Design of Low-Density Parity-Check Codes for Half-Duplex Three-Phase Two-Way Relay Channels

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***Abstract*—In two-way relay channels, two terminal nodes exchange information with the help of a relay node. Designing practical coding schemes for such channels is challenging, especially when messages are encoded into multiple streams and a destination node receives signals from multiple nodes. In this paper, we prove an achievable region for half-duplex three-phase two-way relay channels. Furthermore, we propose low-density parity-check (LDPC) codes for such channels where two source codewords are encoded by systematic LDPC codes at the relay node. To analyze the performance of the codes, discretized density evolution is derived for the joint decoder at terminal nodes. Based on the discretized density evolution, degree distributions are optimized by iterative linear programming in 3 steps. The length of the obtained optimized codes is 26% longer than the theoretic one.**

***Index Terms*—Low-density parity-check (LDPC) codes, two-way relay channel, density evolution.**

I. Introduction

In wireless networks, it has always been a challenge to satisfy high traffic throughput demand. Besides limited power and spectrum resources, interference is also a factor that limits the throughput due to shared medium. In the past decades, various techniques, such as cooperative communications [1], have been developed to achieve higher communication rates.

A typical example of cooperative communications is the communication through relay channels [2] where the source node transmits information to the destination node with the help of a relay node. Although the exact capacity of the relay channel is still unknown, two different relay schemes, known as decode-and-forward and compress-and-forward [2], have been developed. In general, when the source-relay link is reliable, the decode-and-forward scheme is a better choice since noise can be fully eliminated.

A natural extension of the one-way relay channel is the two-way case where two terminal nodes exchange information with the help of a relay node. Some fundamental bounds [3], [4], [5] for two-way relay channels have been proposed by several research groups. In [3], an achievable region of the decode-and-forward scheme based on block Markov superposition coding and an achievable region of the compress-and-forward scheme based on Wyner-Ziv coding for full-duplex two-way relay channels were proposed. In [4], another achievable region for full-duplex two-way relay channels was proved by random binning. In [5], the capacity region of the broadcast phase of the two-way relay channels was determined when destination nodes use side information for decoding.

Besides the research on fundamental bounds, various practical coding schemes have also been proposed for relay channels. Constructing codes for such channels is challenging, especially when messages are encoded into multiple streams and a destination node receives signals from multiple node. Low-density parity-check (LDPC) codes were proposed for one-way relay channels in [6], [7], [8]. In [6], LDPC codes were proposed for one-way relay channels where codes within 1.2 dB of the theoretical limit were found. Furthermore, LDPC codes employing random puncturing were applied to fading relay channels in [7]. In order to improve the performance of one-way relay channels, bilayer LDPC codes were designed based on the bilayer density evolution in [8].

In half-duplex two-phase two-way relay channels, the relay node receives superimposed signals from the two source nodes. Considering this unique property, various coding schemes, such as physical-layer network coding [9], repeat-accumulate codes [10], lattice codes [11] and LDPC codes [12], [13], have been proposed recently.

In this paper, we focus on half-duplex three-phase two-way relay channels. Half-duplex is a practical assumption since it is generally difficult for a node to detect weak received signals when they are mingled with its own strong transmitting signals. Compared with two-phase two-way relay channels, signals from the source node can be utilized for decoding at the destination node. In addition, decoding at the relay node is simpler since no superimposed signals are involved. However, to the best of our knowledge, only a few practical coding schemes [14], [15] have been proposed for three-phase two-way relay channels.

In this paper, we propose LDPC codes for half-duplex three-phase two-way relay channels, first appeared in [16]. LDPC codes are good candidates since they can approach the channel capacities of point-to-point channels. More importantly, they have a comprehensive set of design tools along with their flexible code constructions.

The main contributions of this paper are four-fold. First, we prove an achievable region for half-duplex three-phase two-way relay channels. Second, inspired by the random coding, we propose a code construction which is composed of two irregular LDPC codes at terminal nodes and a systematic LDPC code at the relay node. Note that the relay codeword can be generated by simply adding the two source codewords in GF(2). However, simple addition is not optimal if links between the source node and the relay node are asymmetric

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since equal amount of information from the two codewords is included in the relay codeword. Encoding by systematic LDPC codes at the relay node can be thought as parity forwarding or random binning on multiple sources. This code construction is similar to that of non-systematic low-density generator matrix (LDGM) codes [17], or Luby transform (LT) codes (a class of rateless code) [18], which were originally proposed for point-to-point channels. Third, to analyze the performance of the codes, we employ discretized density evolution for the proposed decoder, where the relationships between the constituent codes are derived. Last, based on the discretized density evolution, we propose a 3-step degree distribution optimization based on iterative linear programming. We show that the length of the obtained optimized codes is only 26% longer than the theoretic one.

This paper is organized as follows. We begin with an introduction of the system model and a proof of an achievable rate region for half-duplex three-phase two-way relay channels in Section II. In Section III, LDPC code constructions and the corresponding message-passing decoding algorithms are proposed. In Section IV, discretized density evolution is derived to analyze the codes. We introduce an iterative linear programming algorithm for code optimization in Section V. An optimized degree distribution is reported and decoding simulation results are included in Section VI. Finally, we conclude this paper in Section VII.

II. System Model and an Achievable Rate Region

A. System Model

In two-way relay channels, two terminal nodes communicate with each other with the help of a relay node. We consider the case when signals from the source node can be utilized for decoding at the destination node. In this case, the transmission over half-duplex two-way relay channels can be modeled as a three-phase transmission. We label the two terminal nodes as node 1 and node 2, respectively, and label the relay node as node 3. In phase 1, node 1 encodes its message and broadcasts the codeword. Both node 2 and node 3 can receive signals. In phase 2, node 2 encodes its message and broadcasts the codeword. Both node 1 and node 3 can receive signals. The relay node can decode the messages of node 1 and node 2 at the end of phase 1 and phase 2, respectively. In phase 3, node 3 encodes the two source codewords to a relay codeword and broadcasts the relay codeword. Both node 1 and node 2 can receive signals. At the end of phase 3, node 1 can jointly decode the message of node 2 from signals received from node 2 in phase 2, signals received from node 3 in phase 3 and its own codeword. Similarly, node 2 can jointly decode the message of node 1. The three phase model is shown in Figure 1.

B. An achievable rate region of half-duplex three-phase two-way relay channels

In this section, we prove an achievable rate region of half-duplex three-phase two-way relay channels. Note that an achievable rate region for full-duplex two-way relay channels was given in [4].

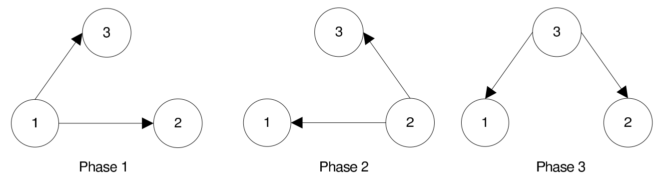

Fig. 1. Three phases in half-duplex two-way relay channels.

The two-way relay channel consists of source input alphabet sets $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{X}_3$, channel output alphabet sets $\mathcal{Y}_1$, $\mathcal{Y}_2$, $\mathcal{Y}_3$ and a set of distributions $p(y_1, y_2, y_3|x_1, x_2, x_3)$. Considering time division, distributions during phase 1, phase 2 and phase 3 are $p(y_2, y_3|x_1)$, $p(y_1, y_3|x_2)$ and $p(y_1, y_2|x_3)$, respectively.

Assume the lengths of codewords in the three phases are n_1, n_2 and n_3, respectively, and $n = \sum_{i=1}^{3} n_i$. Set $\alpha = \frac{n_1}{n}$, $\beta = \frac{n_2}{n}$ and $\gamma = \frac{n_3}{n}$.

A $((2^{nR_1}, 2^{nR_2}), n_1, n_2)$ code for the half duplex three-phase two-way relay channel consists of two sets of integers $\mathcal{W}_1 = \{1, 2, \cdots, 2^{nR_1}\}$ and $\mathcal{W}_2 = \{1, 2, \cdots, 2^{nR_2}\}$, three encoding functions $X_1 : \mathcal{W}_1 \to \mathcal{X}_1^{n_1}$, $X_2 : \mathcal{W}_2 \to \mathcal{X}_2^{n_2}$ and $X_3 : \mathcal{W}_1 \times \mathcal{W}_2 \to \mathcal{X}_3^{n_3}$, and four decoding functions $\mathcal{Y}_3^{n_1} \to \mathcal{W}_1$, $\mathcal{Y}_3^{n_2} \to \mathcal{W}_2$, $\mathcal{Y}_2^{n_1} \times \mathcal{Y}_2^{n_3} \to \mathcal{W}_1$, and $\mathcal{Y}_1^{n_2} \times \mathcal{Y}_1^{n_3} \to \mathcal{W}_2$.

Theorem 1. *For discrete memoryless half-duplex three-phase two-way relay channels, all rate pairs (R_1, R_2) satisfying*

$$R_1 < \min\{\alpha I(X_1; Y_3), \gamma I(X_3; Y_2) + \alpha I(X_1; Y_2)\} \tag{1}$$

and

$$R_2 < \min\{\beta I(X_2; Y_3), \gamma I(X_3; Y_1) + \beta I(X_2; Y_1)\} \tag{2}$$

are achievable for some $p(x_1)p(x_2)p(x_3)$ where $\alpha+\beta+\gamma = 1$.

Proof: Codebook generation: Generate 2^{nR_1} codewords $\mathbf{x}_1 = x_1^{n_1}$ according to $\prod_{i=1}^{n_1} p(x_1)$ and index them as $\mathbf{x}_1(w_1)$, $w_1 \in \{1, 2, \cdots, 2^{nR_1}\}$. Generate 2^{nR_2} codewords $\mathbf{x}_2 = x_2^{n_2}$ according to $\prod_{i=1}^{n_2} p(x_2)$ and index them as $\mathbf{x}_2(w_2)$, $w_2 \in \{1, 2, \cdots, 2^{nR_2}\}$. Generate $2^{n(R_1+R_2)}$ codewords $\mathbf{x}_3 = x_3^{n_3}$ according to $\prod_{i=1}^{n_3} p(x_3)$ and index them as $\mathbf{x}_3(w_1, w_2)$, $w_1 \in \{1, 2, \cdots, 2^{nR_1}\}$, $w_2 \in \{1, 2, \cdots, 2^{nR_2}\}$.

Encoding: In phase 1, to send index w_1, node 1 sends $\mathbf{x}_1(w_1)$. In phase 2, to send w_2, node 2 sends $\mathbf{x}_2(w_2)$. In phase 3, node 3 sends $\mathbf{x}_3(\hat{w}_1, \hat{w}_2)$ after decoding w_1 and w_2 (See the decoding part).

Decoding: Denote $\mathbf{y}_{i,j}$ as the channel output at node i in phase j. At the end of phase 1, node 3 decodes w_1 by finding the unique $\hat{w}_1$ that satisfies the joint typicality check $(\mathbf{x}_1(\hat{w}_1), \mathbf{y}_{3,1}) \in A_\epsilon^{(n_1)}(X_1, Y_3)$ where $A_\epsilon^{(n_1)}(X_1, Y_3)$ is the set of joint typical sequences of X_1 and Y_3. If there is no such or more than one such $\hat{w}_1$, an error is declared. Similarly, at the end of phase 2, node 3 decodes w_2 by finding the unique $\hat{w}_2$ that satisfies $(\mathbf{x}_2(\hat{w}_2), \mathbf{y}_{3,2}) \in A_\epsilon^{(n_2)}(X_2, Y_3)$. If there is no such or more than one such $\hat{w}_2$, an error is declared. At the end of phase 3, node 1 decodes w_2 by finding the unique $\hat{w}_2$ that satisfies $(\mathbf{x}_2(\hat{w}_2), \mathbf{y}_{1,2}) \in A_\epsilon^{(n_2)}(X_2, Y_1)$ and $(\mathbf{x}_3(w_1, \hat{w}_2), \mathbf{y}_{1,3}) \in A_\epsilon^{(n_3)}(X_3, Y_1)$. Node 2 decodes w_1 by finding the unique $\hat{w}_1$ that satisfies $(\mathbf{x}_1(\hat{w}_1), \mathbf{y}_{2,1}) \in A_\epsilon^{(n_1)}(X_1, Y_2)$ and $(\mathbf{x}_3(\hat{w}_1, w_2), \mathbf{y}_{2,3}) \in A_\epsilon^{(n_3)}(X_3, Y_2)$.

Analysis of the probability of error: When node 1 sends $\mathbf{x}_1(w_1)$, the probability that independent $\mathbf{x}_1$ and $\mathbf{y}_{3,1}$ are jointly typical is upper bounded by $2^{-n_1(I(X_1;Y_3)-3\epsilon)}$. There are totally $2^{nR_1}-1$ such $\mathbf{x}_1$. With the union bound, the probability of error at node 3 is upper bounded by $(2^{nR_1}-1)2^{-n_1(I(X_1;Y_3)-3\epsilon)}$, which approaches zero when $n_1 \to \infty$ and $R_1 < \alpha I(X_1;Y_3)$ (from $nR_1 - n_1 I(X_1;Y_3) < 0$ and $\alpha = \frac{n_1}{n}$). Similarly, we need $R_2 < \beta I(X_2;Y_3)$ for node 3 to decode $\mathbf{x}_2$.

When node 2 sends $\mathbf{x}_2(w_2)$, the probability that independent $\mathbf{x}_2$ and $\mathbf{y}_{1,2}$ are jointly typical is upper bounded by $2^{-n_2(I(X_2;Y_1)-3\epsilon)}$. When node 3 sends $\mathbf{x}_3(\hat{w}_1,\hat{w}_2)$, the probability that independent $\mathbf{x}_3$ and $\mathbf{y}_{1,3}$ are jointly typical is upper bounded by $2^{-n_3(I(X_3;Y_1)-3\epsilon)}$. There are totally $2^{nR_2}-1$ such w_2 when node 1 knows w_1. With the union bound, the probability of the event that any independent $\mathbf{x}_2$ and $\mathbf{y}_{1,2}$ are jointly typical and any independent $\mathbf{x}_3$ and $\mathbf{y}_{1,3}$ are jointly typical at node 1 is upper bounded by $(2^{nR_2}-1)2^{-n_2(I(X_2;Y_1)-3\epsilon)}2^{-n_3(I(X_3;Y_1)-3\epsilon)}$, which approaches zero when $n_2 \to \infty$, $n_3 \to \infty$ and $R_2 < \beta I(X_2;Y_1) + \gamma I(X_3;Y_1)$ (from $nR_2 - n_2 I(X_2;Y_1) - n_3 I(X_3;Y_1) < 0$, $\beta = \frac{n_2}{n}$ and $\gamma = \frac{n_3}{n}$). Similarly, $R_1 < \alpha I(X_1;Y_2) + \gamma I(X_3;Y_2)$ is required for node 2 to decode w_1. ∎

For Gaussian half-duplex three-phase two-way relay channels, they can be modeled as follows. In phase 1, $Y_{3,1} = X_1 + Z_{3,1}$ and $Y_{2,1} = X_1 + Z_{2,1}$. In phase 2, $Y_{3,2} = X_2 + Z_{3,2}$ and $Y_{1,2} = X_2 + Z_{1,2}$. In phase 3, $Y_{1,3} = X_3 + Z_{1,3}$ and $Y_{2,3} = X_3 + Z_{2,3}$. $Z_{i,j}$ is a Gaussian distributed random variable with mean zero and variance $\sigma_{i,j}^2$. When binary phase-shift keying (BPSK) is considered, the codeword bit is mapped from $\{0,1\}$ to $\{1,-1\}$. The signal $\mathbf{X}_1 = (X_{1,1},\cdots,X_{1,n_1})$ has a power constraint $\frac{1}{n_1}\sum_{i=1}^{n_1} X_{1,i}^2 \le P_1$. Similarly, $\mathbf{X}_2$ and $\mathbf{X}_3$ have power constraints $\frac{1}{n_2}\sum_{i=1}^{n_2} X_{2,i}^2 \le P_2$ and $\frac{1}{n_3}\sum_{i=1}^{n_3} X_{3,i}^2 \le P_3$.

For Gaussian half-duplex three-phase two-way relay channels, all rate pairs (R_1,R_2) satisfying

$$R_1 < \min\left(\frac{1}{2}\alpha\log_2\left(1+\frac{P_1}{N_{3,1}}\right),\right.$$
$$\left.\frac{1}{2}\alpha\log_2\left(1+\frac{P_1}{N_{2,1}}\right)+\frac{1}{2}\gamma\log_2\left(1+\frac{P_3}{N_{2,3}}\right)\right) \quad (3)$$

and

$$R_2 < \min\left(\frac{1}{2}\beta\log_2\left(1+\frac{P_2}{N_{3,2}}\right),\right.$$
$$\left.\frac{1}{2}\beta\log_2\left(1+\frac{P_2}{N_{1,2}}\right)+\frac{1}{2}\gamma\log_2\left(1+\frac{P_3}{N_{1,3}}\right)\right) \quad (4)$$

are achievable where $N_{i,j} = \sigma_{i,j}^2$ and $\alpha+\beta+\gamma = 1$. Note that (3) and (4) can be easily derived from (1) and (2).

In Figure 2, the achievable rate R_1 is plotted when $\alpha = \beta = \gamma = \frac{1}{3}$ and $\frac{P_1}{N_{3,1}} = 1$. The x-axis and y-axis are signal-to-noise ratios (SNRs) $\frac{P_1}{N_{2,1}}$ and $\frac{P_3}{N_{2,3}}$, respectively. The z-axis is the achievable rate R_1. The flat area is the area where R_1 is limited by the SNR of the source-relay link, while the slope area is the area where R_1 is limited by SNRs of the source-destination link and the relay-destination link.

In Figure 3, the achievable rate R_1 is plotted when $\alpha = \beta = \gamma = \frac{1}{3}$ and $\frac{P_1}{N_{2,1}} = \frac{P_3}{N_{2,3}}$. The x-axis and y-axis are SNRs $\frac{P_1}{N_{3,1}}$ and $\frac{P_1}{N_{2,1}}\left(\frac{P_3}{N_{2,3}}\right)$. The z-axis is the achievable rate R_1. The left slope area is the area where R_1 is limited by the SNR of the source-relay link, while the right slope area is the area where R_1 is limited by SNRs of the source-destination link and the relay-destination link.

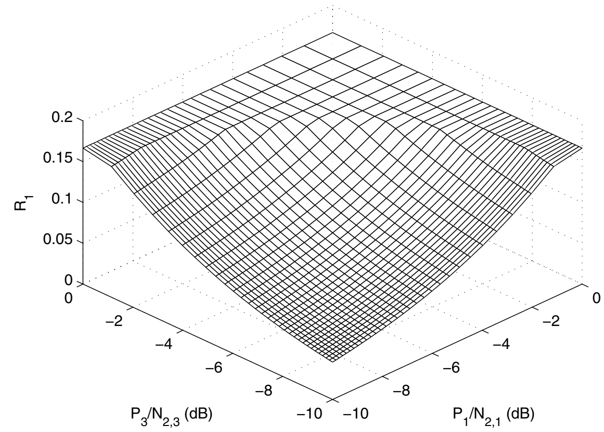

Fig. 2. Achievable rates of R_1 for $\left(\frac{P_1}{N_{2,1}},\frac{P_3}{N_{2,3}}\right)$ pairs.

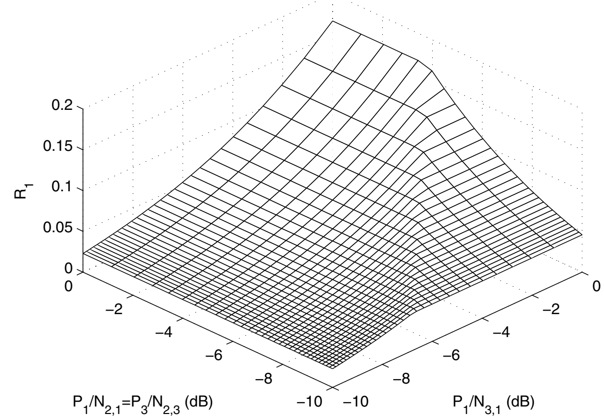

Fig. 3. Achievable rates of R_1 for $\left(\frac{P_1}{N_{3,1}},\frac{P_1}{N_{2,1}}\left(\frac{P_3}{N_{2,3}}\right)\right)$ pairs.

III. Code Constructions and Message-Passing Decoding Algorithms

A. LDPC code constructions and their graph representations

In this section, we propose LDPC code constructions for half-duplex three-phase two-way relay channels and show their graph representations.

At terminal node i for $i = 1,2$, the node encodes a k_i-bit message into an n_i-bit codeword. The codeword is broadcast to the relay node and the other terminal node. Under the decode-and-forward scheme, the relay node can decode the message while the other terminal node cannot decode without the help of the relay node. Intuitively, with additional bits from the relay node, the effective code rate is reduced.

At the relay node, two codewords $\mathbf{c}_1$ and $\mathbf{c}_2$ from terminal nodes are concatenated as a source message $\mathbf{c} = [\mathbf{c}_1\mathbf{c}_2]$. The lengths of $\mathbf{c}_1$, $\mathbf{c}_2$ and $\mathbf{c}$ are n_1, n_2 and n_1+n_2, respectively. An n_3-bit relay codeword $\mathbf{r}$ is generated by a systematic LDPC

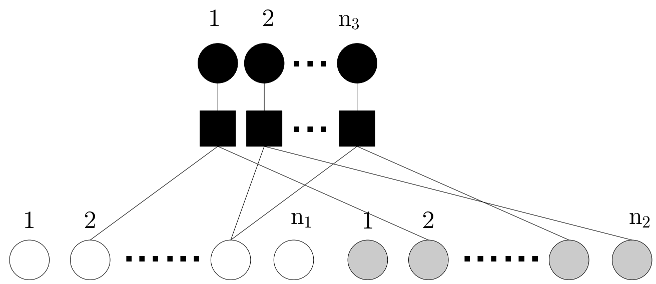

Fig. 4. The graph of the systematic LDPC code at the relay node.

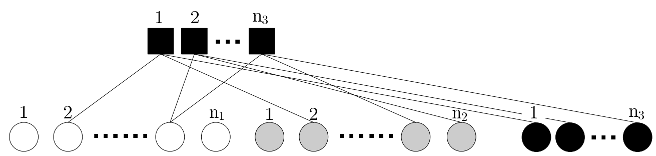

Fig. 5. The equivalent graph of the systematic LDPC code at the relay node.

code where $\mathbf{r} = \mathbf{c}\mathbf{G}$ and $\mathbf{G}$ is a generator matrix with the size $(n_1 + n_2) \times n_3$. These bits are also called relay bits, which are broadcast to both terminal nodes while $\mathbf{c}$ is not sent. Here, $2^{n_1+n_2}$ codeword pairs are mapped to 2^{n_3} codewords. Since n_3 is in general less than $n_1 + n_2$, multiple codeword pairs are mapped to a relay codeword. This code construction is similar to those of non-systematic LDGM codes [17] and LT codes [18]. LDGM codes were initially proposed as an alternative to LDPC codes. In these codes, check bits $\mathbf{c}$ are generated from source bits $\mathbf{s}$ by $\mathbf{c} = \mathbf{s}\mathbf{G}$. For systematic LDGM codes, both source bits and check bits are sent to a destination node. For non-systematic LDGM codes, only check bits are sent. LDGM codes were proposed for channels with known channel parameters and their code rates are fixed. LT codes are the first practical rateless codes, whose idea was originally from Fountain codes [19]. They can be considered as non-systematic LDGM codes, though check bits are continuously generated until a receiver can recover the source message.

In general, any linear code can be represented by a Tanner graph [20]. The graph of the systematic LDPC code at the relay node is shown in Figure 4. Circles are variable nodes and squares are check nodes. The n_3 upper layer variable nodes (in black) represent the relay bits. The lower layer n_1 variable nodes (in white) and n_2 variable nodes (in grey) represent two source codewords from terminal nodes. Check nodes represent parity check constraints among these bits.

Note that the above graph is a bipartite graph. The n_3 upper-layer variable nodes can be moved to the lower layer, as shown in Figure 5. The total number of variable nodes is $n_1+n_2+n_3$. The first two groups of variable nodes represent codewords of two terminal nodes, which are called Group 1 variable nodes and Group 2 variable nodes, respectively. The n_3 right-most variable nodes represent relay bits, which are called Group 3 variable nodes.

At the terminal node, messages are decoded based on three pieces of information: received signals from the source terminal node, received signals from the relay node, and the codeword of the destination node. The graph for decoding is shown in Figure 6. Compared to Figure 5, two groups of check nodes are added to the lower layer. These check

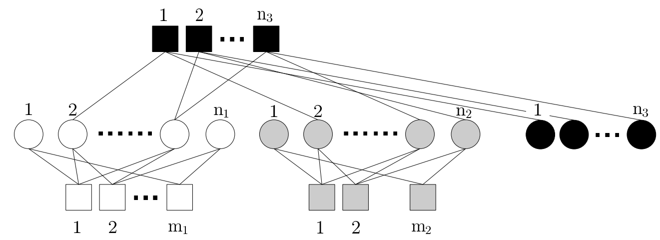

Fig. 6. The graph for decoding at terminal nodes.

nodes represent parity check constraints of LDPC codes at the terminal node, which are called Group 1 check nodes and Group 2 check nodes, respectively. The upper layer check nodes are called Group 3 check nodes.

Group 3 check nodes in Figure 6 can also be moved to the lower layer. In this sense, the decoding algorithm at the terminal node could be similar to those for point-to-point channels.

B. Message-passing algorithms

In this section, we propose message-passing algorithms for decoders in half-duplex three-phase two-way relay channels. In such channels, decoding happens at all three nodes. Since any existing decoding algorithms for point-to-point channels can be used at the relay node, the details are omitted here. In the sequel, we only focus on the message-passing algorithm at destination nodes. Especially, only decoding functions at node 1 are derived since decoding functions at node 2 are similar.

With the help of the graph in Figure 6, the message-passing algorithm can be easily described. The received bit can be represented in an LLR form

$$\log \frac{p(x_j = 1|y_{i,j})}{p(x_j = -1|y_{i,j})} = \frac{2y_{i,j}}{\sigma_{i,j}^2} \tag{5}$$

for $i, j = 1, 2, 3$. Variable nodes and check nodes are associated with decoding functions. Messages flow between variable nodes and check nodes via edges, serving as inputs or outputs of the functions. The algorithm adopts an iterative decoding method by passing messages multiple times between variable nodes and check nodes. In variable nodes, functions are in the form of summation. In check nodes, functions are in the form of $2\tanh^{-1}(\prod \tanh)$. In general, a message passing schedule is required during the iterative decoding. In this work, a flooding schedule is used. In this schedule, all messages from variable nodes are passed to check nodes along all edges, and all output messages from check nodes are passed back to variables nodes thereafter to complete one decoding iteration.

Let v_i^l be a message from a Group i variable node to a Group i check node in the l-th decoding iteration for $i = 1, 2$. Let u_i^l be a message from a Group i check node to a Group i variable node in the l-th decoding iteration for $i = 1, 2$. Let $v_{i,3}^l$ be a message from a Group i variable node to a Group 3 check node in the l-th decoding iteration for $i = 1, 2, 3$. Let $u_{3,i}^l$ be a message from a Group 3 check node to a Group i variable node in the l-th decoding iteration for $i = 1, 2, 3$. For a variable node, a lower/upper variable node degree is defined as the total number of edges connected to a lower/upper layer check node. An upper/lower-degree-i variable node is a variable node with i upper/lower edges. An upper/lower-degree-i variable node edge is an edge connected to an upper/lower-degree-i variable node. Let d_i be a lower degree of a Group i variable node for $i = 1, 2$. Let $d_{3,i}$ be an upper degree of a Group i variable node for $i = 1, 2, 3$. Let g_i be a degree of a Group i check node for $i = 1, 2$. For a Group 3 check node, it has three degrees. Let $g_{3,i}$ be a degree of a Group 3 check node which is the total number of edges connecting to a Group i variable node for $i = 1, 2, 3$. Let $u_{0,i}$ be a channel output LLR associated with a Group i variable node for $i = 1, 2, 3$.

The functions used for decoding messages of terminal node 2 at terminal node 1 are

$$v_2^l = \sum_{i=1}^{d_2-1} u_{2,i}^{l-1} + \sum_{j=1}^{d_{3,2}} u_{3,2,j}^{l-1} + u_{0,2}, \tag{6}$$

$$v_{1,3}^l = u_{0,1}, \tag{7}$$

$$v_{2,3}^l = \sum_{i=1}^{d_2} u_{2,i}^{l-1} + \sum_{j=1}^{d_{3,2}-1} u_{3,2,j}^{l-1} + u_{0,2}, \tag{8}$$

$$v_{3,3}^l = u_{0,3}, \tag{9}$$

$$u_{3,2}^l = 2\tanh^{-1}\left[\prod_{i=1}^{g_{3,1}} \tanh\left(\frac{v_{1,3,i}^l}{2}\right) \prod_{j=1}^{g_{3,2}-1} \tanh\left(\frac{v_{2,3,j}^l}{2}\right) \tanh\left(\frac{v_{3,3}^l}{2}\right)\right], \tag{10}$$

$$u_2^l = 2\tanh^{-1}\left[\prod_{i=1}^{g_2-1} \tanh\left(\frac{v_{2,i}^l}{2}\right)\right]. \tag{11}$$

The function in Group 1 variable nodes is shown in (7). Since terminal node 1 knows its own codeword, intrinsic values of Group 1 variable nodes are $+\infty$ or $-\infty$. Hence, no matter what messages are received from check nodes, Group 1 variable nodes always send $u_{0,1}$ ($+\infty$ or $-\infty$) to upper check nodes. The function in Group 3 variable nodes is shown in (9). Group 3 variable nodes only send $u_{0,3}$ since the degree of Group 3 variable nodes is 1. Functions in Group 2 variable nodes are shown in (6) and (8). Group 2 variable nodes receive messages $u_{3,2,j}^{l-1}$ and $u_{2,i}^{l-1}$ from upper layer check nodes and lower layer check nodes, respectively. These messages are added together with the channel output LLR $u_{0,2} = \frac{2y_{1,2}}{\sigma_{1,2}^2}$. The output $v_{2,3}^l$ is sent to a Group 3 check node in the upper layer. The output v_2^l is sent to a Group 2 check node in the lower layer. The function in Group 3 check nodes at the upper layer is shown in (10). For upper layer check nodes, they only send the output message $u_{3,2}^l$ to a Group 2 variable node. The function in Group 2 check nodes at the lower layer is shown in (11). The output u_2^l is sent to a Group 2 variable node.

IV. Discretized Density Evolution

In this section, density evolution [21] is used as a tool to analyze codes in message-passing algorithms for half-duplex three-phase two-way relay channels.

First, we formally define an ensemble of codes via graph in half-duplex three-phase two-way relay channels. The ensemble is a sequence of codes with the same variable node degree distributions and check node degree distributions.

For systematic LDPC codes at the relay node, we define one variable node degree distribution and two check node degree distributions. These degree distributions are defined from node perspective. Since the relay node only forwards partial information of the source codeword, we allow degree 0 as an upper degree of a variable node. The upper-degree-0 variable node does not connect to any upper layer check nodes. Denote λ_3 as the variable node degree distribution. $\lambda_{3,i,j}$ is the fraction of the total number of upper-degree-j variable nodes in Group i variable nodes to the total number of all 3 groups of variable nodes. $\sum_{i,j} \lambda_{3,i,j} = 1$. Denote $\rho_{3,1}$ and $\rho_{3,2}$ as the two check node degree distributions. $\rho_{3,i,j}$ is the fraction of the total number of upper layer check nodes with degree $d_{3,i} = j$ to the total number of all upper layer check nodes. $\sum_j \rho_{3,i,j} = 1$ for $i = 1, 2$.

The ensemble of codes is defined based on four permutations. π_i is a permutation for codes at terminal node i for $i = 1, 2$. π_3 and π_4 are permutations for the code at the relay node. The definitions of π_1 and π_2 are the same as those in point-to-point channels. Here, we only define π_3 and π_4. Assign some sockets to every Group i variable node according to the degree distribution $\lambda_{3,i}$ for $i = 1, 2$. The sockets on Group i variable nodes are called Group i variable node sockets. Assign two groups of sockets to every upper layer check node according to degree distributions $\rho_{3,1}$ and $\rho_{3,2}$. We call them Group i check node sockets for $i = 1, 2$. Edges connecting to Group i check node sockets are connected to Group i variable nodes. Two groups of variable node sockets and two groups of check node sockets are labeled separately with positive integers starting from 1. Group 1 and Group 2 check node socket labels are permuted by π_3 and π_4. Edges are identified by pairs of sockets, which are denoted as (i, $\pi_3(i)$) and (j, $\pi_4(j)$), where i or j is a Group 1 or Group 2 variable node socket, $\pi_3(i)$ or $\pi_4(j)$ is a check node socket in the two groups of check node sockets, respectively. A code is an element in the permutation space $\pi_1 \times \pi_2 \times \pi_3 \times \pi_4$. All codes in the permutation space are equiprobable.

Density evolution tracks the probability density function of LLR messages. Messages on each edge can be represented by a random variable. The output messages of a check node function and a variable node function can be represented by functions of random variables. If threshold decoding (A bit is decoded as 0 if the message is greater than or equal to zero and decoded as 1 if the message is less than 0) is used, the probability of error is simply the integral of the probability density function from $-\infty$ to 0.

In discretized density evolution, probability density functions are approximated by probability mass functions. Recall that the function of the variable node is a sum of independent random variables, e.g. (6) and (8). The probability mass function of the sum of two independent discrete random variables can be calculated by convolving the probability mass functions of the two random variables by circular discrete convolution. Furthermore, in order to speed up the calculation, the circular discrete convolution can be calculated by discrete Fourier transform and the inverse discrete Fourier transform.

For a variable node with an upper degree i and a lower degree j, denote the probability mass function of input messages on the upper edge and the lower edge as P_i and

P_j, respectively. The probability mass function of the output messages on upper edges is

$$P_v^l = P_0 * \{\otimes_{i-1} P_i^{l-1}\} * \{\otimes_j P_j^{l-1}\} \tag{12}$$

where l is the decoding iteration number, $*$ is discrete convolution, $\otimes_i$ is discrete convolution on i random variables and P_0 is the probability mass function of the channel output LLR message. Similarly, the probability mass function of the output messages on lower edges is

$$P_v^l = P_0 * \{\otimes_i P_i^{l-1}\} * \{\otimes_{j-1} P_j^{l-1}\}. \tag{13}$$

In general, if discrete random variables X_1 and X_2 are independent, the probability mass function of $Z = p(X_1, X_2)$ is

$$P(Z = z) = \sum_{z=p(x_1,x_2)} P(X_1 = x_1)P(X_2 = x_2). \tag{14}$$

The probability mass function of check node output messages in (10) and (11) can be calculated by this way. For the function $Z = 2\tanh^{-1}\left(\tanh\frac{X_1}{2}\tanh\frac{X_2}{2}\right)$, the probability mass function of Z is

$$P(Z = z) = \sum_{z=2\tanh^{-1}\left(\tanh\frac{x_1}{2}\tanh\frac{x_2}{2}\right)} P(X_1 = x_1)P(X_2 = x_2) \tag{15}$$

If the function is in the form of $Z = 2\tanh^{-1}\left(\prod_i \tanh\frac{X_i}{2}\right)$, we can calculate P_Z by recursively calculating the probability mass function of the function of two input random variables with (15).

Denote $P_{2,i}$ as the probability mass function of output messages from upper-degree-i Group 2 variable nodes. The probability mass function of input messages at Group 2 check node sockets is

$$\sum_i \lambda_{3,2,i} P_{2,i}. \tag{16}$$

Denote $Q_{2,i}$ as the probability mass function of output messages from upper layer check nodes with degree $d_{3,2} = i$. The probability mass function of input messages at Group 2 variable node sockets is

$$\sum_i \rho_{3,2,i} Q_{2,i}. \tag{17}$$

V. Code Optimization

In this section, we propose a three-step code optimization to find good codes for half-duplex three-phase two-way relay channels.

In the first two code optimization steps, two irregular LDPC codes for the two source-relay links are designed. Since the underlying channels are point-to-point channels, any existing optimization methods [21], [22], [23], [24] for such channels can be used.

In this work, iterative linear programming [8] is used as the optimization solver. In this solver, the code rate is maximized when $\sigma_{3,i}^2$ is given for $i = 1, 2$. A feasible region is a space on λ_i and ρ_i where λ_i is a variable node degree distribution of irregular LDPC codes at terminal node i and ρ_i is a check node degree distribution of irregular LDPC codes at terminal node i. When the degree of ρ_i is concentrated [23], the optimization problem becomes a sequence of sub-problems: finding an optimal λ_i with a fixed ρ_i. The details of iterative linear programming for point-to-point channels can be found in Appendix II of [8].

In the third code optimization step, systematic LDPC codes at the relay node are optimized. The optimized irregular LDPC codes obtained in the first two steps are used in the third step. The optimization objective is to find the optimal degree distributions that minimize the ratio of the length of the relay codeword to the sum of the lengths of two source codewords

$$\frac{\lambda_{3,3,1}}{\sum_i \lambda_{3,1,i} + \sum_j \lambda_{3,2,j}}. \tag{18}$$

In this optimization problem, the feasible region is a space on $\lambda_{3,1}$, $\lambda_{3,2}$, $\lambda_{3,3}$, $\rho_{3,1}$ and $\rho_{3,2}$. To simplify the optimization problem, the original problem is divided into a sequence of optimization problems on $\lambda_{31}, \lambda_{32}, \lambda_{33}$ with fixed ρ_{31}, ρ_{32}.

The global optimization problem in the third optimization step is

$$\min_{\lambda_{3,1},\lambda_{3,2},\lambda_{3,3}} \lambda_{3,3,1} \tag{19}$$

$$\text{s.t.} \sum_i \lambda_{3,1,i} + \sum_j \lambda_{3,2,j} + \lambda_{3,3,1} = 1 \tag{20}$$

$$0 \le \lambda_{3,1,i}, \lambda_{3,2,j}, \lambda_{3,3,1} \le 1 \tag{21}$$

$$\sum_i i\lambda_{3,1,i} - \left(\sum_j j\rho_{3,1,j}\right)\lambda_{3,3,1} = 0 \tag{22}$$

$$\sum_i i\lambda_{3,2,i} - \left(\sum_j j\rho_{3,2,j}\right)\lambda_{3,3,1} = 0 \tag{23}$$

$$\sum_j j(e_{3,1,j}^{l+1} - e_{3,1}^l)\lambda_{3,1,j} < 0, l = 1, \cdots, L_1 \tag{24}$$

$$\sum_j j(e_{3,2,j}^{l+1} - e_{3,2}^l)\lambda_{3,2,j} < 0, l = 1, \cdots, L_2, \tag{25}$$

where L_1 and L_2 are the total numbers of decoding iterations, $e_{3,i,j}^l$ is the mixture probability of error on upper-degree-j edges of Group i variable nodes in the l-th decoding iteration, and $e_{3,i}^l$ is the probability of error mixture on Group i check node sockets in the l-th decoding iteration. Note that the probability of error is calculated during discretized density evolution by $P(X \le 0) = \sum_{a \le 0} P(X = a)$ where $P(X)$ is the probability mass function of messages. The probability of error mixture $e_{3,i}^l$ can be calculated from the probability mass function of the message mixture at inputs of check nodes.

With constraint (20), (18) becomes (19). Constraint (20) is the condition that the sum of the probability is 1. Constraint (21) is the condition that a probability is upper bounded by 1 and lower bounded by 0.

Constraint (22) comes from

$$\sum_i ni\lambda_{3,1,i} = \sum_j n_3 j\rho_{3,1,j}. \tag{26}$$

The left hand side of (26) is the total number of upper edges connected to Group 1 variable nodes where n is the total number of all 3 groups of variable nodes. In addition, from the upper layer check node perspective, the total number of upper edges connected to Group 1 variable nodes is the right hand side of (26) where n_3 is the total number of upper layer check nodes. The left hand side and the right hand side should be equal and this condition becomes constraint (22) due to $\lambda_{3,3,1} = \frac{n_3}{n}$. Constraint (23) is similar to (22), but it applies to Group 2 variable nodes.

Constraint (24) comes from

$$\sum_i e_{3,1,i}^{l+1} \frac{n i \lambda_{3,1,i}}{\sum_j n j \lambda_{3,1,j}} < e_{3,1}^{l}. \tag{27}$$

The left hand side and the right hand side of (27) are mixtures of probabilities of error of input messages at upper layer check nodes in the $(l+1)$-th and l-th decoding iteration, respectively. (24) is a sequence of constraints on the decoding rule, that is, the probability of error is monotonically decreased during iterative decoding. Constraint (25) is similar to (24) , but it applies to Group 2 variable nodes.

Since the probability of error is a non-linear function of the degree distribution, constraints (24) and (25) are non-linear. However, if the probabilities of error are treated as constants, the non-linear optimization problem becomes a linear optimization problem.

Since the probabilities of error are treated as constants, codes from linear programming might not be decoded. In this case, discretized density evolution can be used to verify whether codes can be decoded. If codes can be decoded, their degree distribution becomes the current best degree distribution. During the discretized density evolution, the probability of error in each decoding iteration can be calculated, which are used in the next optimization iteration. If codes cannot be decoded, the feasible region needs to be shrinked by reducing the value of the right hand side of (24) and (25), denoted as μ, towards $-\infty$. When the feasible region is shrinked, the ratio (18) becomes larger. Hence codes could be easier to decode. As the value μ is reduced, the problem could be infeasible at some point. In other words, no degree distribution satisfies all constraints from (20) to (25). In such a case, we modify the iterative linear programming algorithm proposed in [8] by reducing L_1 and L_2. Since less constraints are applied, the feasible region is enlarged. Note that the probabilities of error in (24) and (25) come from the preceding optimization iteration, which are provided as hints on the boundary of the feasible region in the next optimization iteration. The optimal degree distribution could be inside of the feasible region or outside of the feasible region.

VI. Simulation Results

In this section, two optimized degree distributions for irregular LDPC codes at terminal nodes and an optimized degree distribution for systematic LDPC codes at the relay node are obtained when a half-duplex three-phase two-way relay channel is given. Codes sampled from the optimized degree distributions are simulated. We show that good codes can be found by our proposed three-step optimization. The length of the obtained optimized codes is 26% longer than the theoretic one. In addition, it is shown that the required SNR for a finite-length code converges fast to that for cycle-free infinite-length codes by simulations.

In the first optimization step, irregular LDPC codes for terminal node 1 are optimized, where a k_1-bit source message is encoded into an n_1-bit codeword. The code rate is $\frac{k_1}{n_1}$. The code optimization problem is to maximize the code rate when channel parameter $\sigma_{3,1}$ is given. Codes with rate 0.3277 are found when $\sigma_{3,1}$ is 1.295 and the check node degree is 8. Note that the capacity rate is $\frac{1}{3}$ which can be determined by the equation of the capacity of binary-input additive white Gaussian noise (BIAWGN) channels

$$\begin{aligned} C_{BIAWGN}(\sigma) &= -\int \phi_\sigma(y)\log_2 \phi_\sigma(y)\mathrm{d}y \\ &\quad -\frac{1}{2}\log_2(2\pi e\sigma^2) \end{aligned} \tag{28}$$

where

$$\phi_\sigma(y) = \frac{1}{\sqrt{8\pi\sigma^2}}\left(e^{-\frac{(y+1)^2}{2\sigma^2}} + e^{-\frac{(y-1)^2}{2\sigma^2}}\right) \tag{29}$$

is the probability density function of received signal Y. The optimized variable node degree distribution is shown in Table I.

TABLE I
THE VARIABLE NODE DEGREE DISTRIBUTION FOR CODES WITH RATE 0.3277

i	$\lambda_{1,i}$	i	$\lambda_{1,i}$
2	0.5277	3	0.2903
6	0.0022	7	0.1392
21	0.0199	22	0.0003
100	0.0204		

In the second optimization step, for the link between terminal node 2 and the relay node, codes with rate 0.4852 are found when $\sigma_{3,2}$ is 0.979 and the check node degree is 8. The corresponding capacity rate is $\frac{1}{2}$. The optimized variable node degree distribution is shown in Table II.

TABLE II
THE VARIABLE NODE DEGREE DISTRIBUTION FOR CODES WITH RATE 0.4852

i	$\lambda_{2,i}$	i	$\lambda_{2,i}$
2	0.4928	3	0.2889
5	0.0011	6	0.0517
7	0.1050	8	0.0010
9	0.0007	10	0.0091
11	0.0005	12	0.0004
13	0.0003	14	0.0002
15	0.0001	16	0.0001
22	0.0183	23	0.0275
24	0.0001	25	0.0021

Two codes, Code A and Code B, with the length of 10^5 bits are randomly sampled from the above two degree distributions. Code A (B) is sampled from the degree distribution for irregular LDPC codes at node 1 (node 2). Simulation results are shown in Figure 7 and Figure 8, labeled as *Code A, no relay,* n=10^5 and *Code B, no relay,* n=10^5, respectively. The waterfall curve given in the figure can be considered as the case when no relay node exists. For each code, it is simulated with multiple SNRs. The maximum number of decoding iterations is 200. The corresponding bit error rate (BER) is presented in the logarithmic Y-axis. The equivalent SNRs for $\sigma_{3,1} = 1.295$ and $\sigma_{3,2} = 0.979$ are represented by the vertical lines.

In the third optimization step, in order to optimize the degree distribution of systematic LDPC codes at the relay

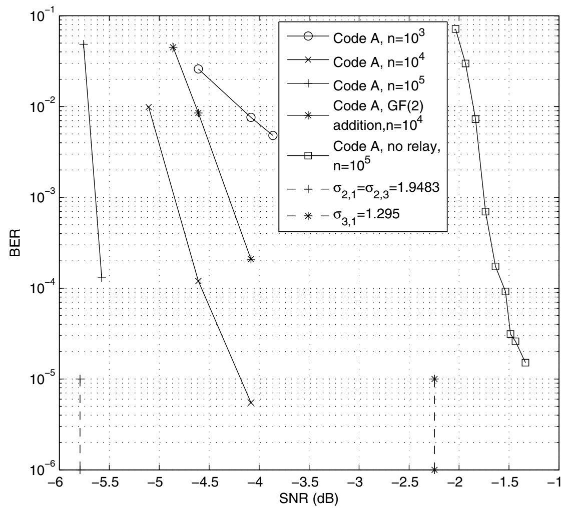

Fig. 7. Simulation results when decoding code A at terminal node 2.

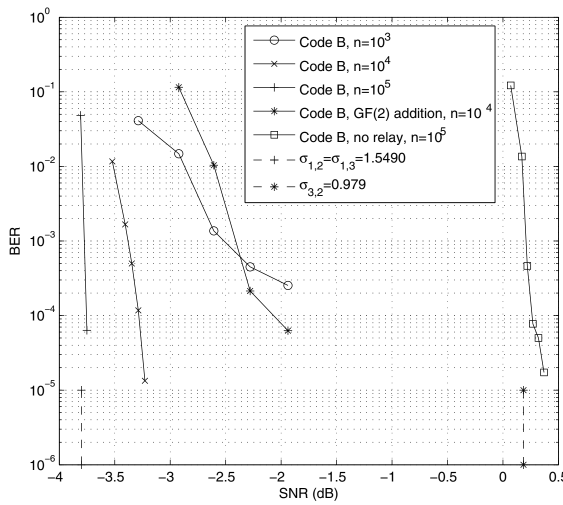

Fig. 8. Simulation results when decoding code B at terminal node 1.

node, channel parameters $\sigma_{1,2}$, $\sigma_{1,3}$, $\sigma_{2,1}$, $\sigma_{2,3}$ are given. We consider the case when $\sigma_{2,1} = \sigma_{2,3} = 1.9483$ and $\sigma_{1,2} = \sigma_{1,3} = 1.5490$. The degree of upper layer check nodes is $g_{3,1} = g_{3,2} = 3$. By iterative linear programming, the optimized $\lambda_{3,3,1}$ is 0.3867 and the optimized $\lambda_{3,1}$, $\lambda_{3,2}$ are shown in Table III and IV. Note that for the given $\sigma_{1,2}$, $\sigma_{1,3}$, $\sigma_{2,1}$, $\sigma_{2,3}$, the lower bound of $\lambda_{3,3,1}$ is $\frac{1}{3}$.

Irregular LDPC codes with the lengths of 10^3, 10^4 and 10^5 bits and systematic LDPC codes with the relay codeword lengths of 1.26×10^3, 1.26×10^4 and 1.26×10^5 bits are randomly sampled from the above degree distributions. Simulation results for decoding Code A and Code B at destination nodes are shown in Figure 7 and Figure 8, respectively. The maximum number of decoding iterations is 500. In the figure, the SNRs are defined as $\frac{1}{\sigma_{1,3}^2}$ and $\frac{1}{\sigma_{2,3}^2}$ for the two decoders at two destination nodes. The BER is defined as the ratio of the total number of erroneous bits to the total number of bits in Group i variable nodes for $i = 1, 2$. In this sense, BERs are

TABLE III
THE VARIABLE NODE DEGREE DISTRIBUTION $\lambda_{3,1}$

i	$\lambda_{3,1,i}$	i	$\lambda_{3,1,i}$
0	0.2238	14	0.0829

TABLE IV
THE VARIABLE NODE DEGREE DISTRIBUTION $\lambda_{3,2}$

i	$\lambda_{3,2,i}$	i	$\lambda_{3,2,i}$
0	0.1482	1	0.0148
2	0.0131	3	0.0121
4	0.0113	5	0.0109
6	0.0107	7	0.0106
8	0.0107	9	0.0108
10	0.0106	11	0.0094
12	0.0059	13	0.0036
14	0.0239		

evaluated at two destination nodes separately. The equivalent SNRs of $\sigma_{2,1} = \sigma_{2,3} = 1.9483$ and $\sigma_{1,2} = \sigma_{1,3} = 1.5490$ are represented by vertical dashed lines. As we can see, with the help of the relay node, the required SNRs are reduced from 0.2 dB to -3.8 dB and from -2.2 dB to -5.7 dB, respectively. In Figure 7 and Figure 8, we also provide the simulation results for the case where two source codewords are added in GF(2) at the relay node. At the BER of 10^{-4}, the required SNR of the GF(2) addition case is around 1.5 dB higher than that of our proposed LDPC code construction.

In density evolution, it is assumed that incoming messages of variable nodes and check nodes are independent. This assumption implies that the bipartite graph has no cycles. However, cycles almost always exist. A natural question is whether the actual density is close to the density in density evolution, especially when deviation is accumulated during the iterative decoding. This question can be empirically answered by Figure 7 and Figure 8. As we can see, when the length of codewords grows from 10^3 to 10^5, the waterfall curve moves closer to the vertical dashed lines, which shows that the required SNR for a finite-length code converges fast to that for cycle-free infinite-length codes.

For the given optimized degree distributions, the evolution of the BER under discretized density evolution is shown in Figure 9. $P_1 = P_2 = P_3 = 1$, $\sigma_{1,2} = \sigma_{1,3} = 1.5490$ and $\sigma_{2,1} = \sigma_{2,3} = 1.9483$ are used. The two BER curves are monotonically decreasing during iterative decoding. The required decoding iterations at two destination nodes are close to 100 and 300, respectively. The decrease of the BER as a function of the current BER is shown in Figure 10. The critical point [22] is the point where the decrease of the BER is a local minimum. As shown in Figure 10, the critical points of two codes at two destination nodes are close to 0.12 and 0.19, respectively.

VII. CONCLUSION

In half-duplex three-phase two-way relay channels, codewords are broadcast and signals are received from the source node and the relay node. In this work, we constructed systematic LDPC codes at the relay node to encode two source codewords. At the destination node, signals from the source node and the relay node are used for joint decoding. We

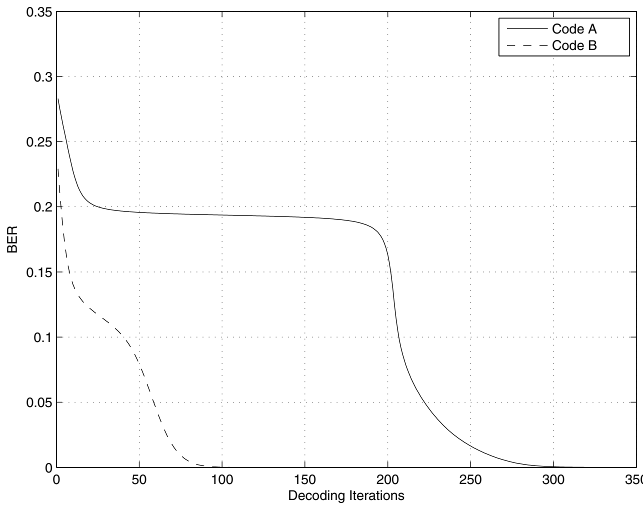

Fig. 9. Evolution of the bit error rate during iterative decoding.

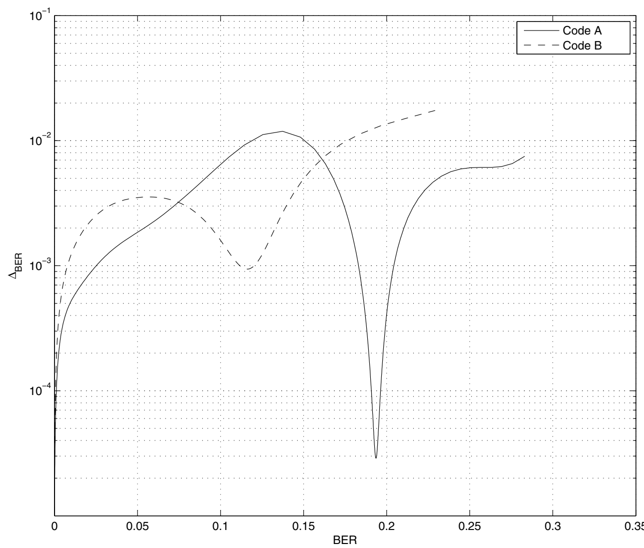

Fig. 10. The decrease of the bit error rate as a function of the current bit error rate.

designed the codes with discretized density evolution and iterative linear programing, and demonstrated that good codes can be found within our framework. For future work, we will extend our work to fading channels and wireless relay networks.

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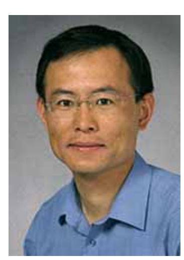

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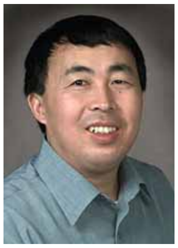

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