Conclusions**

In this paper, an optimal resource (relay, subcarrier and power) allocation algorithm for co-channel deployed OFDMA based femtocell systems has been proposed. As the original formulated resource allocation problem for the femtocell system is non-convex, the optimal resource allocation algorithm has been derived utilizing the relaxed resource allocation problem. The proposed resource allocation algorithm outperforms the resource allocation scheme proposed in [4] and achieves throughput improvement of 9% at the expense of a slight increment of the complexity.

**References**


